ENGN0030: Advanced Section

Electromagnetism: What You Need to Know

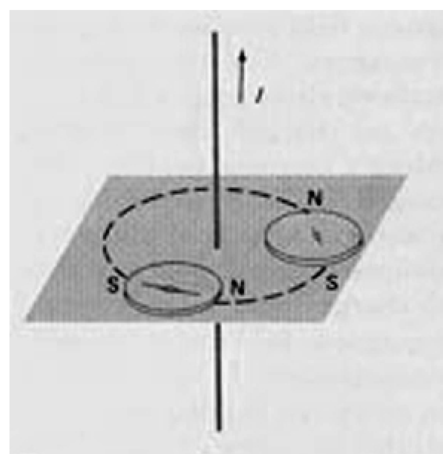
I. Introduction

The interdependence of electricity and magnetism is one of great importance. Motors, generators, transformers; all depend entirely on this wondrous phenomenon. For your design project you will have to undertake certain E&M calculations in order to achieve optimal running conditions. Some of you may have previously examined the relationship between electricity and magnetism before, most likely in a high school physics class. Or it may be that you have never learned a single thing about E&M, but I can tell you're super excited to start. In any case, this handout should serve as a brief overview to the concepts you will need to understand so as to effectively design your system.

II. Basic Electromagnetism ("Right Hand Rule")

By now, you should know something about what goes on inside a wire when there is a direct current (DC) passing through it: electrons flow, they have energy, and eureka! the light bulb turns on. Equally exciting, however, is what occurs OUTSIDE of the wire. As the current runs through a wire, a magnetic field is created, encircling the wire.

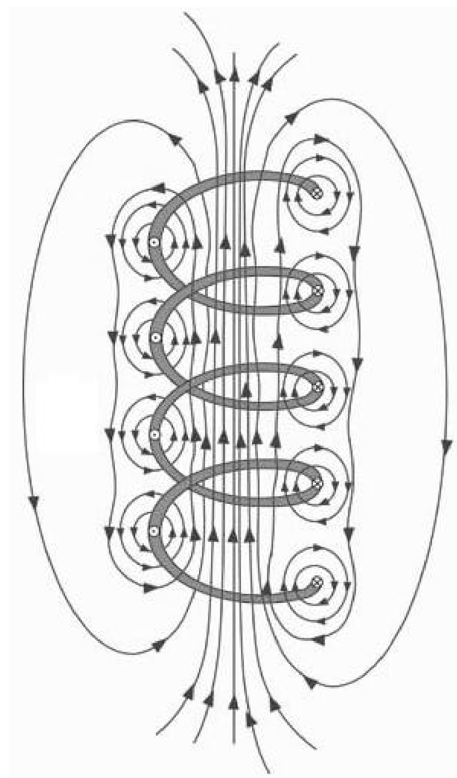
If you wish to see for yourself this magnetic field in action, place a compass near a current carrying wire. To observe the full effect, the face of the compass must be placed at a right angle to the wire. The compass, which is just a magnet, will align itself with the magnetic field around the wire, pointing in the field's direction rather than pointing north. You don't need a compass to discover the direction of the magnetic field though. You can use the first of E&M's many "right hand rules". This right hand rule, often referred to as "the right hand grip rule", requires that you pretend to grip the wire and point your thumb in the direction of current flow. If you've done this and your hand kind of looks like you're giving a thumbs up, your curled front fingers are now pointing in the direction of the magnetic field. The strength of the field is proportional to the amount of current running through the wire. The exact calculations for these examples are given by the Biot-Savart law, which we will not go into.



Deflection of a compass needle near a current-carrying wire, showing the presence and direction of the magnetic field.

If you now imagine this straight piece of wire being bent, picture what this would do to the magnetic field surrounding it. Think of the magnetic field lines as hoops spaced evenly along the wire, kind of like a slinky but the hoops are not connected to one another in a spiral. As the wire bends in upon itself, parts of the hoops are forced together in a smaller area and become more crowded. At the same time, the parts of the hoops on the outside of the bend have more space and move further away from one another. Making a loop with the wire, the magnetic field lines will look something like a doughnut. However, the field within the loop, where the magnetic lines are packed in close and overlapping, will be much stronger than a field next to a straight wire. Conversely, the field on the outside, with less dense field lines, will be weaker. The direction of these fields can be determined using the right hand grip rule.

Taking it one step further, if one were to take the wire and make multiple loops and stack them on top of one another (making a sort of spiral) what do you think would happen to the field inside? Each loop of wire has the same magnetic field around it so adding a second loop will add more field lines and further augment the total field intensity within the coils. Not only that, but placing one loop on top of another will negate the part of the field where they meet. Keeping the right hand rule in mind, think of the direction of the magnetic field at the top of a loop of wire. Next, think of the direction of the magnetic field at the bottom of an identical loop of wire. They are opposite in direction and, if the current running through them is the same, placing one on top of the other, the two components will cancel each other out completely. As you add more and more coils, the only part of the magnetic field remaining will be the portion contained within the column; even the field outside of the column can be proven to approach zero. This device with stacked coils of wire is called a solenoid. Solenoids are the most basic electromagnets and can be turned off or strengthened by respectively decreasing or increasing the current running through the coils. Oftentimes a core made up of magnetic metal will be placed in the center of the solenoid so as to better propagate the magnetic field. The second right hand rule, which can be verified using the first right hand rule, requires that your fingers act as the coils of a solenoid with the tips pointing in the direction of the current. Your thumb then points in the direction of the magnetic field generated by the loops.



III. Electromagnetic Induction

So we've seen how a current can create a magnetic field. Now we will examine how a changing magnetic field can induce a current in a solenoid, as it will in your design project. Faraday discovered this aspect of electromagnetism noticing that only a changing magnetic field would cause any current flow in a closed circuit. Through his experiments, he derived this relationship, known as Faraday's law:

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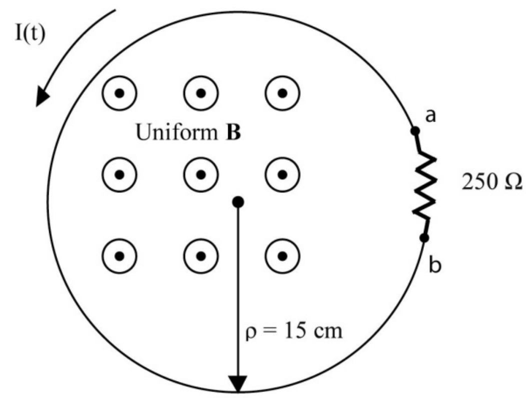
This equation implies a closed path (such as a loop of wire). Here, Φ is the magnetic flux passing through the surface defined by the closed path. ε (or emf) is "electromotive force", the energy gained per unit charge, and is measured in volts. One way in which a change in flux can occur is a time varying magnetic field through a stationary closed path, like in your generator design projects. Or, to put it more simply, as the rotating magnets pass over the stationary loops of wire, a current will be induced.

Here is an example problem using Faraday's law:

In the figure to the right, let $B = 0.2\cos 120\pi \cdot t$ **Teslas and assume that the conductor joining the two ends of the resistor is perfect. It may be assumed that the magnetic field produced by** ***I(t)*** **is negligible. Find: (a)** $V_{ba}(t)$ **; (b)** ***I(t)*****.**



Solution:

Ultimately, we want to find the change in flux through the loop with time. However we are not given an equation for the flux Φ, but rather an equation for the magnetic field, *B*. This quantity is also known as the "Magnetic Flux Density" and can be used to calculate the flux: we merely need to multiply the density by the area of the loop. In this situation, it means integrating over the surface created by the loop:

$$emf = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_S B \cdot dS$$

This can be further simplified since the magnetic flux density does not vary along the surface over which we are integrating; the field changes with time, but within the loop it is uniform. (NOTE: This may not be the case for your design project.) Because of this, the magnetic flux density can be pulled out of the integral and calculated without affecting the final result:

$$-\frac{dB}{dt}\int_S dS = -(-24\pi \sin 120\pi \cdot t)\int_S dS$$

Now the problem simply becomes a matter of finishing off the surface integral which you should notice has the same result as just multiplying by the area of the loop. If the magnetic field were NOT uniform, this integral would be slightly more complex.

$$(24\pi \sin 120\pi \cdot t)\int_S dS = (24\pi \sin 120\pi \cdot t)\int_0^{\rho}\int_0^{2\pi} \rho \cdot d\phi d\rho$$

$$(24\pi \sin 120\pi \cdot t)(2\pi)(\frac{1}{2}\rho^2)$$

Plugging in the given values we get

$$emf = V_{ba}(t) = 5.33 \sin 120\pi \cdot t \text{ Volts}$$

Next, to find the current *I(t)*, we simply need to make use of ohm's law:

$$V = IR$$

$$I(t) = \frac{V_{ba}(t)}{R} = 21.3 \sin 120\pi \cdot t \text{ milliamps}$$

Check your answer with the right hand rule and make sure everything makes sense.

In this example, the problem involved only a single loop but many solenoids have multiple coils that affect the *emf* generated by a magnetic field. Luckily, calculating the contribution of those extra loops is trivial. You simply calculate the effect of one loop, as done above, and multiply by the number of loops. A modified Faraday's law:

$$emf = -N \frac{d\Phi}{dt}$$

where N is the number of loops in the solenoid.