Hydrostatics and Bernoulli's Principle

Slide Notes

Hydrostatics and Bernoulli's Principle

1. Fluid mechanics = science that deals with the behavior of fluids at rest (**hydrostatics**) or in motion (**fluid dynamics**), and the interaction of fluids with solids or other fluids at the boundaries.

2. EN0810 = core course generally taken in junior year that focuses on the details of fluid mechanics

3. This lecture is a 1 hour tutorial on the most basics. Mostly, we will be focused on **hydraulics** which deals with **liquid fluid flow in pipes and open channels**.

Fluid Dynamics Basic Definitions

4. Couple of terms and common assumptions needed for fluid analysis as opposed to solid body analysis:

    a. *Fluid* – A substance in the liquid or gas phase

    b. *Steady* – No change at a point in time

        i. This is important because it defines a state of **consistency** that allows for basic calculations under normal parameters. **Analysis** becomes incredibly more **difficult under varying** or oscillatory fluid **behavior**.

    c. *Incompressible flow* – A fluid flow where variations in density are negligible

        i. Can also be stated that **changes in density = 0**. Generally, incompressible flow is a required assumption/parameter for systems using volumetric flow and single values of density. For

liquids, this is an acceptable assumption so most hydraulic problems disregard any minute changes in density.

    d. ***Control Volume*** – Properly selected region in space that usually encloses a device that involves mass flow

        i. Important to define the CV because it sets up the framework for analysis. It also allows parameters to be defined relative to a set position.

    e. ***Irrotational Flow*** - Flow that is void of vorticities or irregularities common in turbulent flows

        i. Common assumption that allows certain equations to exclude the effects that result from vortexes and irregular frictional effects.

Hydrostatic Pressure

5. Pressure = normal force exerted by a fluid per unit area

6. Pressure and stress both are forces per unit area. The only difference is that stress is a directed force while pressure is **universally applied** to a body or point.

7. Four main types of pressure:

    a. Gage Pressure → Difference between atmospheric and absolute pressure

    b. Vacuum Pressure → Occasionally used for a negative gage pressure

    c. Atmospheric Pressure = 101.325 kPa or 14.696 psi

    d. Absolute Pressure =

        i. Gage Pressure + Atmospheric Pressure

            OR

ii. Atmospheric Pressure - Vacuum Pressure

8. Gage pressure is pressure relative to the atmospheric pressure and what is mostly used for analysis in high pressure situations.

   a. $P_{gage} = P_{abs} - P_{atm}$

9. Pressure of a fluid at rest increases with depth as a result of the added weight from the fluid above. Gage pressure is plotted with sea level being approximately zero (because atmospheric pressure is not included). *[below]*
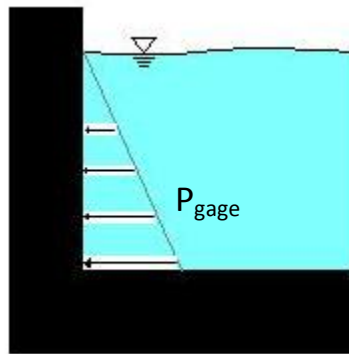
10. Basic proof/understanding of hydrostatic pressure:

   a. **W = mg = ρghwl**

      i. Where h = height, w = width, l = length of a given body. In this case, the body is a fluid element within a given control volume defined by the height, width, and length.

      ii. This, however, can be put into a relative sense because there is no pressure difference in the horizontal direction.

      iii. As a result, width and length become negligible because these have no impact on the pressure.

      iv. Also, the units work out to be F/A as opposed to just force.

11. THEREFORE, **pressure difference between two points** in a constant density fluid is **proportional to the vertical distance Δz** (or h) between the points and the density ρ of the fluid.

12. The **SI unit** for pressure is the **Pascal**, named after Blaise Pascal who – among other things – proved the idea of a vacuum and developed the hydraulic press.

A Hydraulic Press Example

13. Pascal's principle = pressure within a closed system is constant

14. As a result, you can take advantage of different areas to produce a greater force with less input force. A car jack in mechanic's workshops and smaller bottle jacks are good examples of this application.

15. The ratios of the areas determine how much you can lift. Let's say, for example, you had an area of ratio of 10 – you could lift 1000 Newtons of force with a mere 100 Newtons. However, it also takes 10 times the distance for the smaller piston to lift the larger piston. In other words, although you only need 100 Newtons to lift 1000 Newtons, you would need to push the small piston one meter to raise the large piston 10 centimeters.

Velocity for a Fluid

16. Similar to the relationship between the force of weight and hydrostatic pressure, the velocity for a fluid is very similar to the velocity of a falling particle through air.

   a. To assess and calculate the velocity of the water, the first step to analyzing the flow relative to a given height is to set a base level elevation from which all relative heights will be derived.

   b. After that's completed, depending on the problem, you determine what is the peak height of the water level and calculate the velocity using the equation: $V = \sqrt{2gh}$.

    c. Note that based on the simple proof on the slide, that this velocity relationship is the same as the velocity of a particle after it has travelled from a stationary position to a certain distance *h* under solely gravitational force.

    d. In this manner, if you neglect frictional forces, you should be able to calculate the output velocity of the water (and any humans it might be carrying) from a water slide with a fixed height.

DAT WUZ…THE BEST WATUR SLIDE EVAR!!!

17. No falling asleep in lecture.

Bernoulli's Equation

18. Equivalent to the **Conservation of Energy** principle.

    a. P/ρ ~ potential energy → PRESSURE HEAD

    b. $V^2/2$ ~ kinetic energy → VELOCITY HEAD

    c. gz ~ gravitational potential energy → ELEVATION HEAD

19. Bernoulli's Principle shows the relationship between pressure, velocity, elevation in a moving fluid.

    a. Higher Pressure = _____ Velocity or _____ Elevation

    b. Higher Velocity = _____Pressure or _____ Elevation

20. Bernoulli's Equation Analysis

    a. Similar to hydrostatic pressure examples, a base level needs to be defined so the elevation head can be described by a single variable *z* or *h*.

    b. A starting point for the first side of the equation and an ending point for the other side of the equation need to be defined. Use common assumptions like steady, incompressible, and irrotational flow. Then,

simply balance the forces to equal each other by solving for the one unknown.

c. The units for Bernoulli's Equation can vary based on where the constants for density and gravity are located. Many forms of Bernoulli's equation divide everything through by gravity so the units are merely units of length (or head). This is supposed to represent the elevation which equals all of the energies combined. This value is called the energy grade line. Our equation does not put everything in terms of head, but the units do not change anything as long as they are identical on either side of the equation.

21. Example 12-3 - Fundamentals of Thermal-Fluid Sciences

Water is flowing from a hose attached to a water main at 400 kPa gage. A child places his thumb to cover most of the hose outlet, increasing the pressure upstream of his thumb, causing a thin jet of high-speed water to emerge. If the hose is held upward, what is the maximum height that the jet could achieve?

Solution:

Assumptions:

1. The flow exiting into the air is steady, incompressible, and irrotational.

2. The water pressure in the hose near the outlet is equal to the main water pressure.

3. The surface tension effects are negligible.

4. The friction between the water and air is negligible

5. The irreversibilities that may occur at the outlet of the hose due to abrupt expansion are negligible.

Constants:

$\rho_{water} = 1000 \text{ kg/m}^3$

<u>Calculations:</u>

The water height will be at a maximum under the assumptions. The water velocity in the hose is relatively low ($V_1 = 0$) and we take the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory, the velocity reaches zero ($V_2 = 0$) and atmospheric pressure pertains.
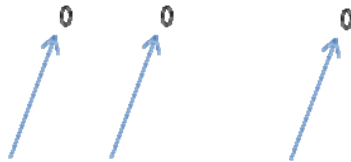
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$\longrightarrow \frac{P_1}{\rho} = \frac{P_{atm}}{\rho} + gz_2$$

$$\longrightarrow z_2 = \frac{P_1 - P_{atm}}{\rho g} = \frac{P_{1,gage}}{\rho g} = \frac{400\ kPa}{\left(1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)}\left(\frac{1000\frac{N}{m^2}}{1kPa}\right)\left(\frac{1kg \cdot \frac{m}{s^2}}{1N}\right) = 40.8\ m$$

Therefore, the water jet can rise as high as 40.8 m into the sky in this case.

<u>Discussion</u>

The result obtained by the Bernoulli equation represents the upper limit and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.8 m, and , in all likelihood, the rise will be much less than 40.8 m due to irreversible losses that we neglected.

Limitations of the Bernoulli Equation

22. **Steady Flow** – Should not be used during the transient start-up or shut-down periods of a machine and should never be used during periods of change in flow conditions.

23. **Frictionless Flow** – Every flow involves some friction, but many situations can have a negligible amount of friction. Long thin pipes or regions where solid objects create a lot of turbulence cannot be satisfied with Bernoulli's equation. Another example is something where wakes play a major roll such as an airplane wing or fish swimming. A lot of situations with sudden expansion or contractions require extra terms to make Bernoulli's equation valid.

24. **No shaft work** – Because the Bernoulli equation is based on the conservation of energy in a closed system, extra work added to the fluid negates the balance of the Bernoulli equation.

25. **Incompressible flow** – If the flow can be compressed, volumes do not remain constant so density does not remain constant. The Mach number characterizes the level of compressibility in response to pressure variations in the flow. This is outside the scope of the lecture, but a Mach number of less than about 0.3 is considered incompressible.

26. **No heat transfer** – Similar to the shaft work requirement, heat transfer will add or remove heat from a control volume and thus negates the validity of the Bernoulli equation.

27. **Flow along a streamline** – In other words, the flow needs to be irrotational. Irrotational flow introduces vorticities, which distorts consistent flow and makes Bernoulli's equation worthless.

An Example of Bernoulli's Principle

28. How does Bernoulli's Principle describe this phenomenon?

Volumetric Flow Balance

29. With the common assumptions of steady state flow and an incompressible fluid, the **volumetric flow** of a moving body of fluid is the **cross-sectional area multiplied by the velocity**. The unit of volumetric flow is cubic length per unit time (i.e. cubic meters/second).

30. Equating volumetric flows can be done in a similar manner to Bernoulli's equation. Choose a starting and end point with defined control volumes and find both cross-sectional areas and one velocity to find the last remaining velocity. In pipe flow for a **steady state system** with incompressible flow, a change in pipe diameter will result in a velocity proportional to the change in areas multiplied by the input velocity regardless of the elevation change (see diagram on slide). If this last statement is true, what variable changes to compensate for the energy transferred by the elevation gain?

Mass Flow

31. Multiply volumetric flow by density to get a mass per unit second.

Pressure and Velocity Relationship

32. Intuitively, does this make sense?

33. Why is P_3 less than P_1?

Minor Losses

34. In fluid mechanics, viscosity and friction are always present. Viscosity is the inability of a fluid to flow. Remember in the limitations of Bernoulli's equation that long thin sections of pipe and expansions/contractions were not permitted. In actuality, these values just require another term added to the end of the Bernoulli equation. This term is called **head loss** because it is the theoretical loss **head** (or elevation) resulting from friction or minor losses.

35. **Minor losses** are deemed as frictional effects from piping items like **fittings, valves, bends, elbow, tees**, etc. Each item has a **loss coefficient**, K_L, associated with it. Although these are generally associated with piping systems, comparable analogies can be made with many non-piping system applications of fluid mechanics.

36. For a sharp-edged orifice opening, the loss coefficient, K_L, is equal to **0.5**.

Drag

37. Varies dramatically whether the flow is turbulent or laminar. In order to use the simple drag equation, the flow must be turbulent around the object. Aside from that assumption, the normal requirements of steady, incompressible flow are also required.

38. All of the variables that you would expect to be in the drag equation are there: frontal surface area (surface projected onto a plane), velocity of the fluid, density, and a drag coefficient.

39. Drag coefficients range from like 0.04 of a streamlined body to a semicircular shell with a drag coefficient of 2.3 like the one below.

V →

Cd=1,1   Cd=0,34

Cd=1,4   Cd=0,05

Cd=0,45   Cd=0,1-0,2