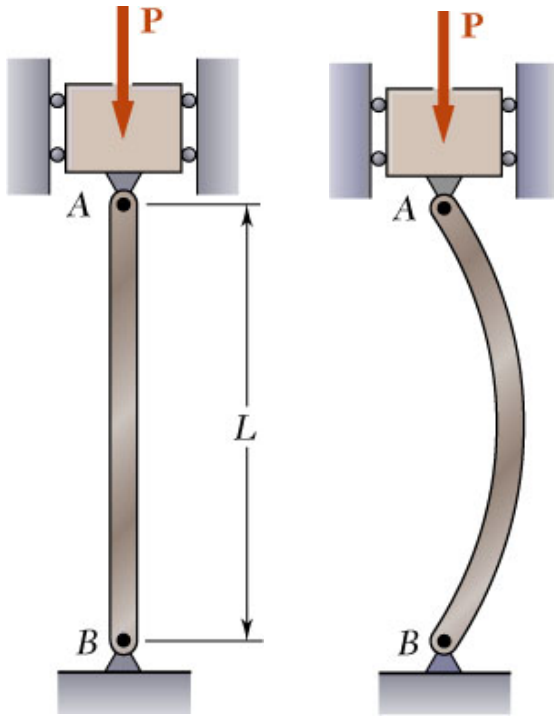


Stability of Structures



- In the design of columns, cross-sectional area is selected such that

- allowable stress is not exceeded

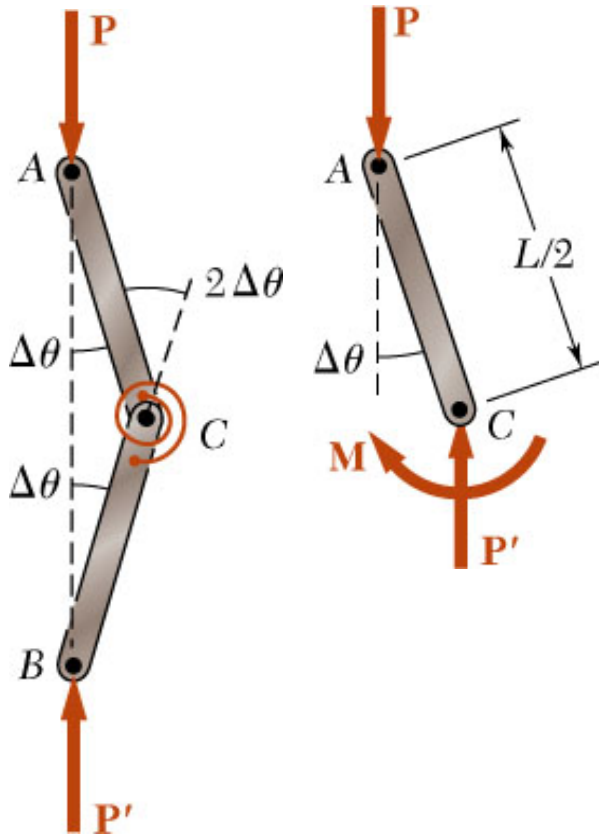
$$\sigma = \frac{P}{A} \leq \sigma_{all}$$

- deformation falls within specifications

$$\delta = \frac{PL}{AE} \leq \delta_{spec}$$

- After these design calculations, may discover that the column is unstable under loading and that it suddenly becomes sharply curved or buckles.

Stability of Structures



- Consider model with two rods and torsional spring. After a small perturbation,

$$K(2\Delta\theta) = \text{restoring moment}$$

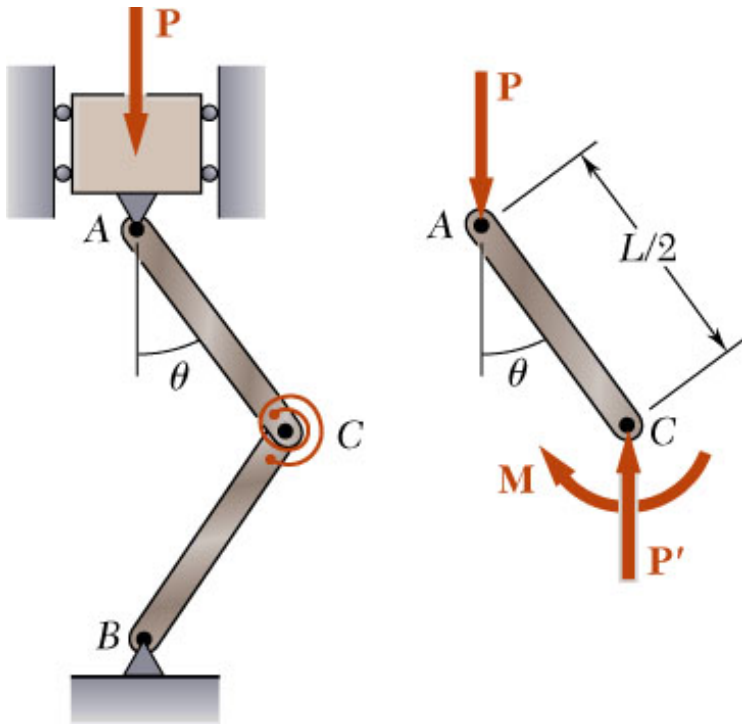
$$P \frac{L}{2} \sin \Delta\theta = P \frac{L}{2} \Delta\theta = \text{destabilizing moment}$$

- Column is stable (tends to return to aligned orientation) if

$$P \frac{L}{2} \Delta\theta < K(2\Delta\theta)$$

$$P < P_{cr} = \frac{4K}{L}$$

Stability of Structures



- Assume that a load P is applied. After a perturbation, the system settles to a new equilibrium configuration at a finite deflection angle.

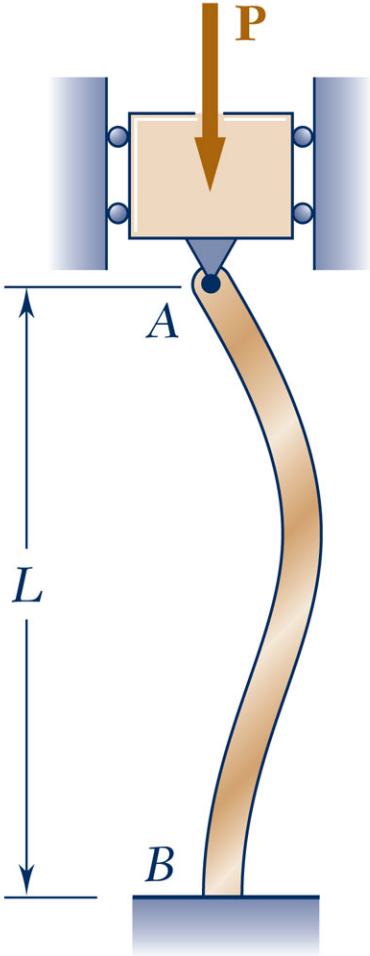
$$P \frac{L}{2} \sin \theta = K(2\theta)$$

$$\frac{PL}{4K} = \frac{P}{P_{cr}} = \frac{\theta}{\sin \theta}$$

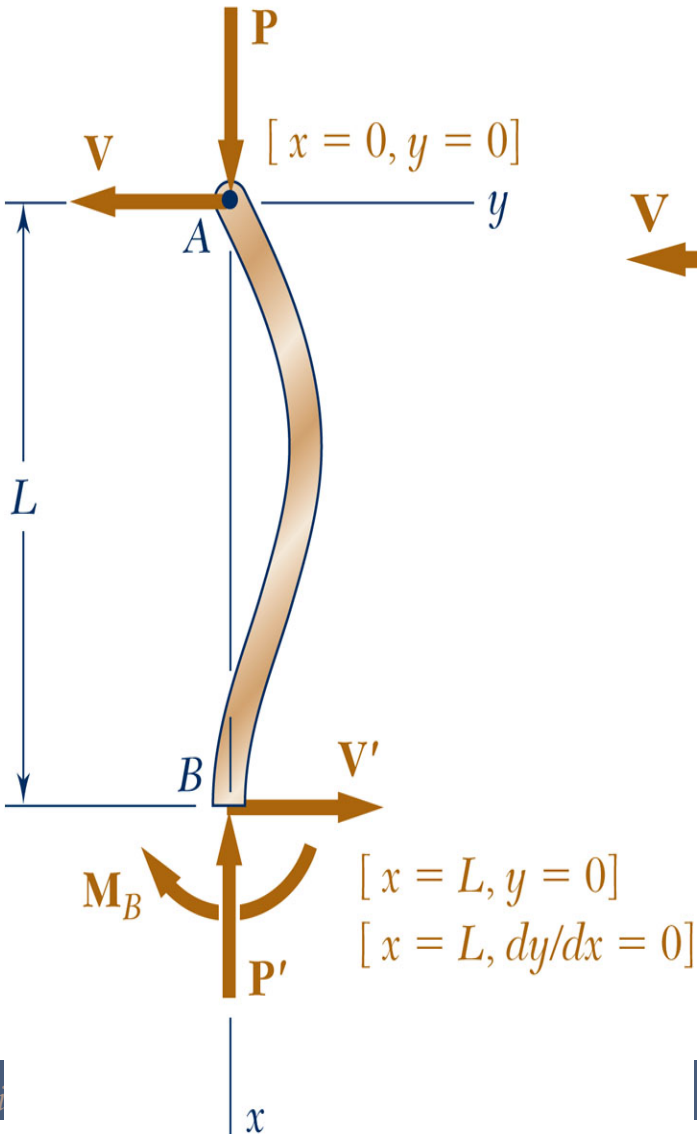
- Noting that $\sin \theta < \theta$, the assumed configuration is only possible if $P > P_{cr}$.

Extension of Euler's Formula

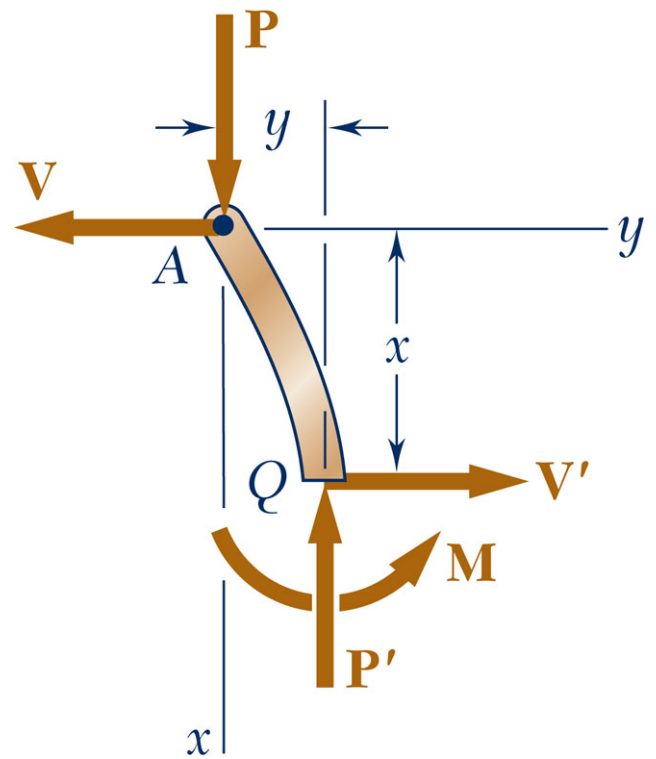
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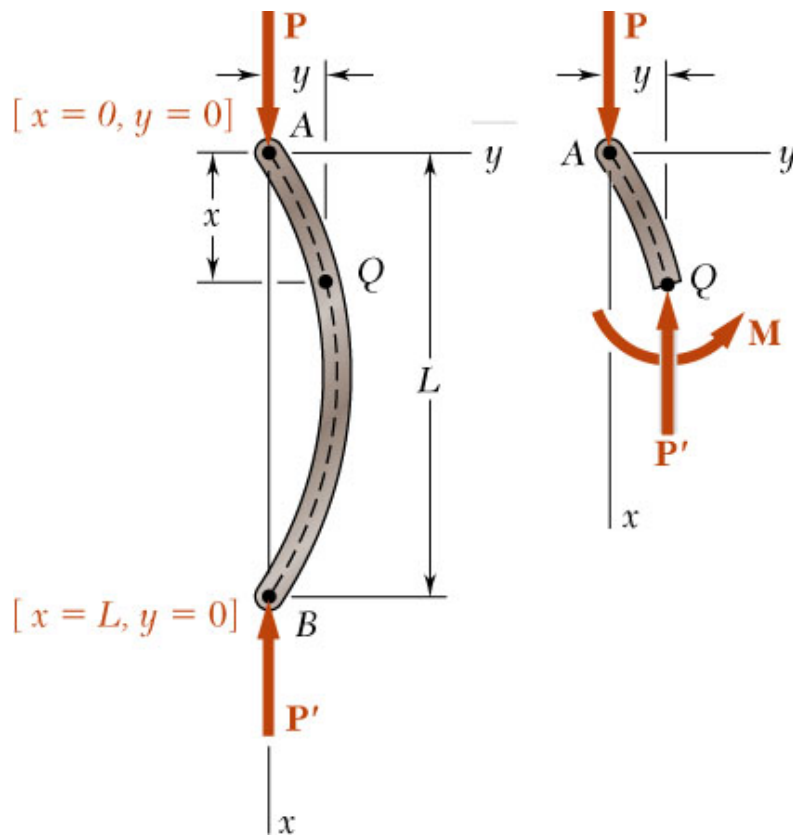


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End

Euler's Formula for Pin-Ended Beams



- Consider an axially loaded beam. After a small perturbation, the system reaches an equilibrium configuration such that

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} = -\frac{P}{EI} y$$

$$\frac{d^2 y}{dx^2} + \frac{P}{EI} y = 0$$

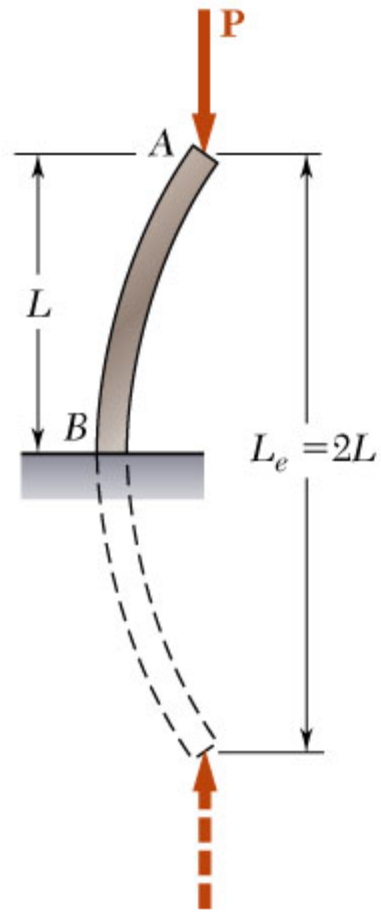
- Solution with assumed configuration can only be obtained if

$$P > P_{cr} = \frac{\pi^2 EI}{L^2}$$

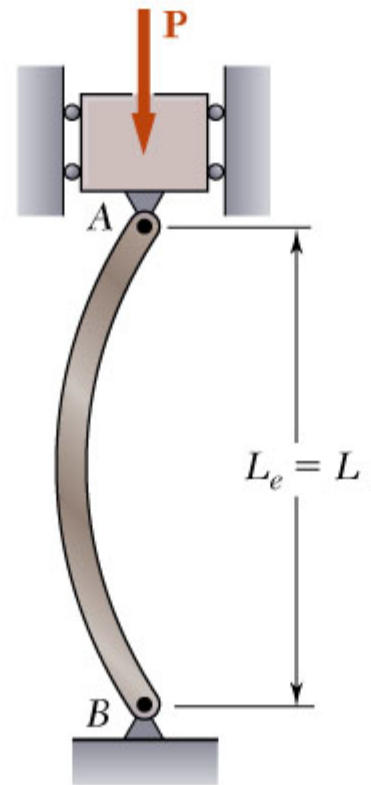
$$\sigma = \frac{P}{A} > \sigma_{cr} = \frac{\pi^2 E (Ar^2)}{L^2 A} = \frac{\pi^2 E}{(L/r)^2}$$

Extension of Euler's Formula

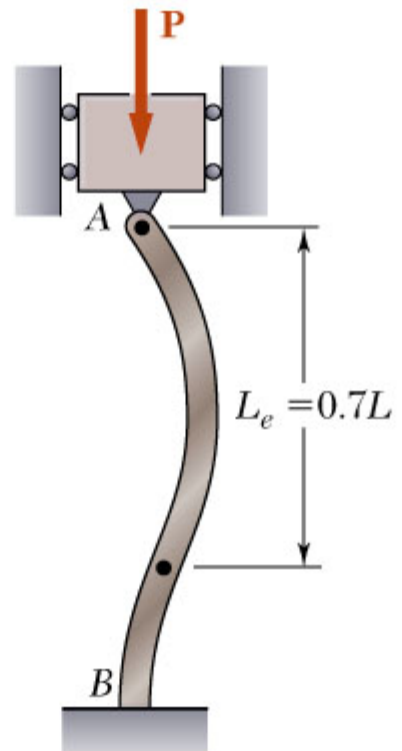
(a) One fixed end, one free end



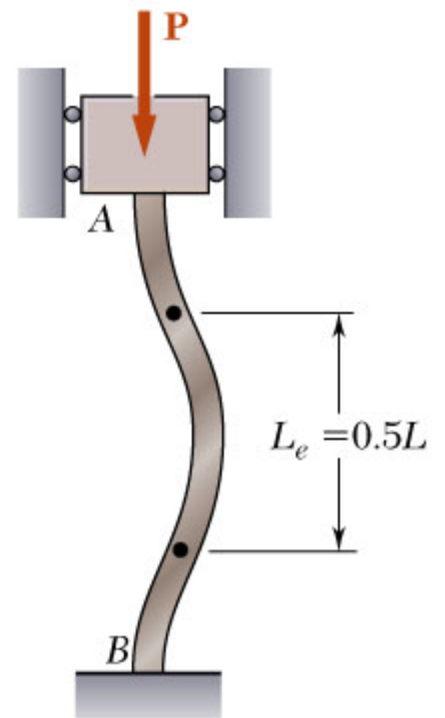
(b) Both ends pinned



(c) One fixed end, one pinned end



(d) Both ends fixed

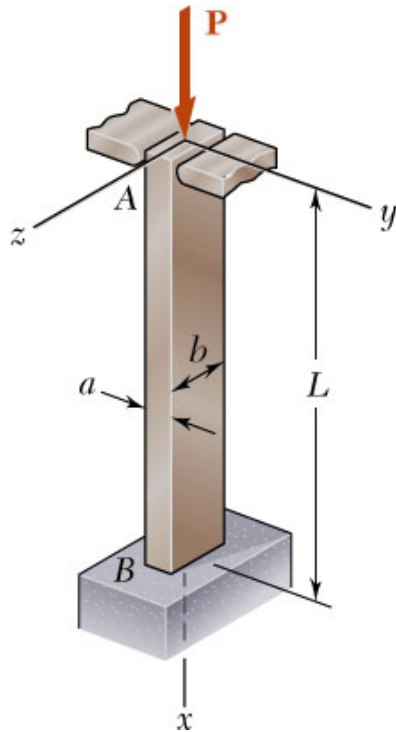


$$P > P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\sigma = \frac{P}{A} > \sigma_{cr} = \frac{\pi^2 E (Ar^2)}{L_e^2 A} = \frac{\pi^2 E}{(L_e/r)^2}$$

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Sample Problem 10.1



$$L = 20 \text{ in.}$$

$$E = 10.1 \times 10^6 \text{ psi}$$

$$P = 5 \text{ kips}$$

$$FS = 2.5$$

An aluminum column of length L and rectangular cross-section has a fixed end at B and supports a centric load at A . Two smooth and rounded fixed plates restrain end A from moving in one of the vertical planes of symmetry but allow it to move in the other plane.

- Determine the ratio a/b of the two sides of the cross-section corresponding to the most efficient design against buckling.
- Design the most efficient cross-section for the column.

$$P > P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\sigma = \frac{P}{A} > \sigma_{cr} = \frac{\pi^2 E (Ar^2)}{L_e^2 A} = \frac{\pi^2 E}{(L_e/r)^2}$$

Sample Problem 10.1

SOLUTION:

The most efficient design occurs when the resistance to buckling is equal in both planes of symmetry. This occurs when the slenderness ratios are equal.

- Buckling in xy Plane:

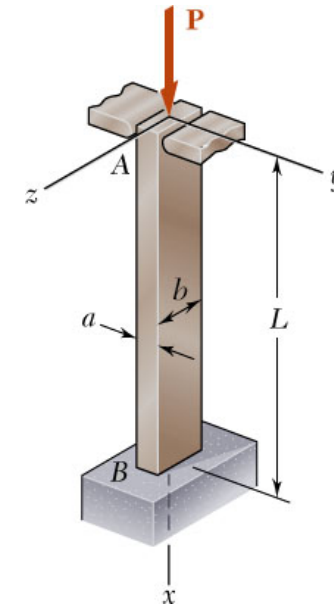
$$r_z^2 = \frac{I_z}{A} = \frac{\frac{1}{12}ba^3}{ab} = \frac{a^2}{12} \quad r_z = \frac{a}{\sqrt{12}}$$

$$\frac{L_{e,z}}{r_z} = \frac{0.7L}{a/\sqrt{12}}$$

- Buckling in xz Plane:

$$r_y^2 = \frac{I_y}{A} = \frac{\frac{1}{12}ab^3}{ab} = \frac{b^2}{12} \quad r_y = \frac{b}{\sqrt{12}}$$

$$\frac{L_{e,y}}{r_y} = \frac{2L}{b/\sqrt{12}}$$



- Most efficient design:

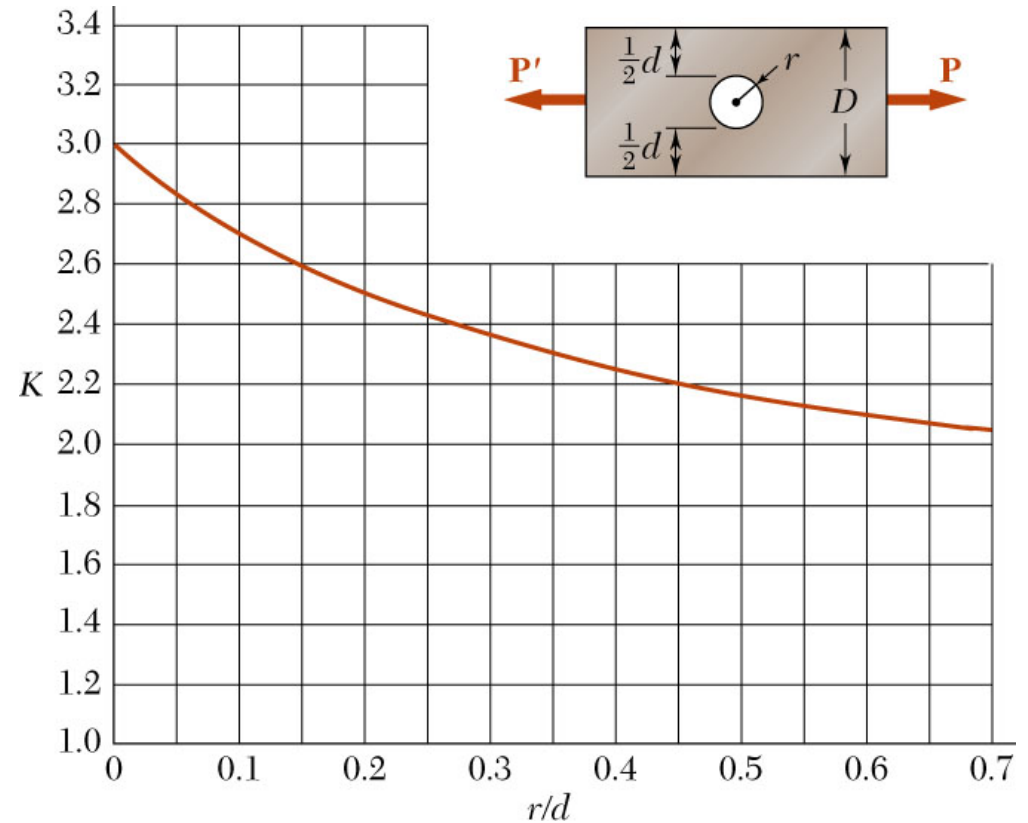
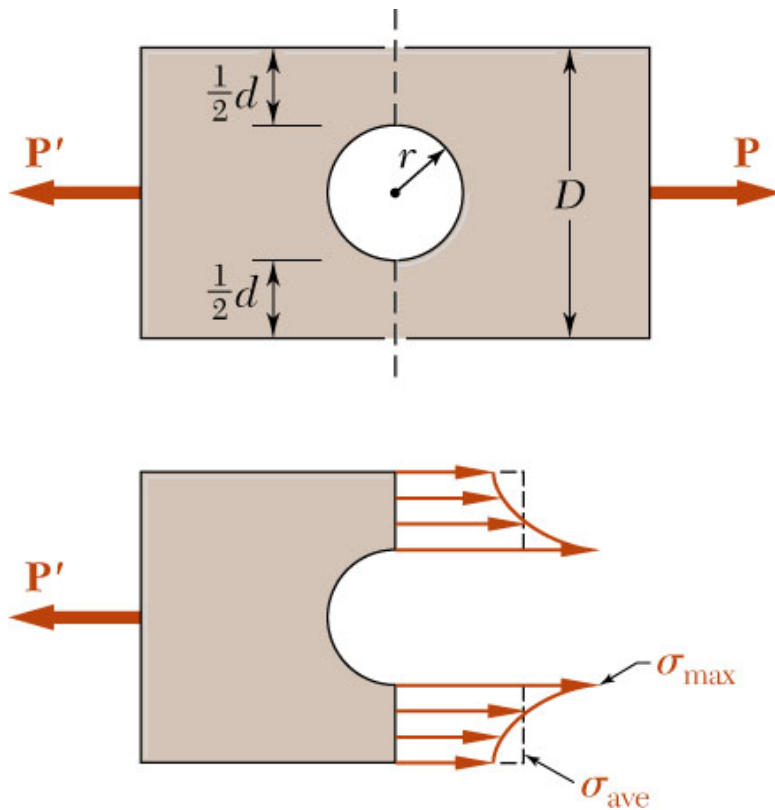
$$\frac{L_{e,z}}{r_z} = \frac{L_{e,y}}{r_y}$$

$$\frac{0.7L}{a/\sqrt{12}} = \frac{2L}{b/\sqrt{12}}$$

$$\frac{a}{b} = \frac{0.7}{2}$$

$$\frac{a}{b} = 0.35$$

Stress Concentration: Hole

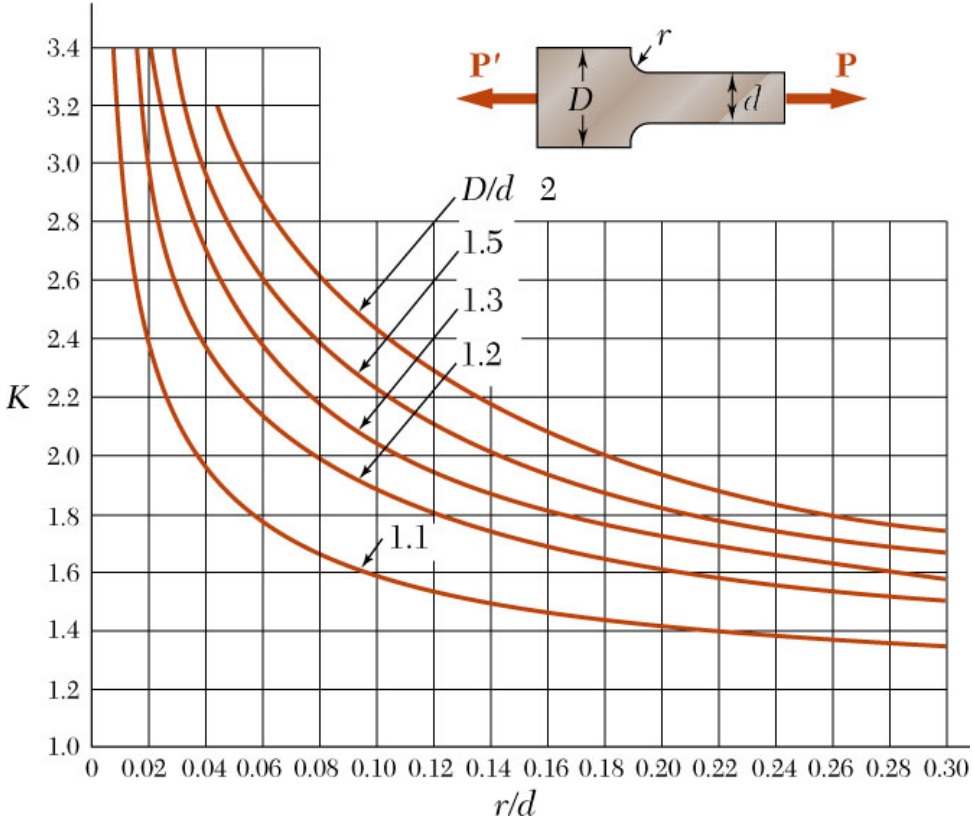
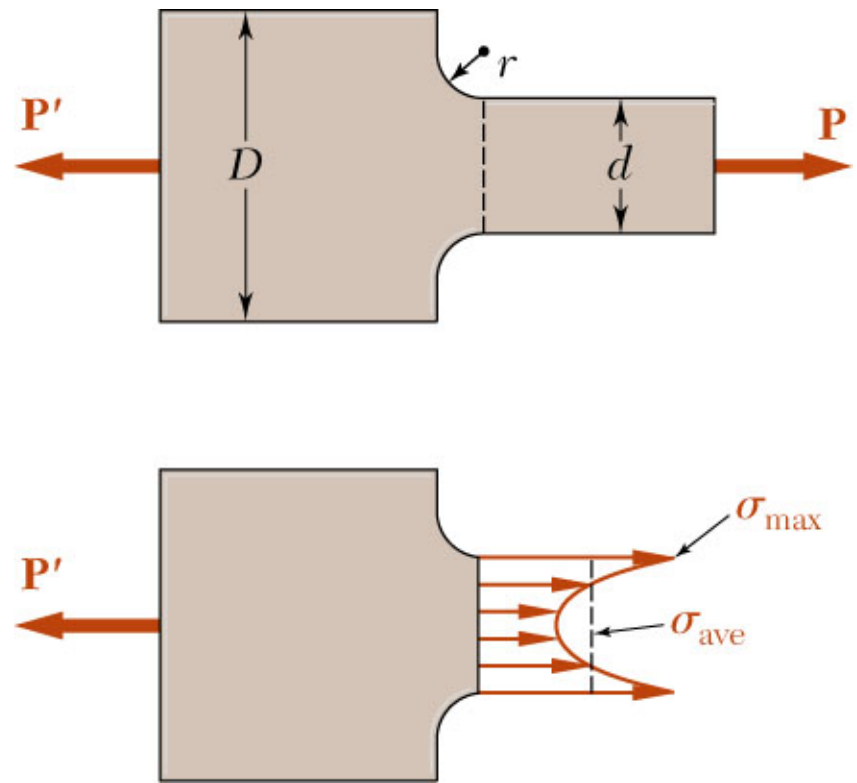


(a) Flat bars with holes

Discontinuities of cross section may result in high localized or *concentrated* stresses.

$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

Stress Concentration: Fillet



(b) Flat bars with fillets