Stress & Strain: Axial Loading

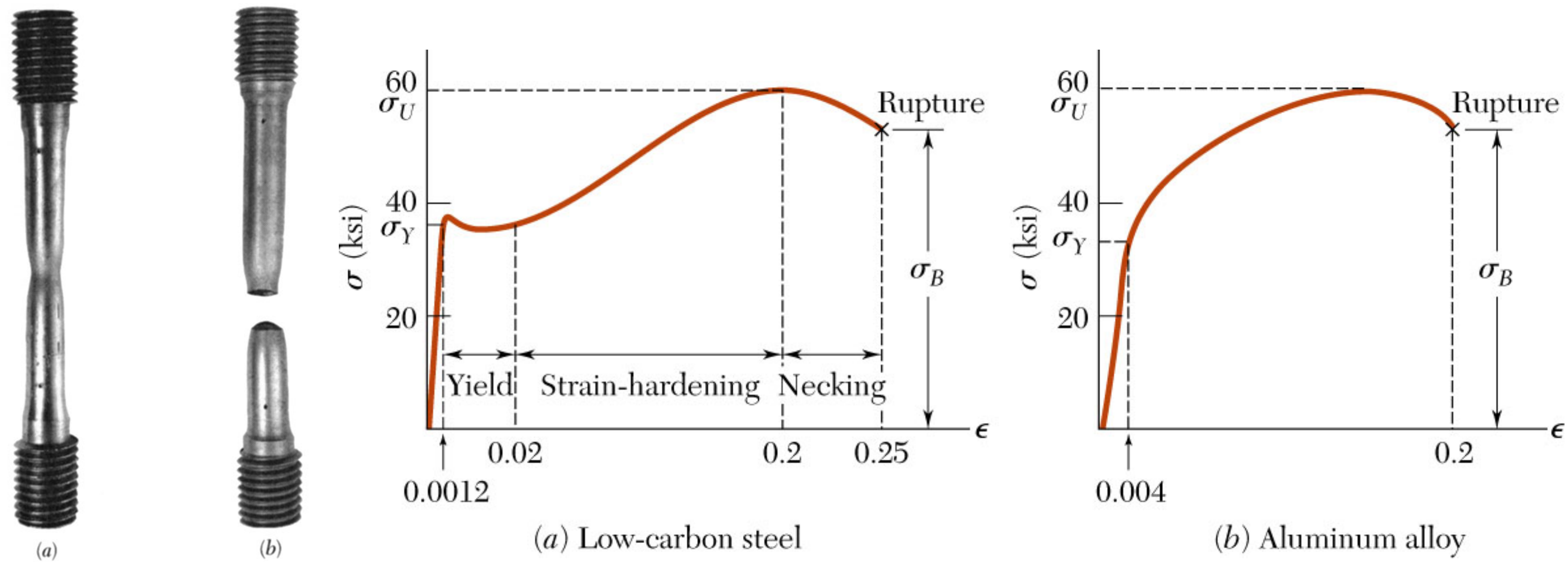
Suitability of a structure or machine may depend on the deformations in the structure as well as the stresses induced under loading. Statics analyses alone are not sufficient.

Considering structures as deformable allows determination of member forces and reactions which are statically indeterminate.

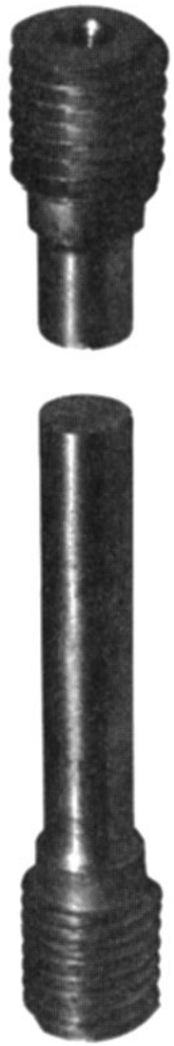
Determination of the stress distribution within a member also requires consideration of deformations in the member.

Chapter 2 is concerned with deformation of a structural member under axial loading. Later chapters will deal with torsional and pure bending loads.

Stress-Strain Diagram: Ductile Materials



Stress-Strain Diagram: Brittle Materials



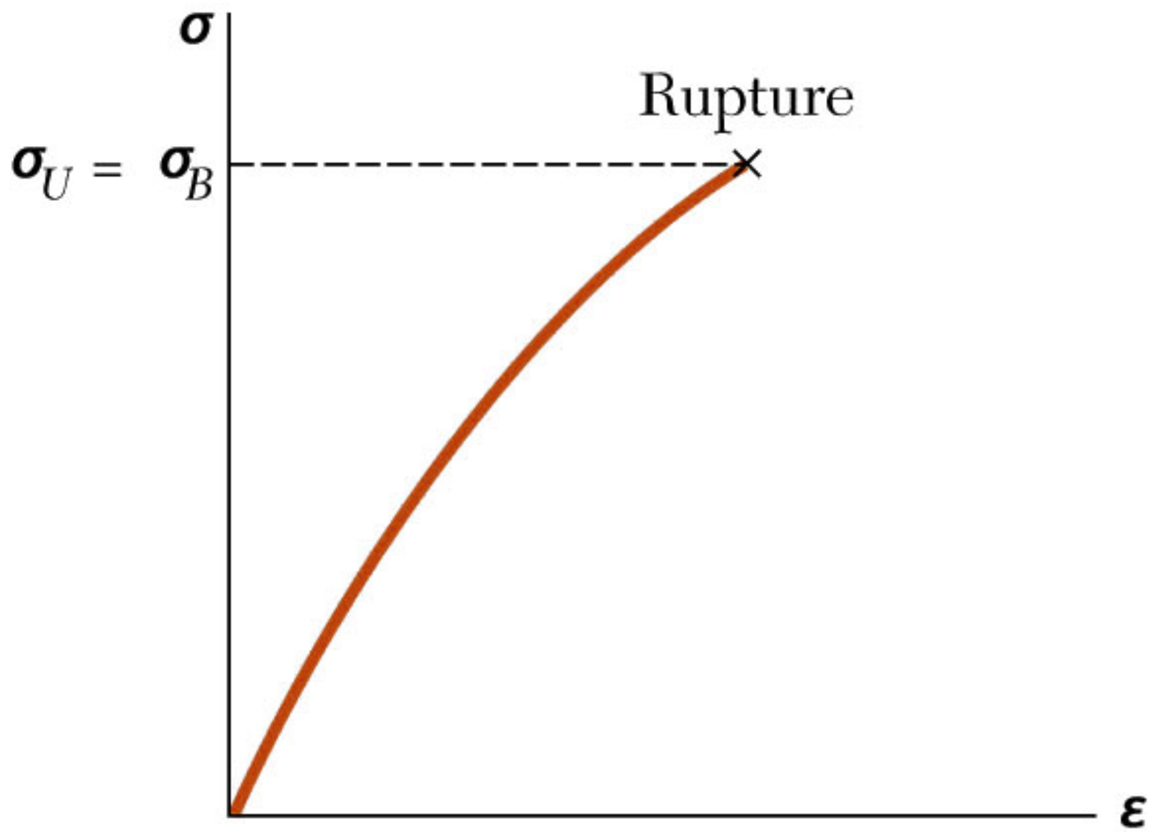


Fig 2.1 Stress-strain diagram for a typical brittle material.

Hooke's Law: Modulus of Elasticity

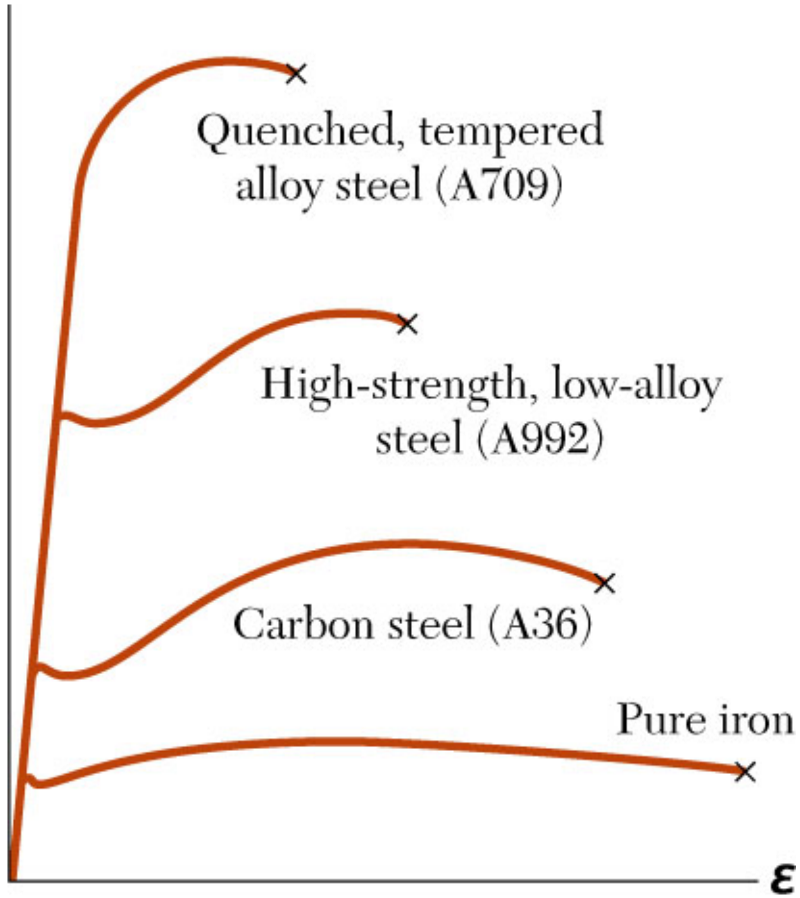


Fig 2.16 Stress-strain diagrams for iron and different grades of steel.

Below the yield stress

$$\sigma = E\varepsilon$$

E = Youngs Modulus or Modulus of Elasticity

Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Elastic vs. Plastic Behavior

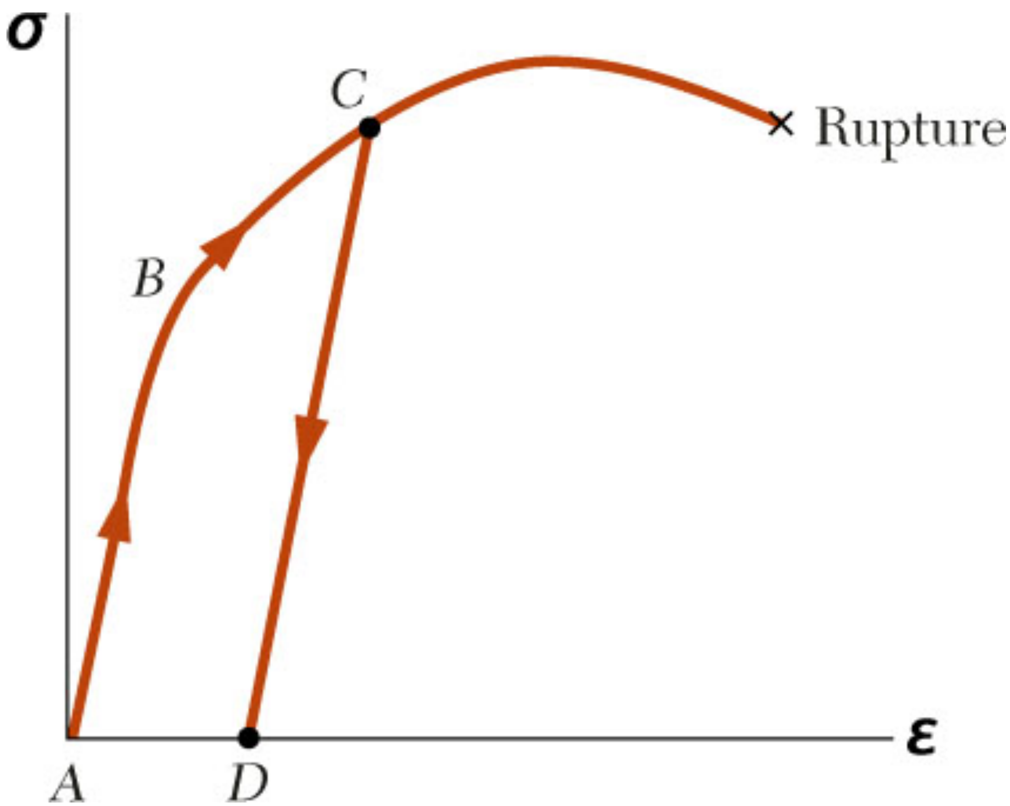


Fig. 2.18

If the strain disappears when the stress is removed, the material is said to behave *elastically*.

The largest stress for which this occurs is called the *elastic limit*.

When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

Deformations Under Axial Loading

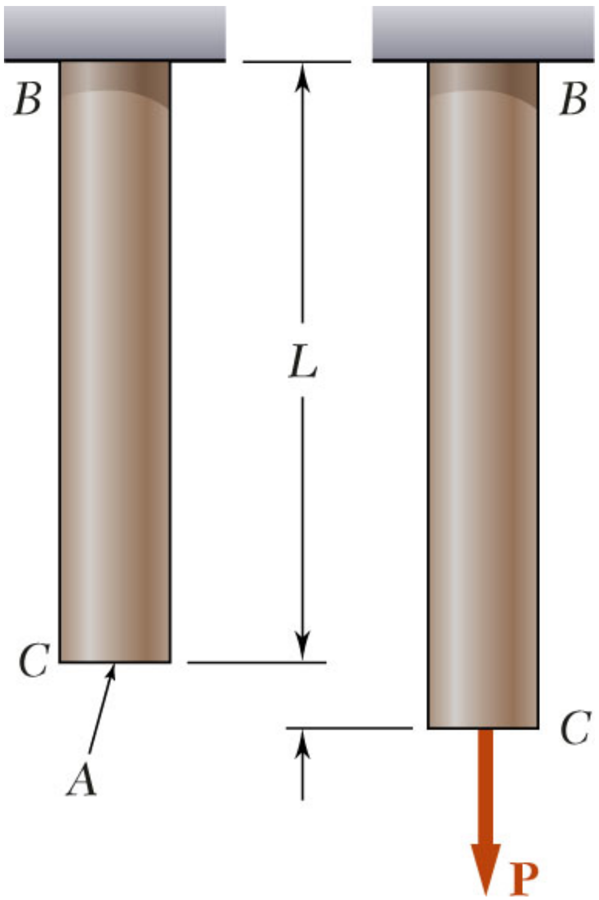


Fig. 2.22

From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

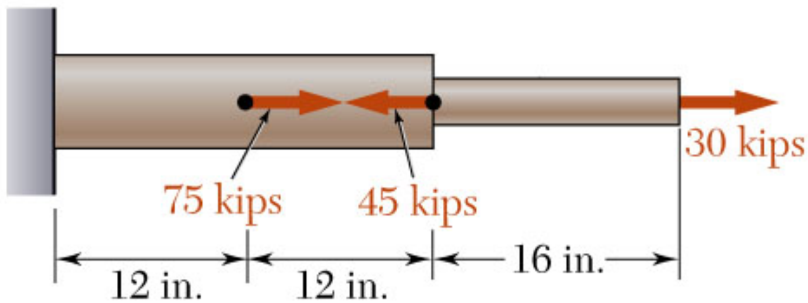
Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Example



$$E = 29 \times 10^6 \text{psi}$$

$$D = 1.07 \text{ in.} \quad d = 0.618 \text{ in.}$$

Determine the deformation of the steel rod shown under the given loads.

SOLUTION:

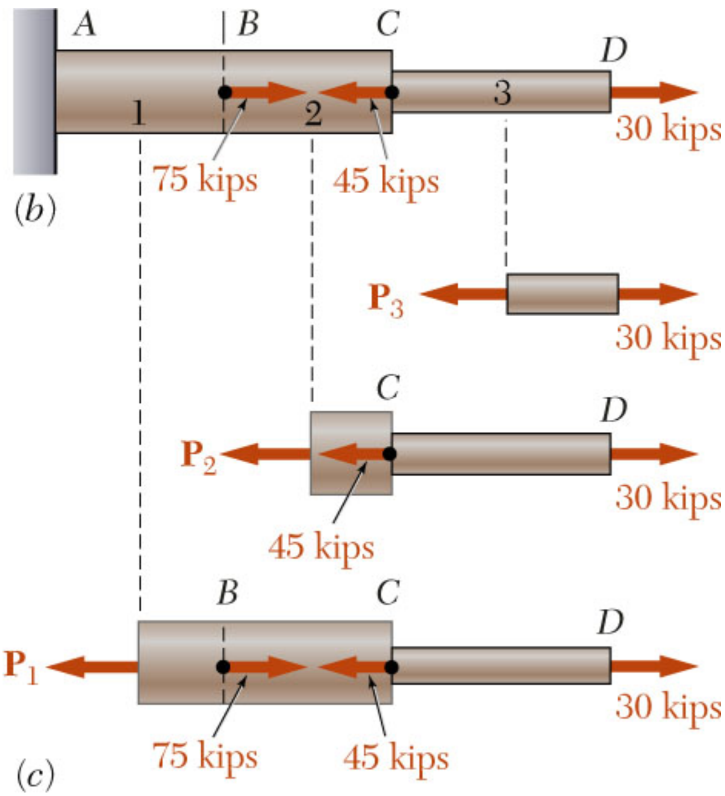
Divide the rod into components at the load application points.

Apply a free-body analysis on each component to determine the internal force

Evaluate the total of the component deflections.

SOLUTION:

Divide the rod into three components:



$L_1 = L_2 = 12\text{ in.}$ $\quad L_3 = 16\text{ in.}$

$A_1 = A_2 = 0.9\text{ in}^2$ $\quad A_3 = 0.3\text{ in}^2$

Apply free-body analysis to each component to determine internal forces,

$$P_1 = 60\times10^3\text{ lb}$$

$$P_2 = -15\times10^3\text{ lb}$$

$$P_3 = 30\times10^3\text{ lb}$$

Evaluate total deflection,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

$$= \frac{1}{29\times10^6}\left[\frac{\left(60\times10^3\right)12}{0.9} + \frac{\left(-15\times10^3\right)12}{0.9} + \frac{\left(30\times10^3\right)16}{0.3}\right]$$

$$= 75.9\times10^{-3}\text{ in.}$$

$$\boxed{\delta = 75.9\times10^{-3}\text{ in.}}$$

Statically indeterminate problems

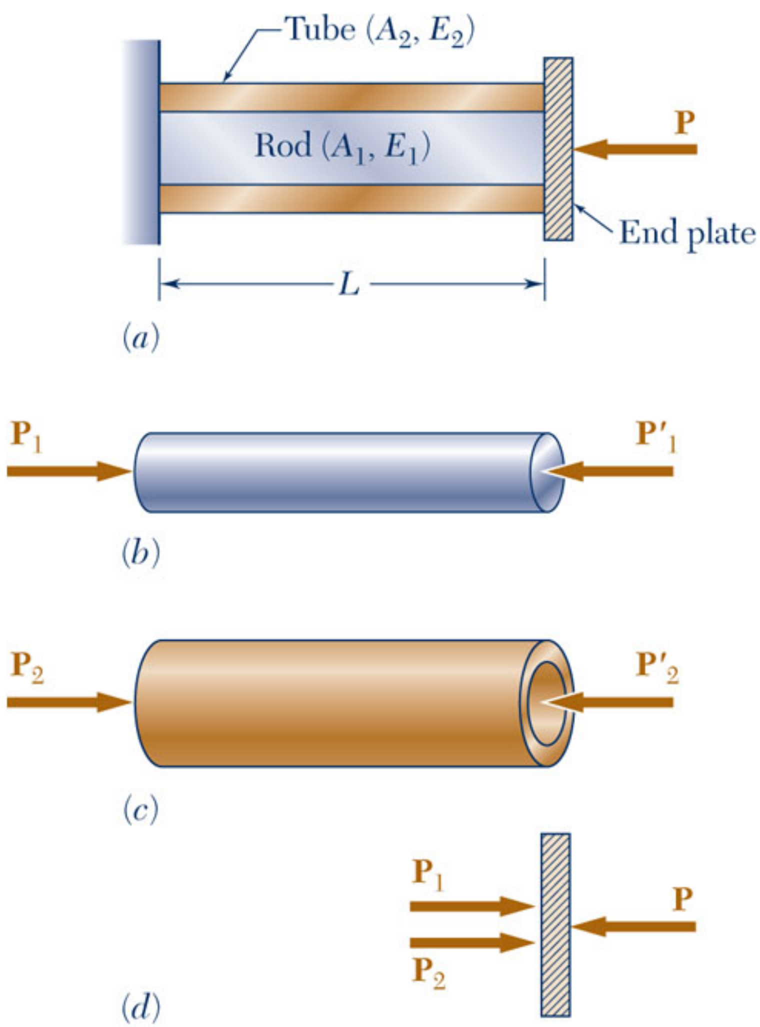
STATICS/EQUILIBRIUM (FORCE AND MOMENT BALANCE) ALONE CANNOT GIVE FORCES IN ALL PARTS OF THE STRUCTURE

HAVE TO INCLUDE STRESS-STRAIN RELATIONS TO OBTAIN FORCES

STEPS FOR SOLVING PROBLEMS

(1) Static Equilibrium Equations (Force and Moment Balance)
(2) Geometry of Deformation (Compatibility)
(3) Force-Deformation Relations
Combine (1-3) and Solve

Example



What is the deformation in the rod and tube when a force P is applied at the rigid end plate ?