Sign up for lab near the homework drop-off box

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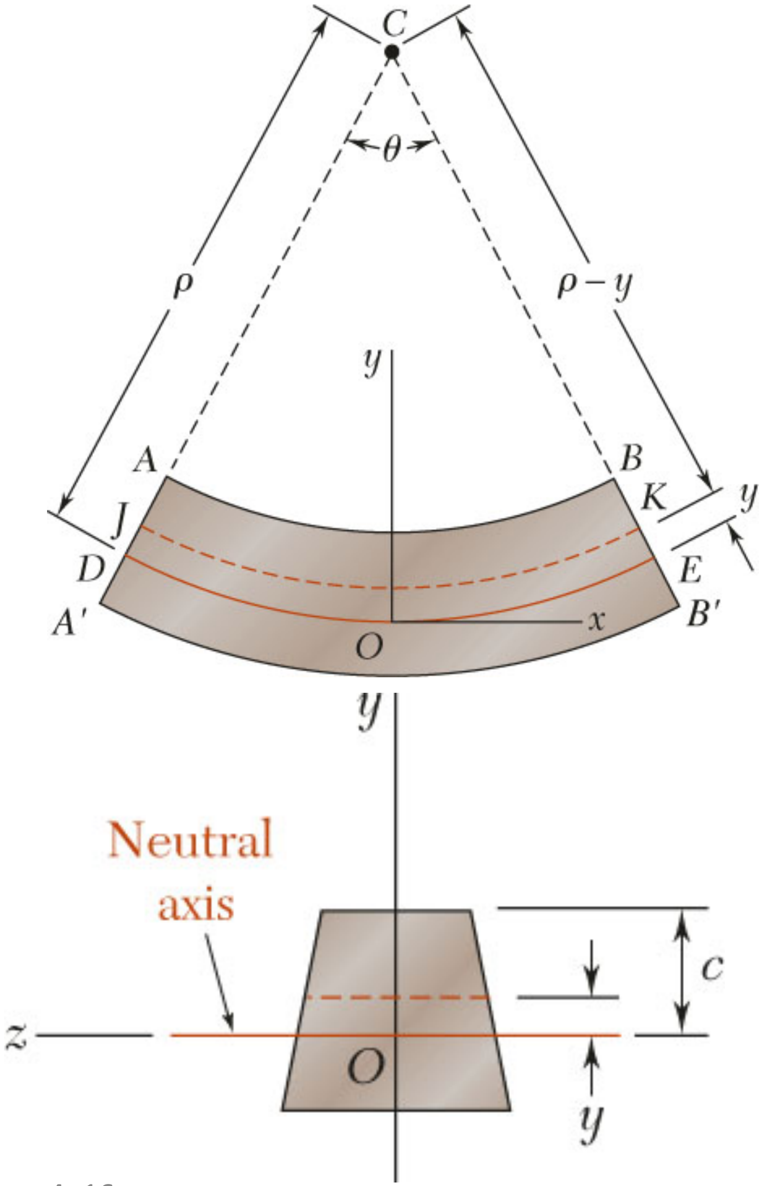
Questions about grading: see me Fri 4 – 5 PM (office hr this week)

Summary of the key results for bending



Consider a beam segment of length *L*.
After deformation, the length of the neutral surface remains *L*. At other sections,

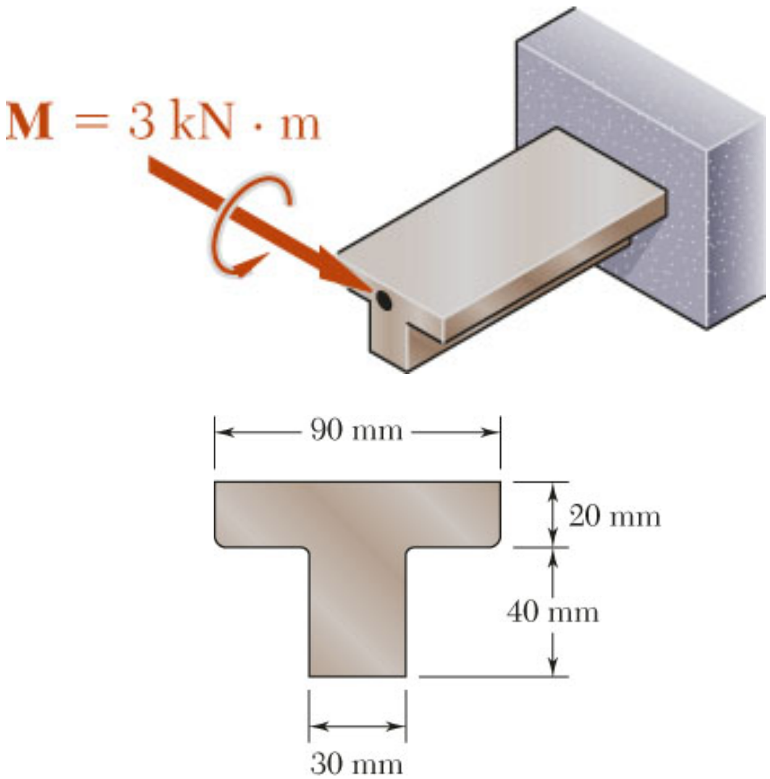
$$\varepsilon_x = -\frac{y}{\rho} \qquad \text{(strain varies linearly)}$$

$$\frac{M}{EI} = \frac{1}{\rho} \quad \textit{and} \quad \sigma_x = -\frac{My}{I}$$

$$\textit{where} \quad I = \int y^2 \, dA$$

$$|\sigma_x|_{max} = \frac{Mc}{I}$$

Sample Problem 4.2



A cast-iron machine part is acted upon by a 3 kN-m couple. Knowing *E* = 165 GPa and neglecting the effects of fillets, determine (a) the maximum tensile and compressive stresses, (b) the radius of curvature.

SOLUTION:

Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} \qquad I_{x'} = \sum\left(\bar{I} + A\,d^2\right)$$

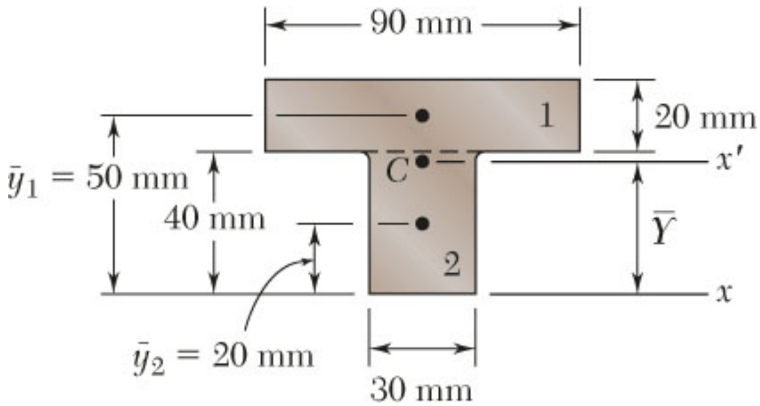
Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

Calculate the curvature

$$\frac{1}{\rho} = \frac{M}{EI}$$

Sample Problem 4.2

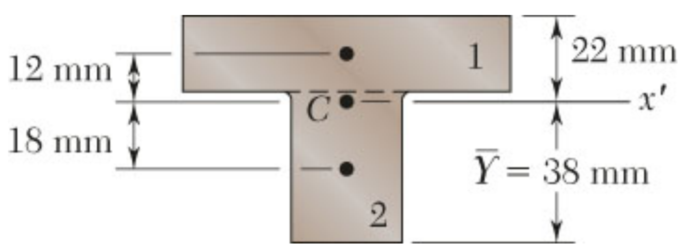


SOLUTION:

Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$20 \times 90 = 1800$	50	90×10^3
2	$40 \times 30 = 1200$	20	24×10^3
	$\sum A = 3000$		$\sum \bar{y}A = 114 \times 10^3$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{114 \times 10^3}{3000} = 38\text{ mm}$$

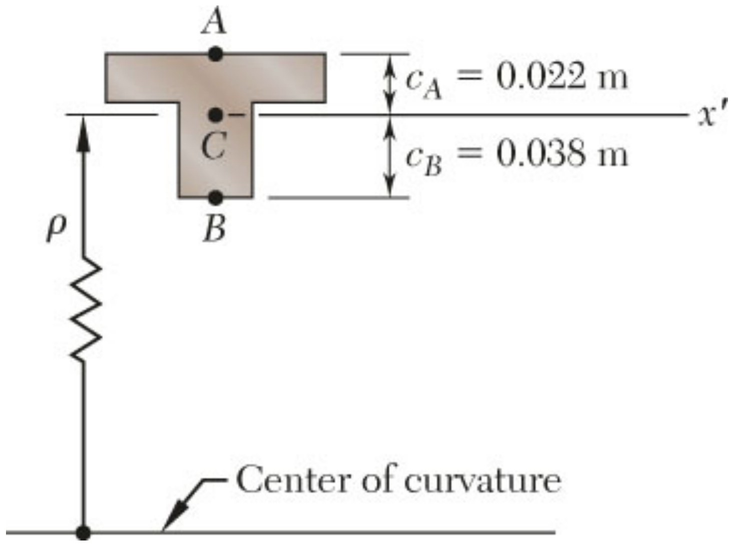


$$I_{x'} = \sum\left(\bar{I} + A d^2\right) = \sum\left(\tfrac{1}{12} bh^3 + A d^2\right)$$

$$= \left(\tfrac{1}{12} 90 \times 20^3 + 1800 \times 12^2\right) + \left(\tfrac{1}{12} 30 \times 40^3 + 1200 \times 18^2\right)$$

$$I = 868 \times 10^3 \text{mm}^4 \;=\; 868 \times 10^{-9} \text{m}^4$$

Sample Problem 4.2



Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

$$\sigma_A = \frac{M c_A}{I} = \frac{3\,\text{kN}\cdot\text{m}\times 0.022\,\text{m}}{868\times 10^{-9}\,\text{m}^4}$$

$$\boxed{\sigma_A = +76.0\ \text{MPa}}$$

$$\sigma_B = -\frac{M c_B}{I} = -\frac{3\,\text{kN}\cdot\text{m}\times 0.038\,\text{m}}{868\times 10^{-9}\,\text{m}^4}$$

$$\boxed{\sigma_B = -131.3\ \text{MPa}}$$

Calculate the curvature

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$= \frac{3\,\text{kN}\cdot\text{m}}{(165\ \text{GPa})\left(868\times 10^{-9}\,\text{m}^4\right)}$$

$$\frac{1}{\rho} = 20.95\times 10^{-3}\,\text{m}^{-1}$$

$$\rho = 47.7\ \text{m}$$

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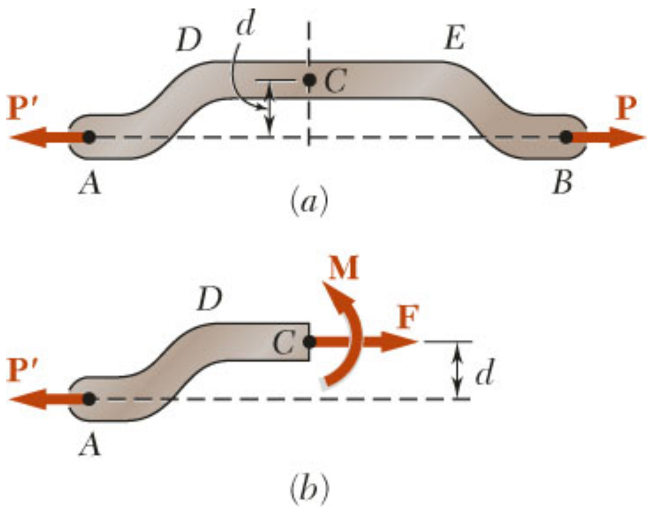
Eccentric Axial Loading in a Plane of Symmetry



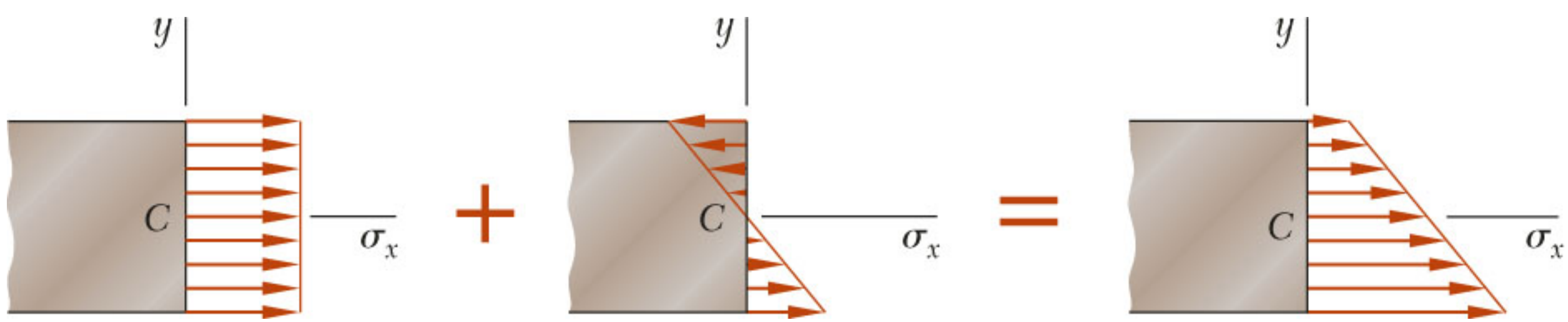
Eccentric loading

$$F = P$$
$$M = Pd$$

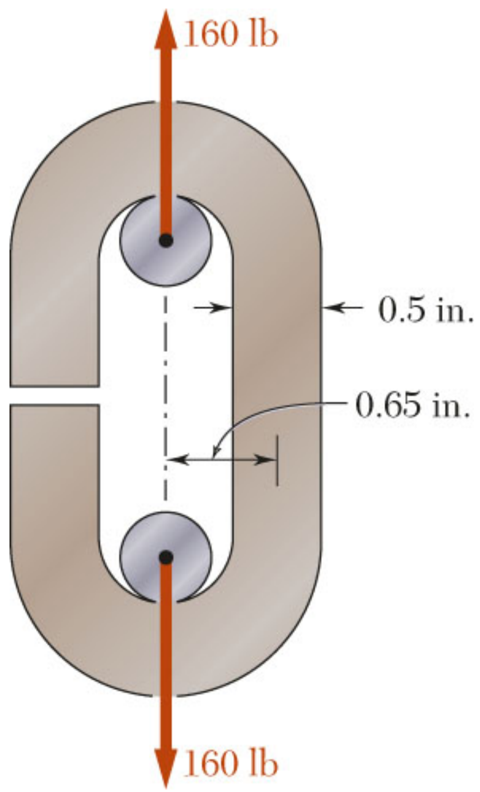
Stress due to eccentric loading found by superposing the uniform stress due to a centric load and linear stress distribution due a pure bending moment

$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$
$$= \frac{P}{A} - \frac{My}{I}$$

Validity requires stresses below proportional limit, deformations have negligible effect on geometry, and stresses not evaluated near points of load application.



Example 4.07



An open-link chain is obtained by bending low-carbon steel rods into the shape shown. For 160 lb load, determine (a) maximum tensile and compressive stresses, (b) distance between section centroid and neutral axis

SOLUTION:

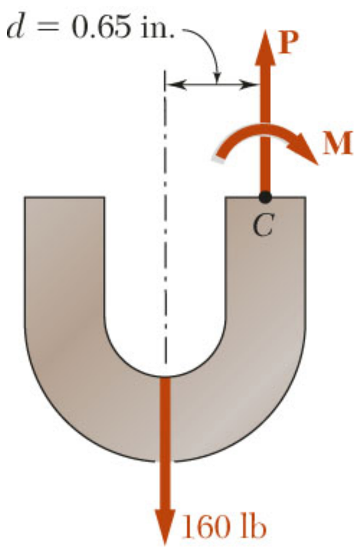
Find the equivalent centric load and bending moment

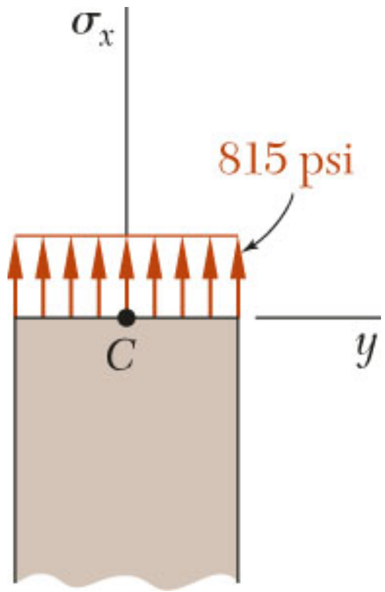
Superpose the uniform stress due to the centric load and the linear stress due to the bending moment.

Evaluate the maximum tensile and compressive stresses at the inner and outer edges, respectively, of the superposed stress distribution.

Find the neutral axis by determining the location where the normal stress is zero.

Example 4.07





Normal stress due to a centric load

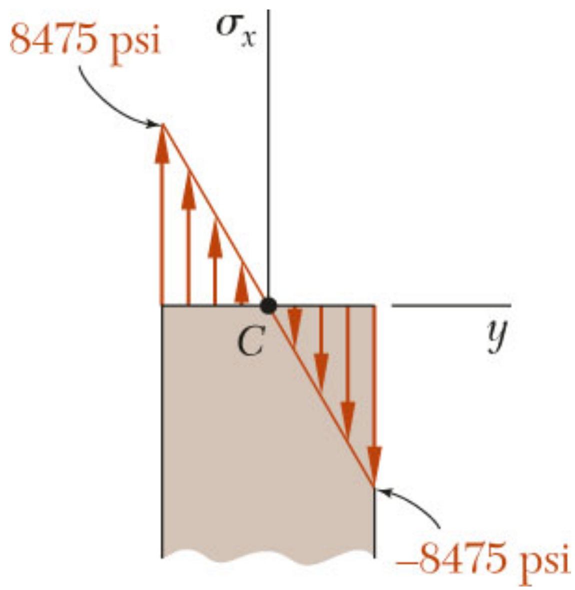
$$A = \pi c^2 = \pi (0.25\,\text{in})^2 = 0.1963\,\text{in}^2$$

$$\sigma_0 = \frac{P}{A} = \frac{160\,\text{lb}}{0.1963\,\text{in}^2} = 815\,\text{psi}$$

Equivalent centric load and bending moment

$$P = 160\,\text{lb}$$

$$M = Pd = (160\,\text{lb})(0.65\,\text{in}) = 104\,\text{lb}\cdot\text{in}$$

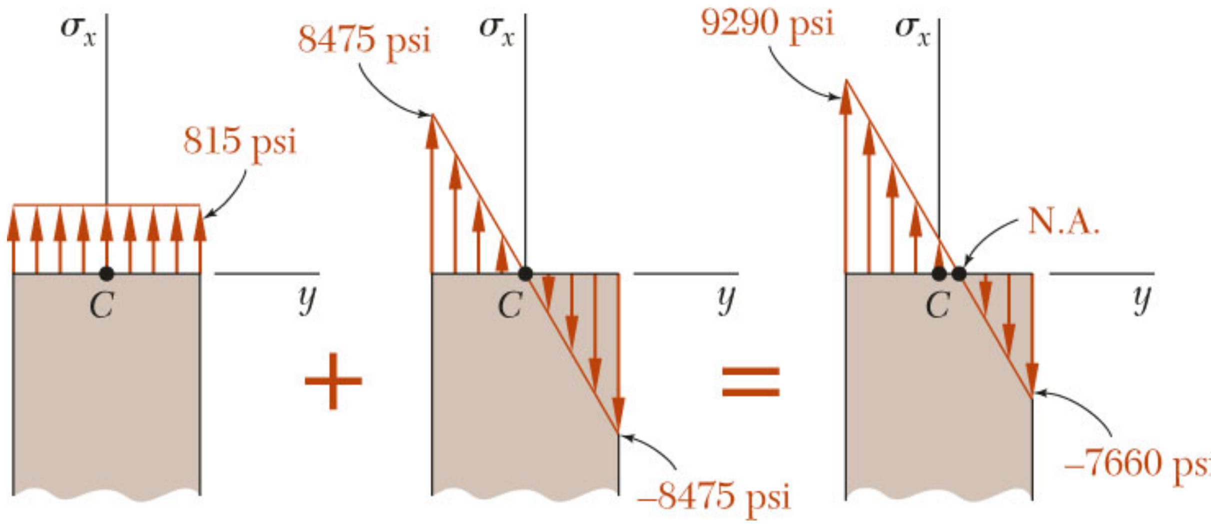


Normal stress due to bending moment

$$I = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi (0.25)^4 = 3.068\times10^{-3}\,\text{in}^4$$

$$\sigma_m = \frac{Mc}{I} = \frac{(104\,\text{lb}\cdot\text{in})(0.25\,\text{in})}{3.068\times10^{-3}\,\text{in}^4} = 8475\,\text{psi}$$

Example 4.07



Maximum tensile and compressive stresses

$$\sigma_t = \sigma_0 + \sigma_m$$
$$= 815 + 8475$$
$$\boxed{\sigma_t = 9260\,\text{psi}}$$

$$\sigma_c = \sigma_0 - \sigma_m$$
$$= 815 - 8475$$
$$\boxed{\sigma_c = -7660\,\text{psi}}$$

Neutral axis location

$$0 = \frac{P}{A} - \frac{My_0}{I}$$

$$y_0 = \frac{P}{A}\frac{I}{M} = (815\,\text{psi})\frac{3.068\times10^{-3}\,\text{in}^4}{105\,\text{lb}\cdot\text{in}}$$

$$\boxed{y_0 = 0.0240\,\text{in}}$$