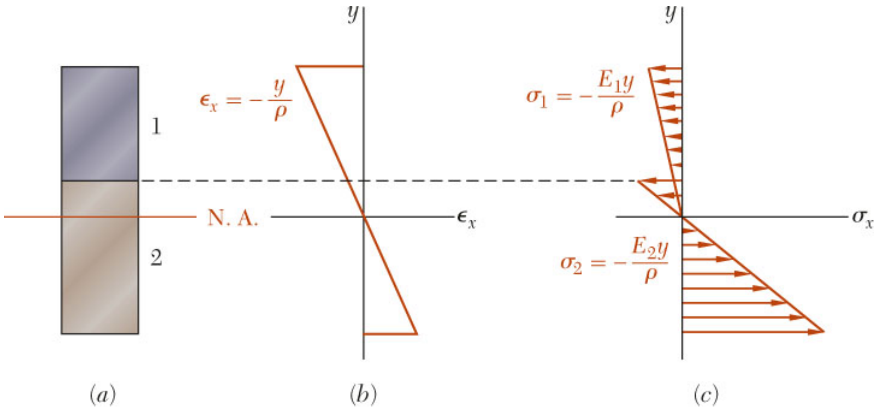
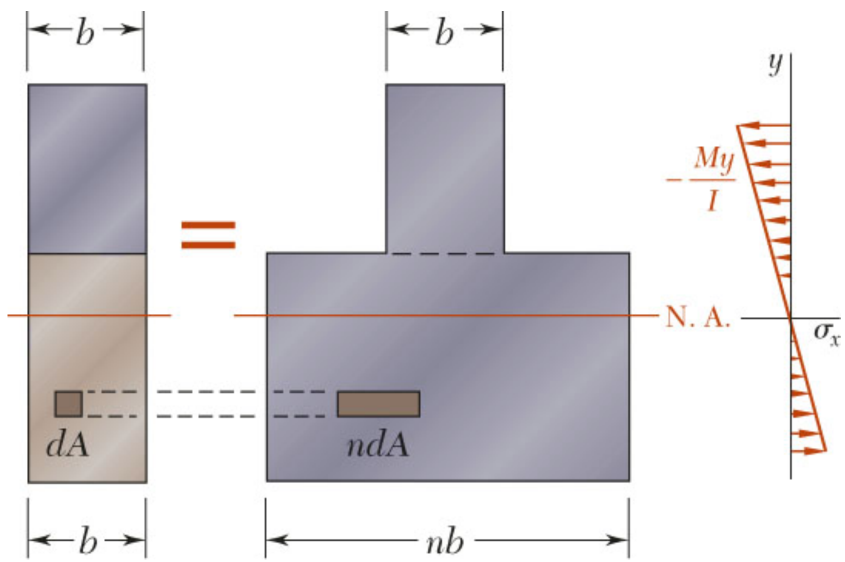
Bending of Members Made of Several Materials





$$\sigma_x = -\frac{My}{I}$$

$$\sigma_1 = \sigma_x \qquad \sigma_2 = n\sigma_x$$

Consider a composite beam formed from two materials with E_1 and E_2.

Normal strain varies linearly.

$$\varepsilon_x = -\frac{y}{\rho}$$

Piecewise linear normal stress variation.

$$\sigma_1 = E_1\varepsilon_x = -\frac{E_1 y}{\rho} \qquad \sigma_2 = E_2\varepsilon_x = -\frac{E_2 y}{\rho}$$

Neutral axis does not pass through section centroid of composite section.

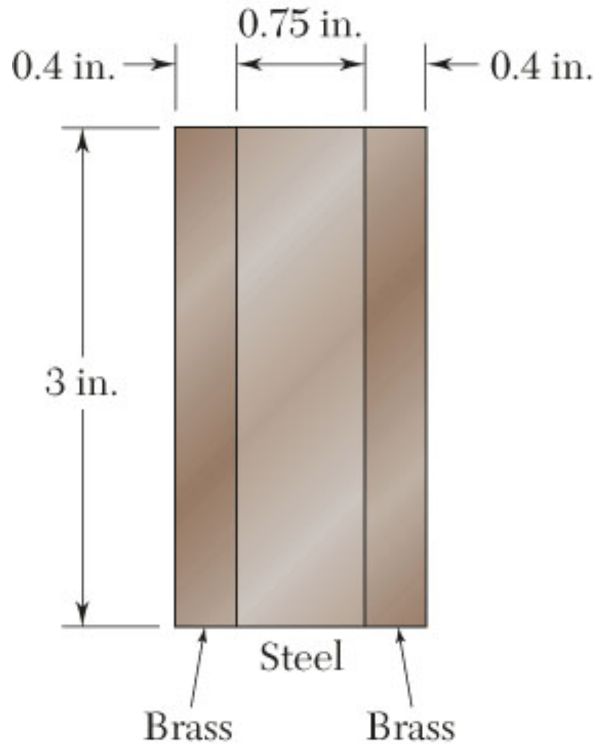
Elemental forces on the section are

$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA \quad dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA$$

Define a transformed section such that

$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho}(n\,dA) \qquad n = \frac{E_2}{E_1}$$

Example 4.03



Bar is made from bonded pieces of steel (E_s = 29x10^6 psi) and brass (E_b = 15x10^6 psi). Determine the maximum stress in the steel and brass when a moment of 40 kip*in is applied.

SOLUTION:

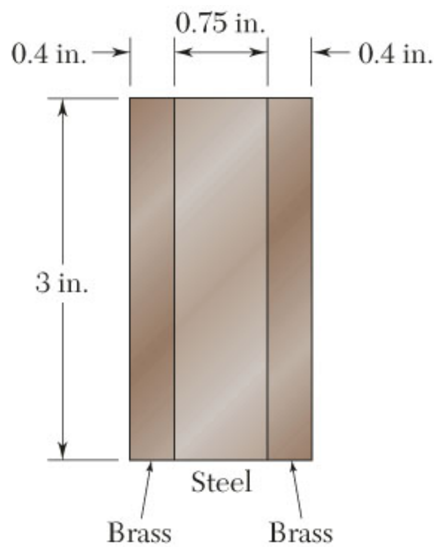
Transform the bar to an equivalent cross section made entirely of brass

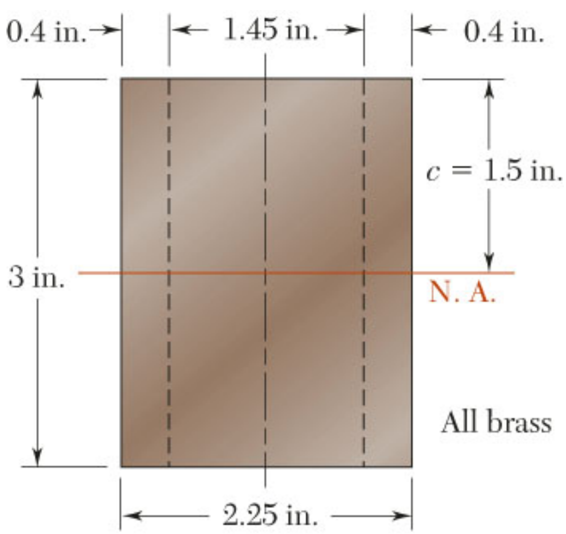
Evaluate the cross sectional properties of the transformed section

Calculate the maximum stress in the transformed section. This is the correct maximum stress for the brass pieces of the bar.

Determine the maximum stress in the steel portion of the bar by multiplying the maximum stress for the transformed section by the ratio of the moduli of elasticity.

Example 4.03





SOLUTION:

Transform the bar to an equivalent cross section made entirely of brass.

$$n = \frac{E_s}{E_b} = \frac{29\times 10^6\,\text{psi}}{15\times 10^6\,\text{psi}} = 1.933$$

$$b_T = 0.4\text{ in} + 1.933\times 0.75\text{ in} + 0.4\text{ in} = 2.25\text{ in}$$

Evaluate the transformed cross sectional properties

$$I = \tfrac{1}{12} b_T h^3 = \tfrac{1}{12}(2.25\text{ in.})(3\text{ in.})^3$$

$$= 5.063\text{ in.}^4$$

Calculate the maximum stresses

$$\sigma_m = \frac{Mc}{I} = \frac{(40\text{ kip}\cdot\text{in.})(1.5\text{ in.})}{5.063\text{ in.}^4} = 11.85\text{ ksi}$$

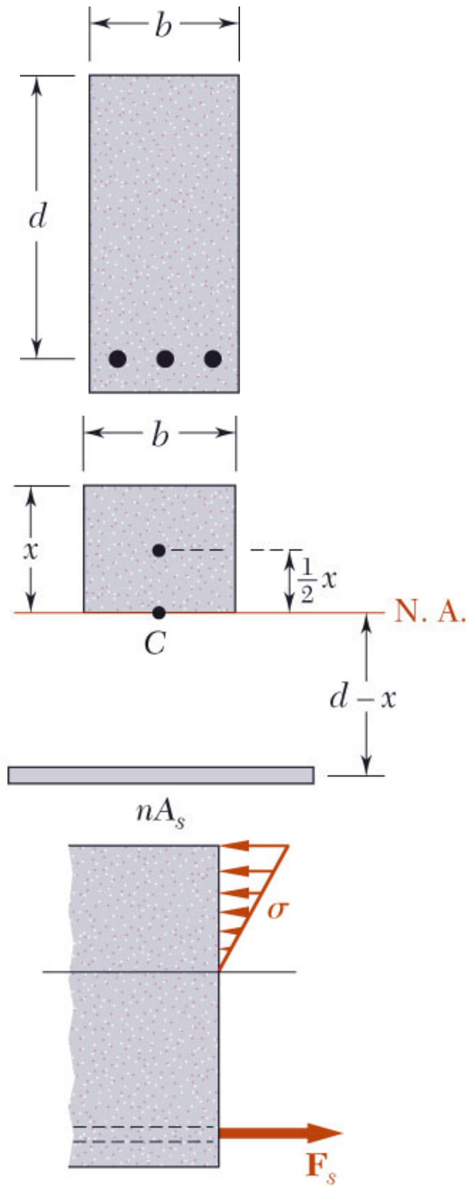
$$(\sigma_b)_{max} = \sigma_m$$

$$(\sigma_s)_{max} = n\sigma_m = 1.933\times 11.85\text{ ksi}$$

$$(\sigma_b)_{max} = 11.85\text{ ksi}$$

$$(\sigma_s)_{max} = 22.9\text{ ksi}$$

Reinforced Concrete Beams



Concrete beams subjected to bending moments are reinforced by steel rods.

The steel rods carry the entire tensile load below the neutral surface. The upper part of the concrete beam carries the compressive load.

In the transformed section, the cross sectional area of the steel, A_s, is replaced by the equivalent area nA_s where $n = E_s/E_c$.

To determine the location of the neutral axis,

$$(bx)\frac{x}{2} - n\,A_s(d - x) = 0$$

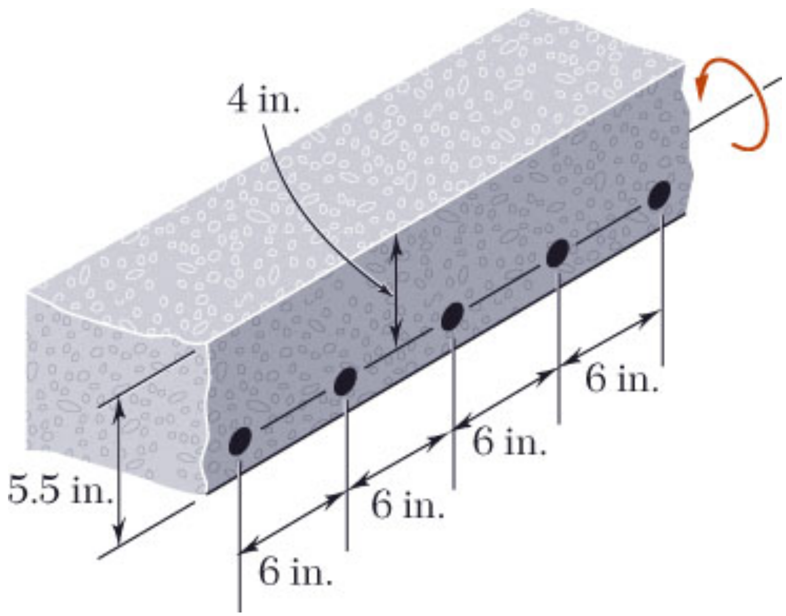
$$\frac{1}{2}b\,x^2 + n\,A_s x - n\,A_s d = 0$$

The normal stress in the concrete and steel

$$\sigma_x = -\frac{My}{I}$$

$$\sigma_c = \sigma_x \qquad \sigma_s = n\sigma_x$$

Sample Problem 4.4



A concrete floor slab is reinforced with 5/8-in-diameter steel rods. The modulus of elasticity is 29x10^{6}psi for steel and 3.6x10^{6}psi for concrete. With an applied bending moment of 40 kip*in for 1-ft width of the slab, determine the maximum stress in the concrete and steel.

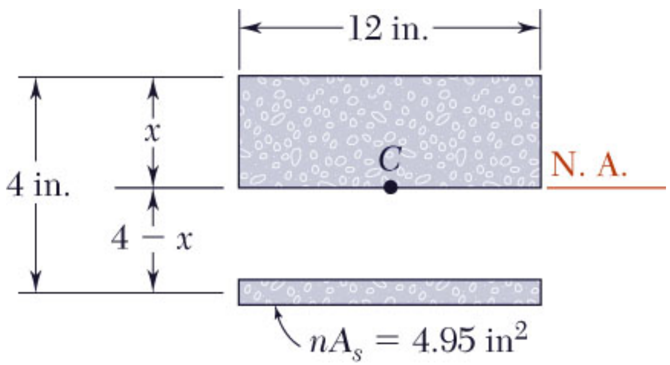
SOLUTION:

Transform to a section made entirely of concrete.

Evaluate geometric properties of transformed section.

Calculate the maximum stresses in the concrete and steel.

Sample Problem 4.4



SOLUTION:

Transform to a section made entirely of concrete.

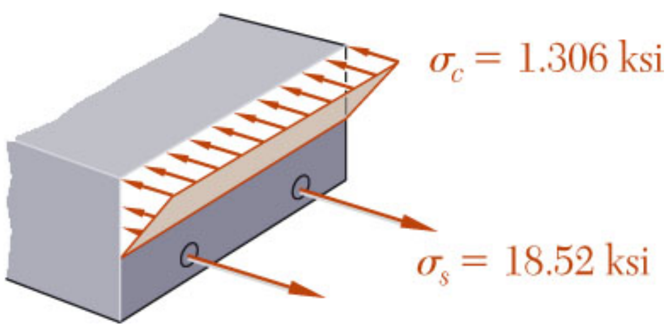
$$n = \frac{E_s}{E_c} = \frac{29\times10^6\ \text{psi}}{3.6\times10^6\ \text{psi}} = 8.06$$

$$nA_s = 8.06\times2\left[\tfrac{\pi}{4}\left(\tfrac{5}{8}\ \text{in}\right)^2\right] = 4.95\,\text{in}^2$$

Evaluate the geometric properties of the transformed section.

$$12x\left(\frac{x}{2}\right) - 4.95(4-x) = 0 \qquad x = 1.450\,\text{in}$$

$$I = \tfrac{1}{3}(12\,\text{in})(1.45\,\text{in})^3 + \left(4.95\,\text{in}^2\right)(2.55\,\text{in})^2 = 44.4\,\text{in}^4$$

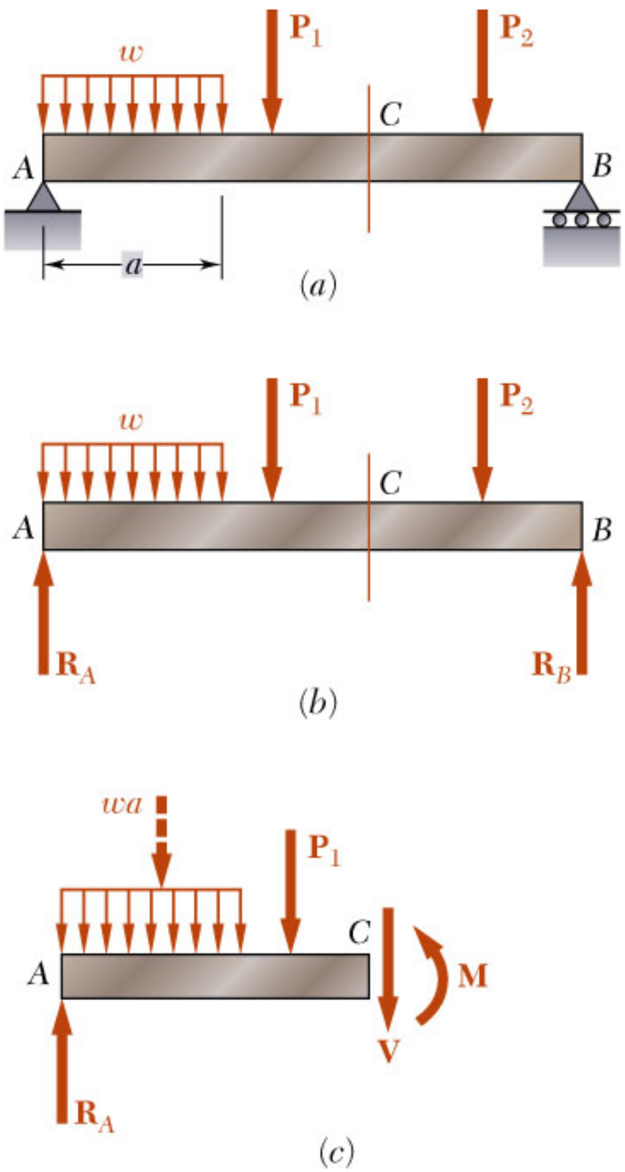


Calculate the maximum stresses.

$$\sigma_c = \frac{Mc_1}{I} = \frac{40\,\text{kip}\cdot\text{in}\times1.45\,\text{in}}{44.4\,\text{in}^4} \qquad \boxed{\sigma_c = 1.306\,\text{ksi}}$$

$$\sigma_s = n\frac{Mc_2}{I} = 8.06\,\frac{40\,\text{kip}\cdot\text{in}\times2.55\,\text{in}}{44.4\,\text{in}^4} \qquad \boxed{\sigma_s = 18.52\,\text{ksi}}$$

Chapter 5: Analysis and Design of Beams for Bending



Objective - Analysis and design of beams

Beams - structural members supporting loads at various points along the member

Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads

Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)

Normal stress is often the critical design criteria

$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{|M|c}{I} = \frac{|M|}{S}$$

Requires determination of the location and magnitude of largest bending moment

Introduction

Classification of Beam Supports