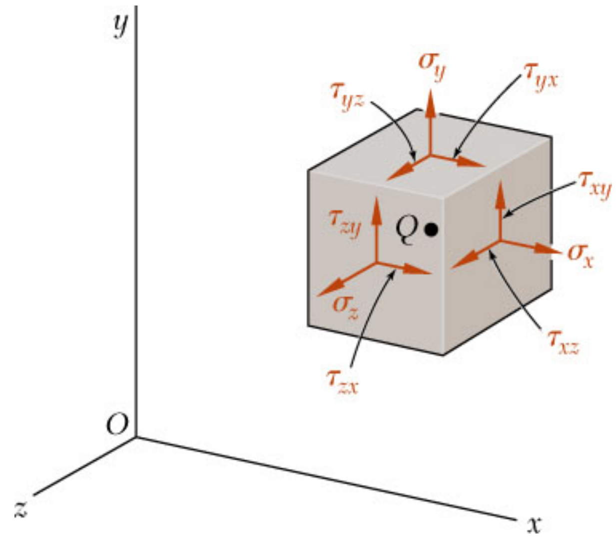
CHAPTER 7: Transformation of Stress and Strain



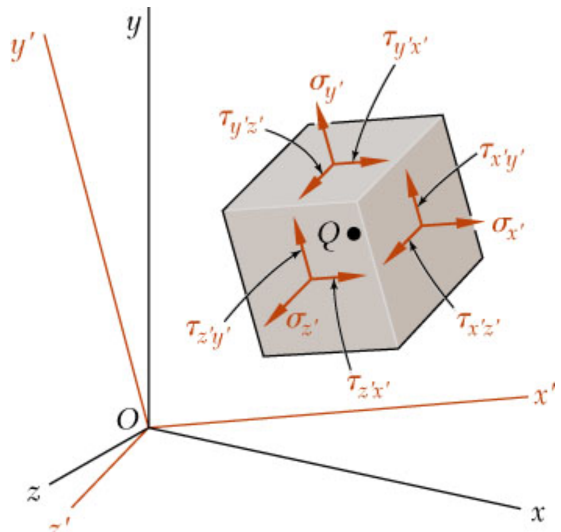
The most general state of stress at a point may be represented by 6 components,

$\sigma_x, \sigma_y, \sigma_z$ normal stresses

$\tau_{xy}, \tau_{yz}, \tau_{zx}$ shearing stresses

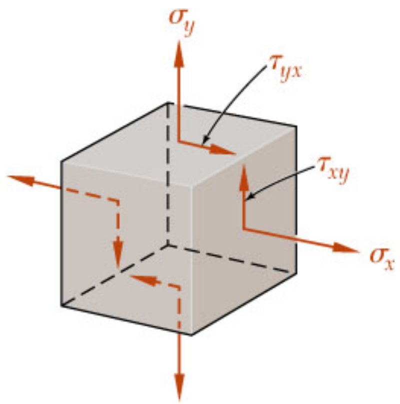
(Note : $\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xz}$)

Same state of stress is represented by a different set of components if axes are rotated.



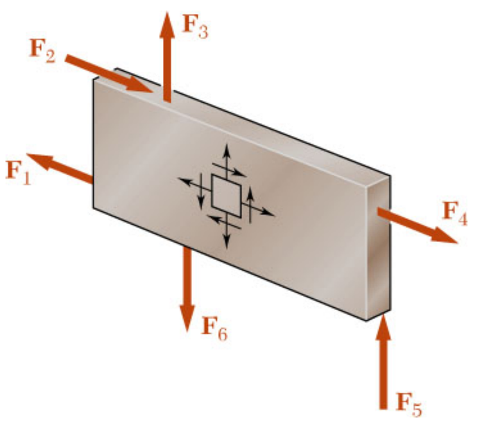
The first part of the chapter is concerned with how the components of stress are transformed under a rotation of the coordinate axes. The second part of the chapter is devoted to a similar analysis of the transformation of the components of strain.

Introduction

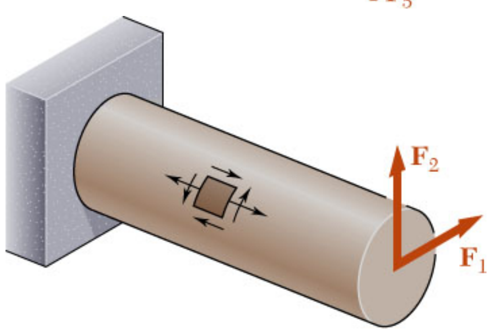


Plane Stress - state of stress in which two faces of the cubic element are free of stress. For the illustrated example, the state of stress is defined by

$$\sigma_x, \sigma_y, \tau_{xy} \quad \text{and} \quad \sigma_z = \tau_{zx} = \tau_{zy} = 0.$$

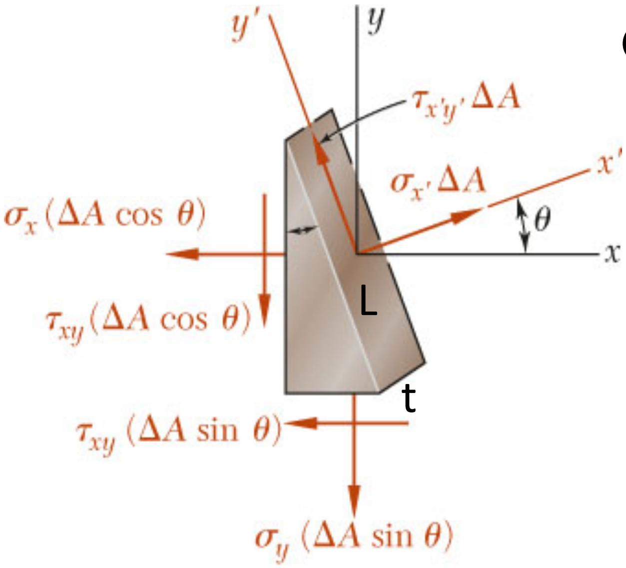


State of plane stress occurs in a thin plate subjected to forces acting in the midplane of the plate.



State of plane stress also occurs on the free surface of a structural element or machine component, i.e., at any point of the surface not subjected to an external force.

Transformation of Plane Stress



Consider the conditions for equilibrium of a prismatic element with faces perpendicular to the *x*, *y*, and *x'* axes.

$$\sum F_{x'} = 0 = \sigma_{x'}\Delta A - \sigma_x(\Delta A\cos\theta)\cos\theta - \tau_{xy}(\Delta A\cos\theta)\sin\theta - \sigma_y(\Delta A\sin\theta)\sin\theta - \tau_{xy}(\Delta A\sin\theta)\cos\theta$$

$$\sum F_{y'} = 0 = \tau_{x'y'}\Delta A + \sigma_x(\Delta A\cos\theta)\sin\theta - \tau_{xy}(\Delta A\cos\theta)\cos\theta - \sigma_y(\Delta A\sin\theta)\cos\theta + \tau_{xy}(\Delta A\sin\theta)\sin\theta$$

The equations may be rewritten to yield

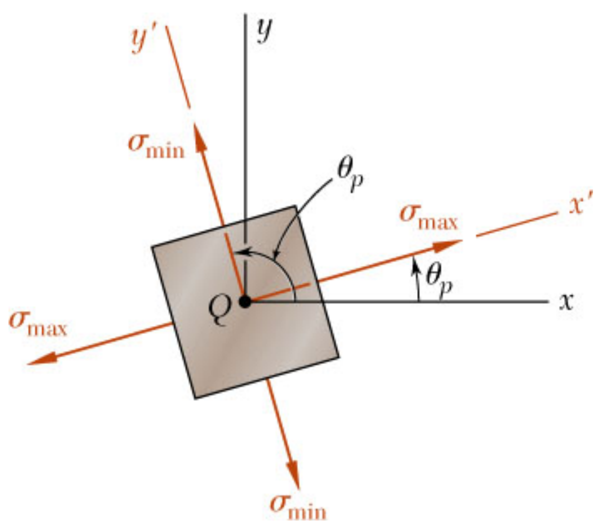
$\Delta A = L\,t$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stresses



The previous equations are combined to yield parametric equations for a circle,

$$(\sigma_{x'} - \sigma_{ave})^2 + \tau_{x'y'}^2 = R^2$$

where

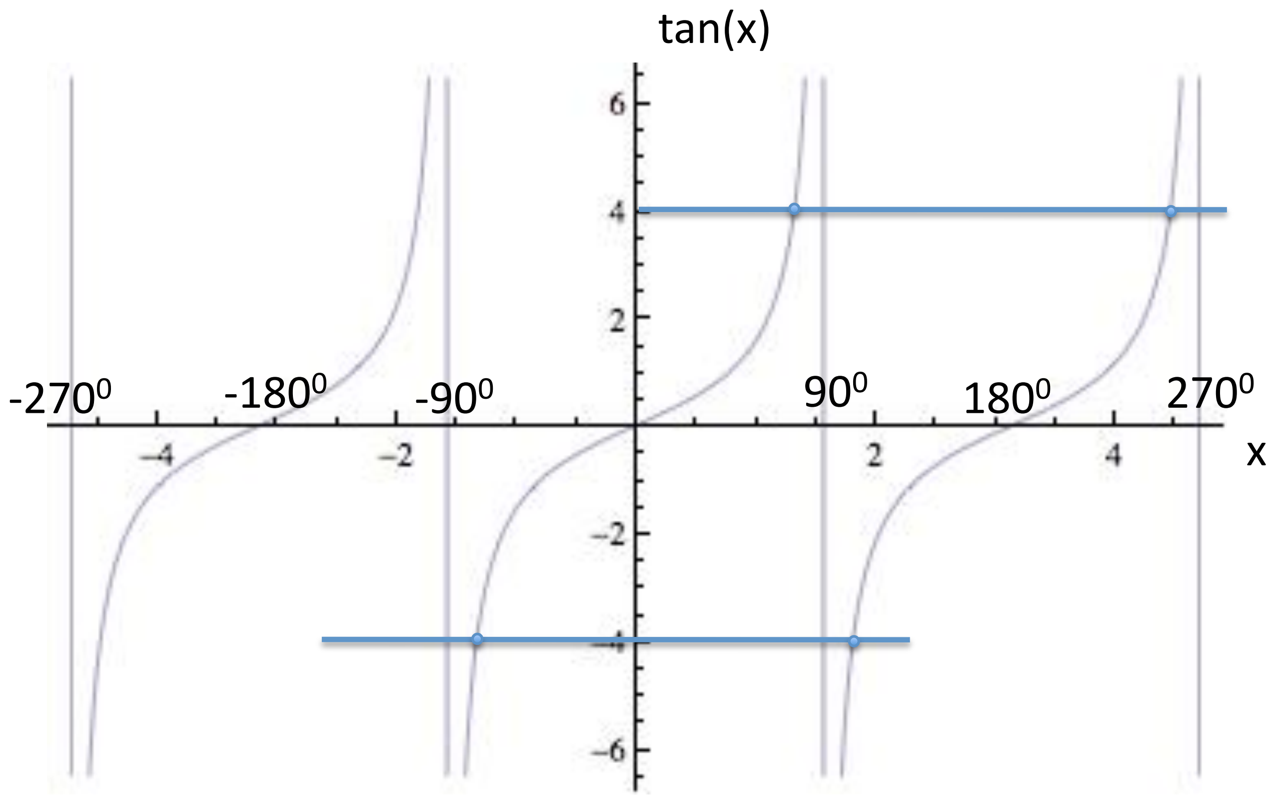
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Principal stresses occur on the *principal planes of stress* with zero shearing stresses.

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

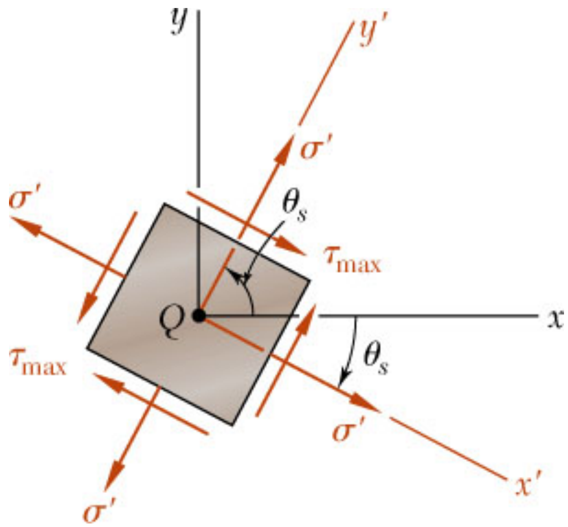
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Note : defines two angles separated by 90°


tan(x)
6
4
2
-2700
-1800
-900
900
1800
2700
-4
-2
2
4
x
−2
−4
−6

Maximum Shearing Stress

Maximum shearing stress occurs for $\sigma_{x'} = \sigma_{ave}$



$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Note : defines two angles separated by 90° and offset from θ_p by 45°

$$\sigma' = \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$\tan 2\theta_p \tan 2\theta_s = -1$$

$$2(\theta_p - \theta_s) = 90^0$$

$$\theta_p - \theta_s = 45^0$$