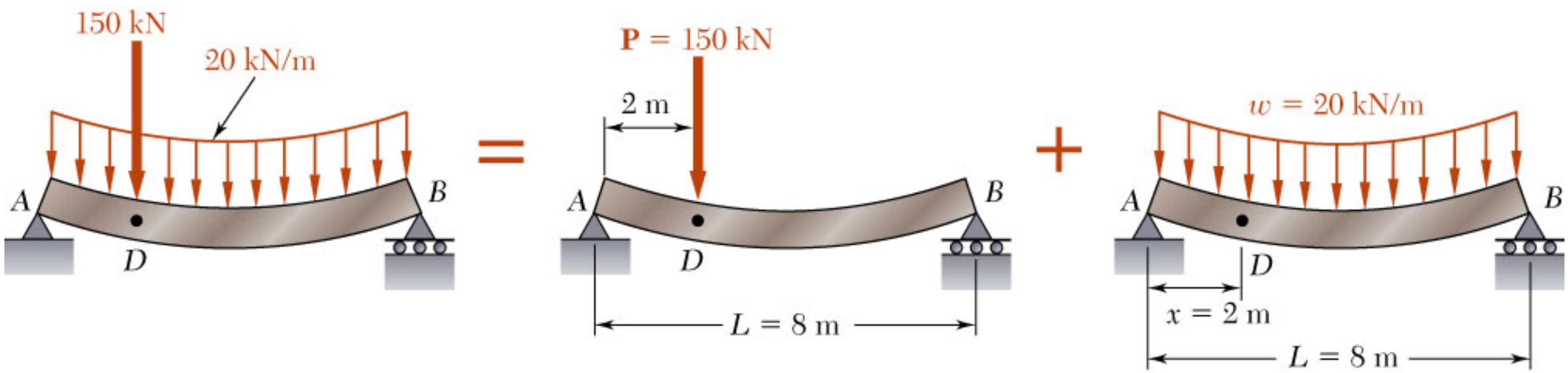
Method of Superposition



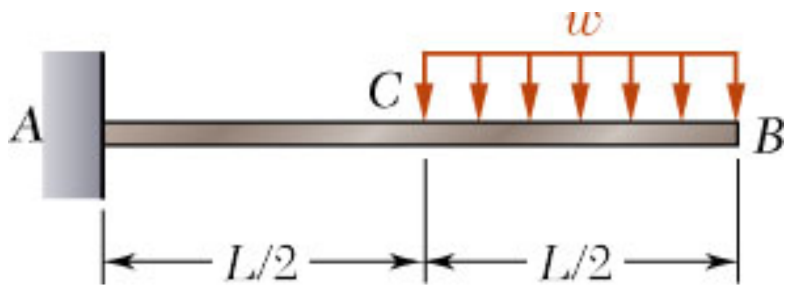
Principle of Superposition:

Deformations of beams subjected to combinations of loadings may be obtained as the linear combination of the deformations from the individual loadings

Procedure is facilitated by tables of solutions for common types of loadings and supports.

See Appendix D, Page A28

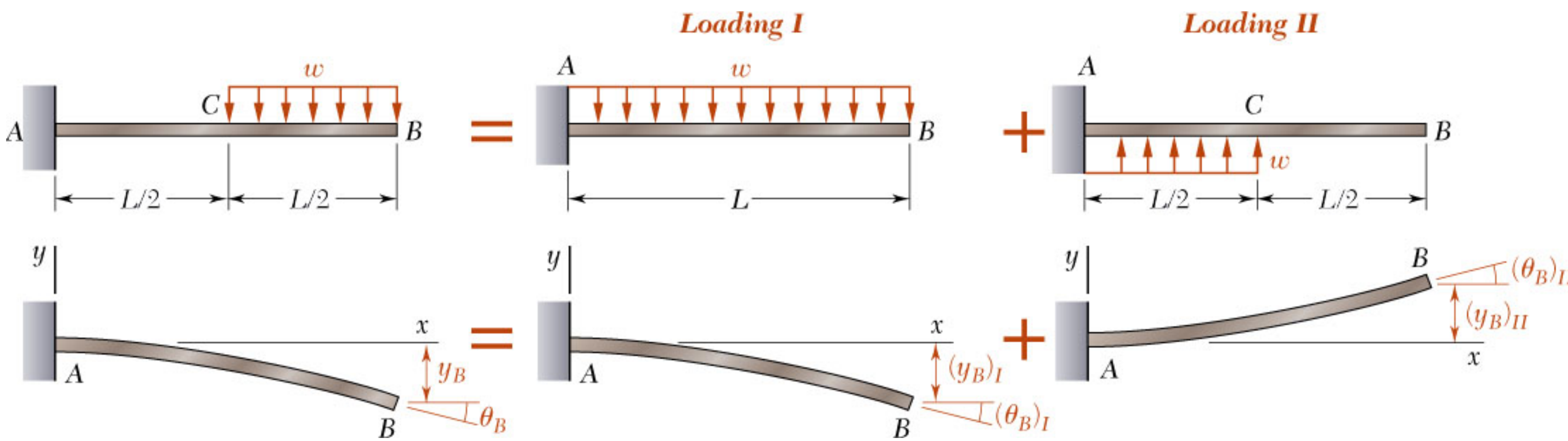
Sample Problem 9.7



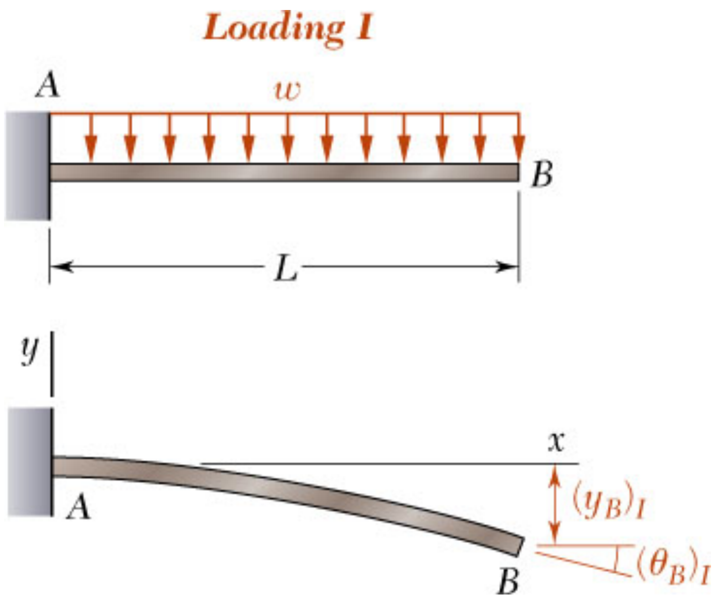
For the beam and loading shown, determine the slope and deflection at point *B*.

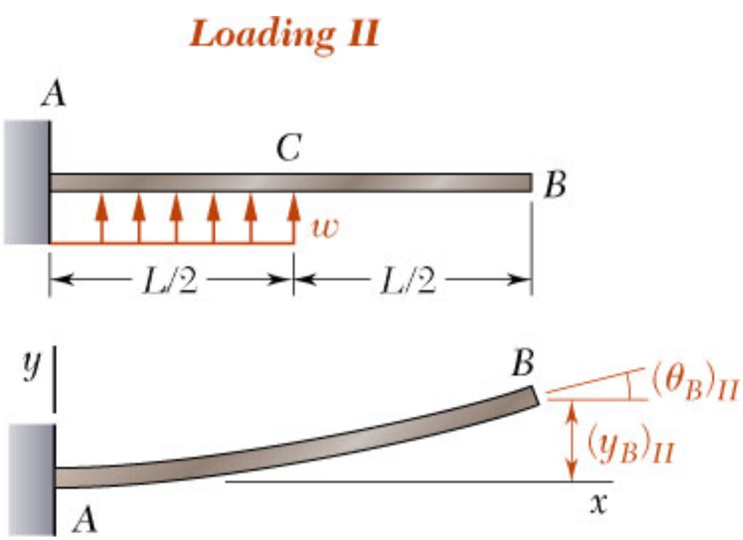
SOLUTION:

Superpose the deformations due to *Loading I* and *Loading II* as shown.



Sample Problem 9.7





Loading I

$$(\theta_B)_I = -\frac{wL^3}{6EI} \qquad (y_B)_I = -\frac{wL^4}{8EI}$$

Loading II

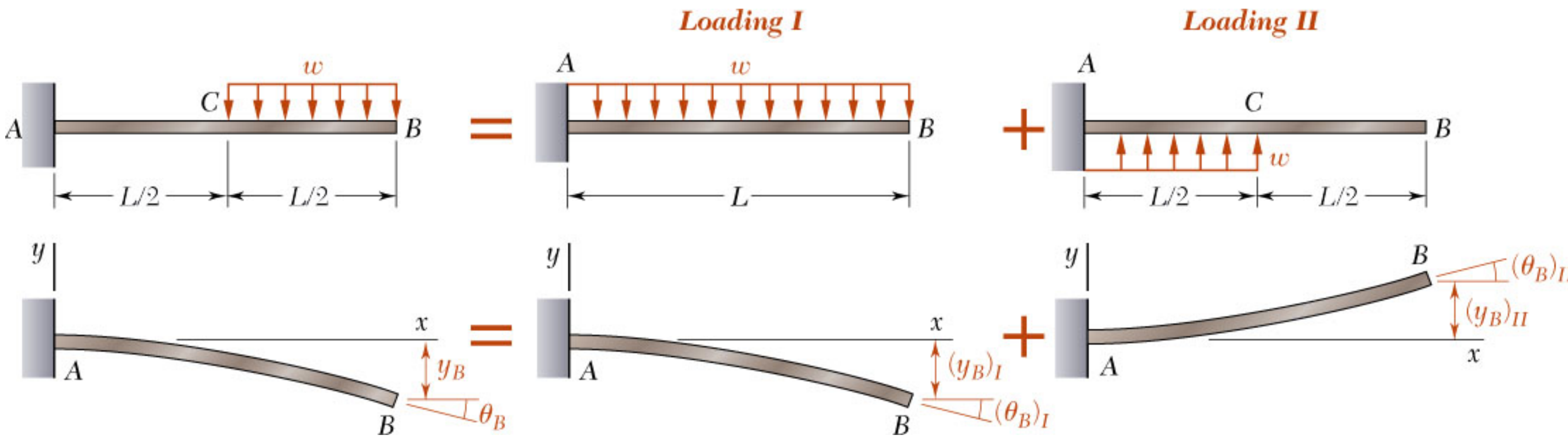
$$(\theta_C)_{II} = \frac{wL^3}{48EI} \qquad (y_C)_{II} = \frac{wL^4}{128EI}$$

In beam segment CB, the bending moment is zero and the elastic curve is a straight line.

$$(\theta_B)_{II} = (\theta_C)_{II} = \frac{wL^3}{48EI}$$

$$(y_B)_{II} = \frac{wL^4}{128EI} + \frac{wL^3}{48EI}\left(\frac{L}{2}\right) = \frac{7wL^4}{384EI}$$

Sample Problem 9.7



Combine the two solutions,

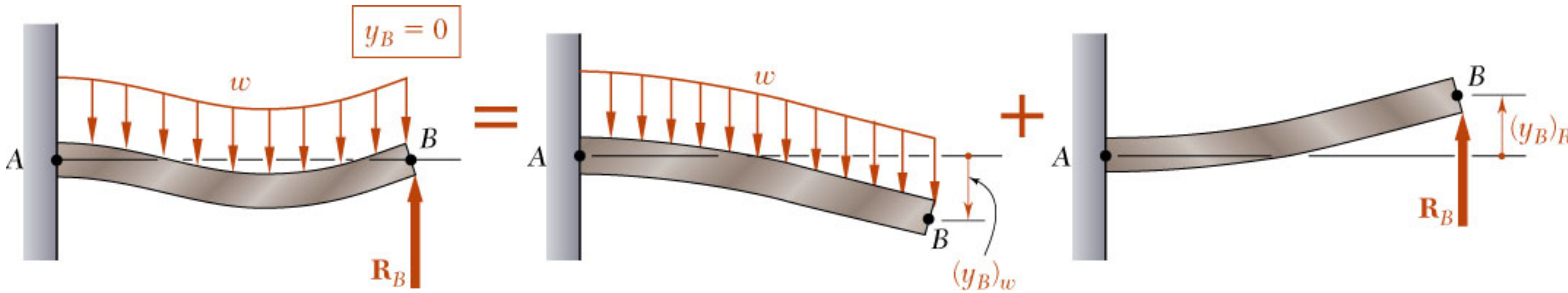
$$\theta_B = (\theta_B)_I + (\theta_B)_{II} = -\frac{wL^3}{6EI} + \frac{wL^3}{48EI}$$

$$\boxed{\theta_B = -\frac{7wL^3}{48EI}}$$

$$y_B = (y_B)_I + (y_B)_{II} = -\frac{wL^4}{8EI} + \frac{7wL^4}{384EI}$$

$$\boxed{y_B = -\frac{41wL^4}{384EI}}$$

Application of Superposition to Statically Indeterminate Beams



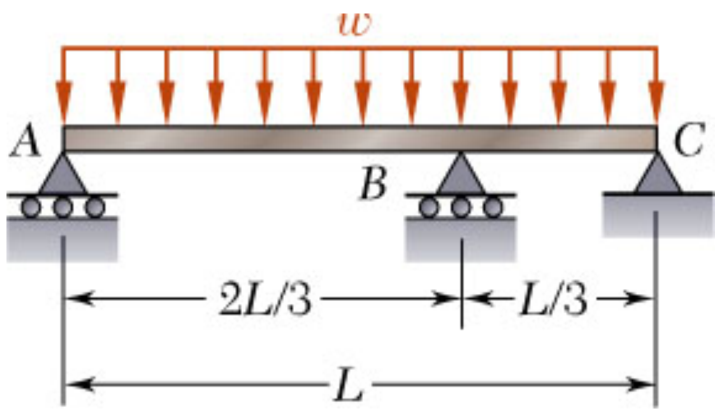
Method of superposition may be applied to determine the reactions at the supports of statically indeterminate beams.

Designate one of the reactions as redundant and eliminate or modify the support.

Determine the beam deformation without the redundant support.

Treat the redundant reaction as an unknown load which, together with the other loads, must produce deformations compatible with the original supports.

Sample Problem 9.8

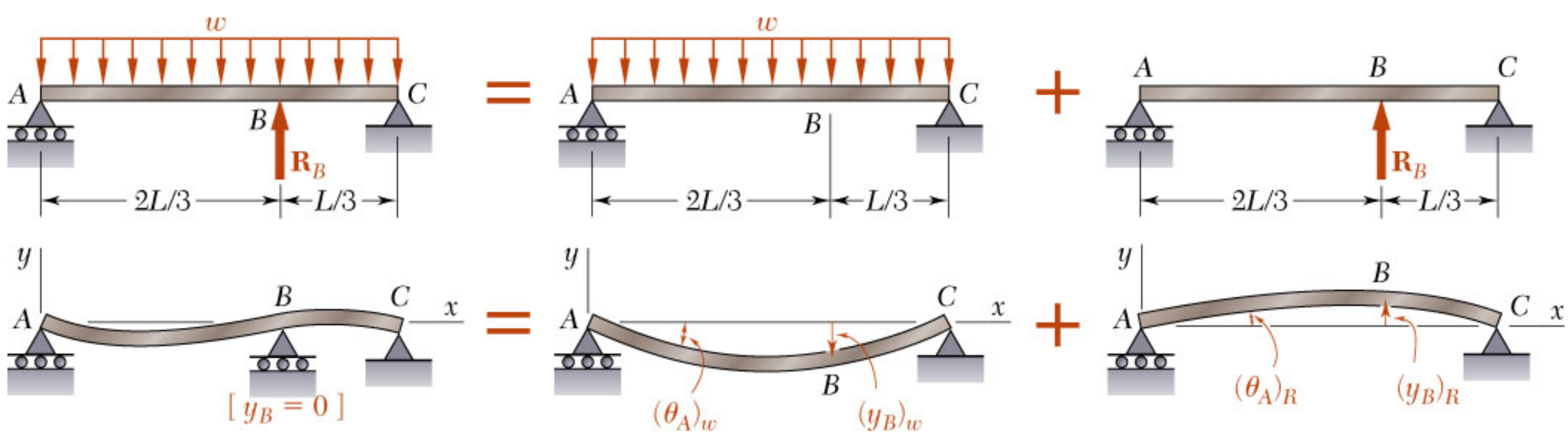


For the uniform beam and loading shown, determine the reaction at each support and the slope at end *A*.

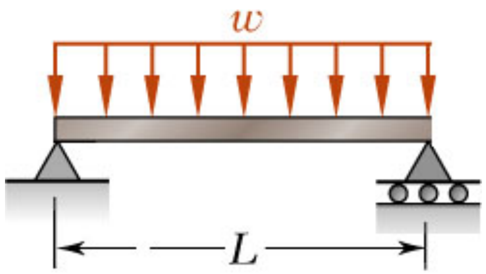
SOLUTION:

Release the "redundant" support at B, and find deformation.

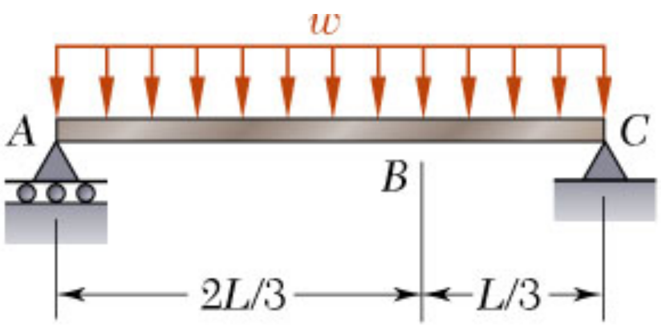
Apply reaction at *B* as an unknown load to force zero displacement at *B*.

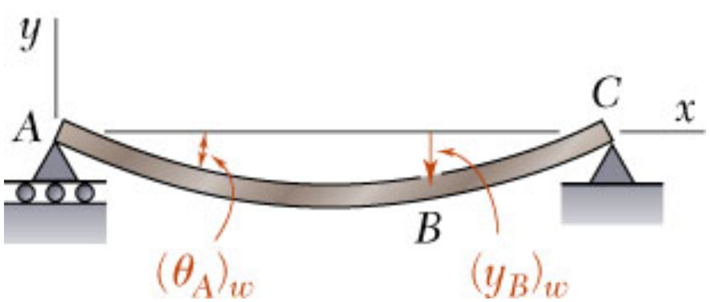


Sample Problem 9.8



$$(y_B)_w = -\frac{w}{24EI}\left[x^4 - 2Lx^3 + L^3 x\right]$$





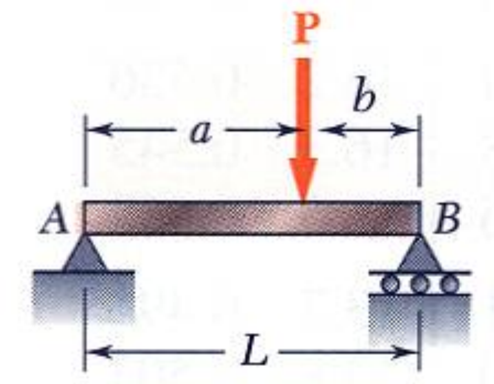
Distributed Loading:

At point *B*, $x = \frac{2}{3}L$

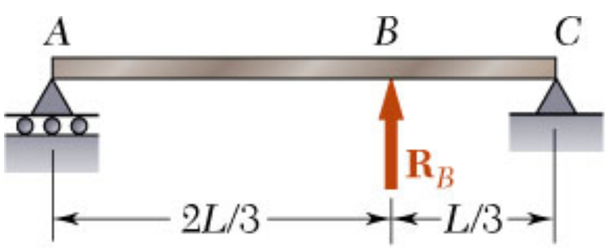
$$(y_B)_w = -\frac{w}{24EI}\left[\left(\frac{2}{3}L\right)^4 - 2L\left(\frac{2}{3}L\right)^3 + L^3\left(\frac{2}{3}L\right)\right]$$

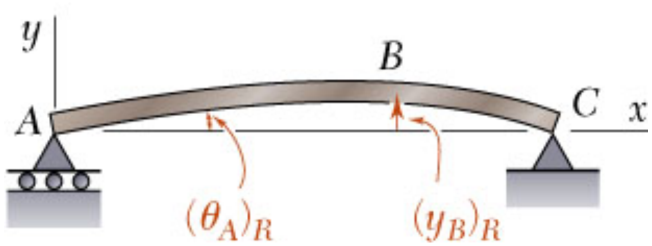
$$= -0.01132\frac{wL^4}{EI}$$

Sample Problem 9.8



$$\text{At } x = a, \quad y = -\frac{Pa^2b^2}{3EIL}$$



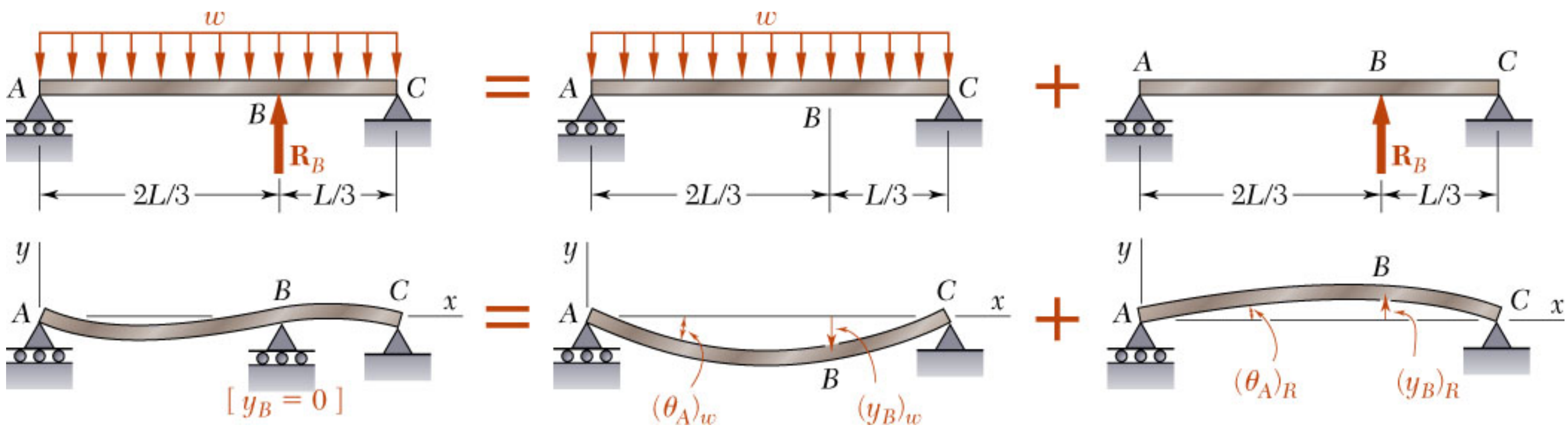


For $a = \frac{2}{3}L$ and $b = \frac{1}{3}L$

$$(y_B)_R = \frac{R_B}{3EIL}\left(\frac{2}{3}L\right)^2\left(\frac{L}{3}\right)^2$$

$$= 0.01646\frac{R_BL^3}{EI}$$

Sample Problem 9.8



For compatibility with original supports, $y_B = 0$

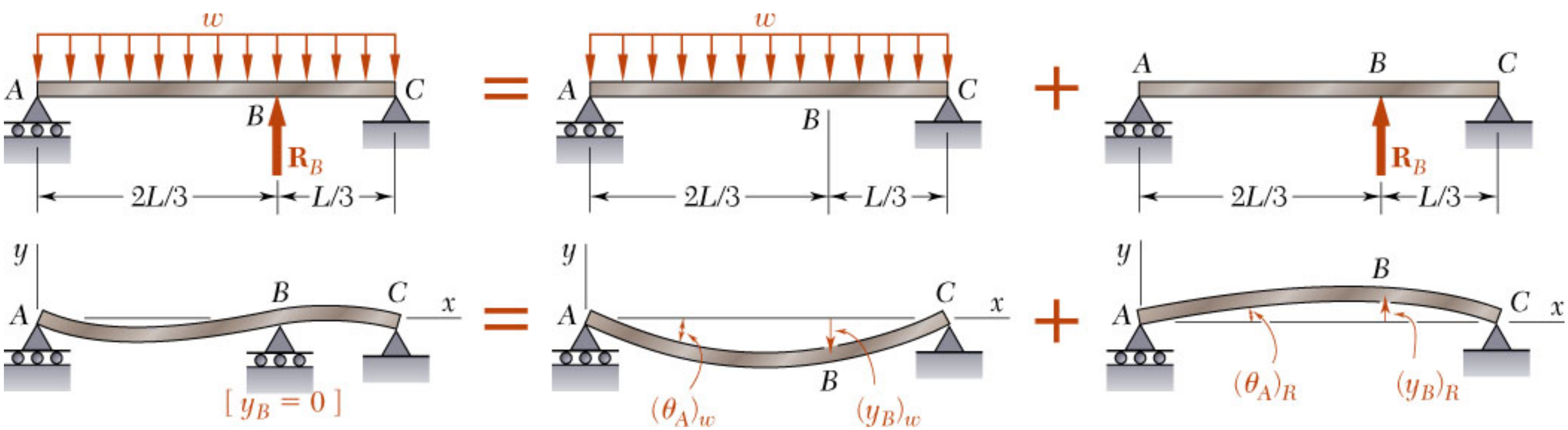
$$0 = (y_B)_w + (y_B)_R = -0.01132\frac{wL^4}{EI} + 0.01646\frac{R_B L^3}{EI}$$

$$R_B = 0.688wL \uparrow$$

From statics,

$$R_A = 0.271wL \uparrow \qquad R_C = 0.0413wL \uparrow$$

Sample Problem 9.8



Slope at end *A*,

$$(\theta_A)_w = -\frac{wL^3}{24EI} = -0.04167\frac{wL^3}{EI}$$

$$(\theta_A)_R = -\frac{Pb\left(L^2 - b^2\right)}{6EIL} = \frac{0.0688wL}{6EIL}\left(\frac{L}{3}\right)\left[L^2 - \left(\frac{L}{3}\right)^2\right] = 0.03398\frac{wL^3}{EI}$$

$$\theta_A = (\theta_A)_w + (\theta_A)_R = -0.04167\frac{wL^3}{EI} + 0.03398\frac{wL^3}{EI}$$

$$\boxed{\theta_A = -0.00769\frac{wL^3}{EI}}$$