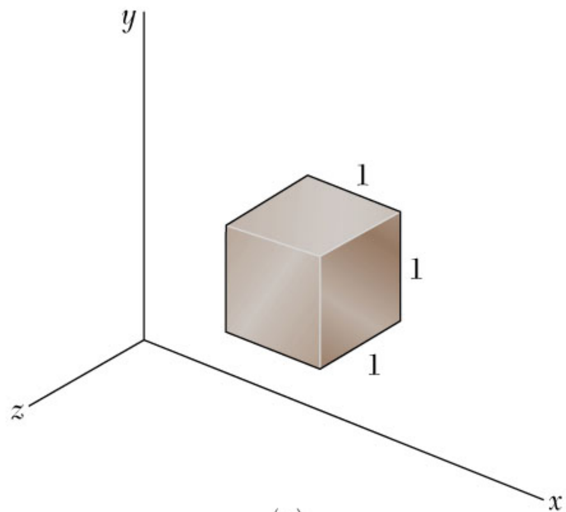
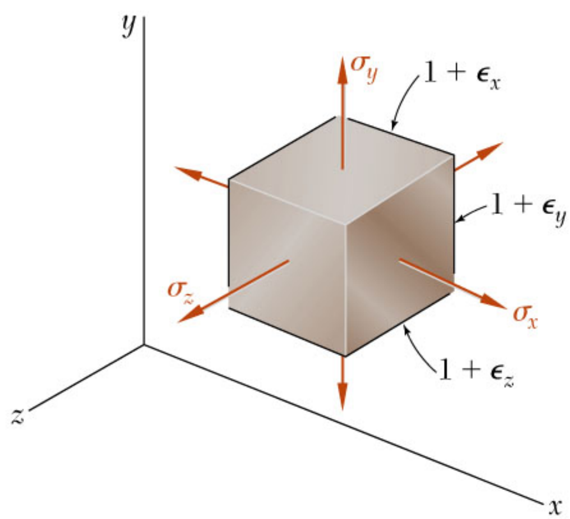
Generalized Hooke's Law



(a)



(b)

For an element subjected to multi-axial loading, the normal strain components resulting from the stress components may be determined from the *principle of superposition*. This requires:

1) strain is linearly related to stress
2) deformations are small

With these restrictions:

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

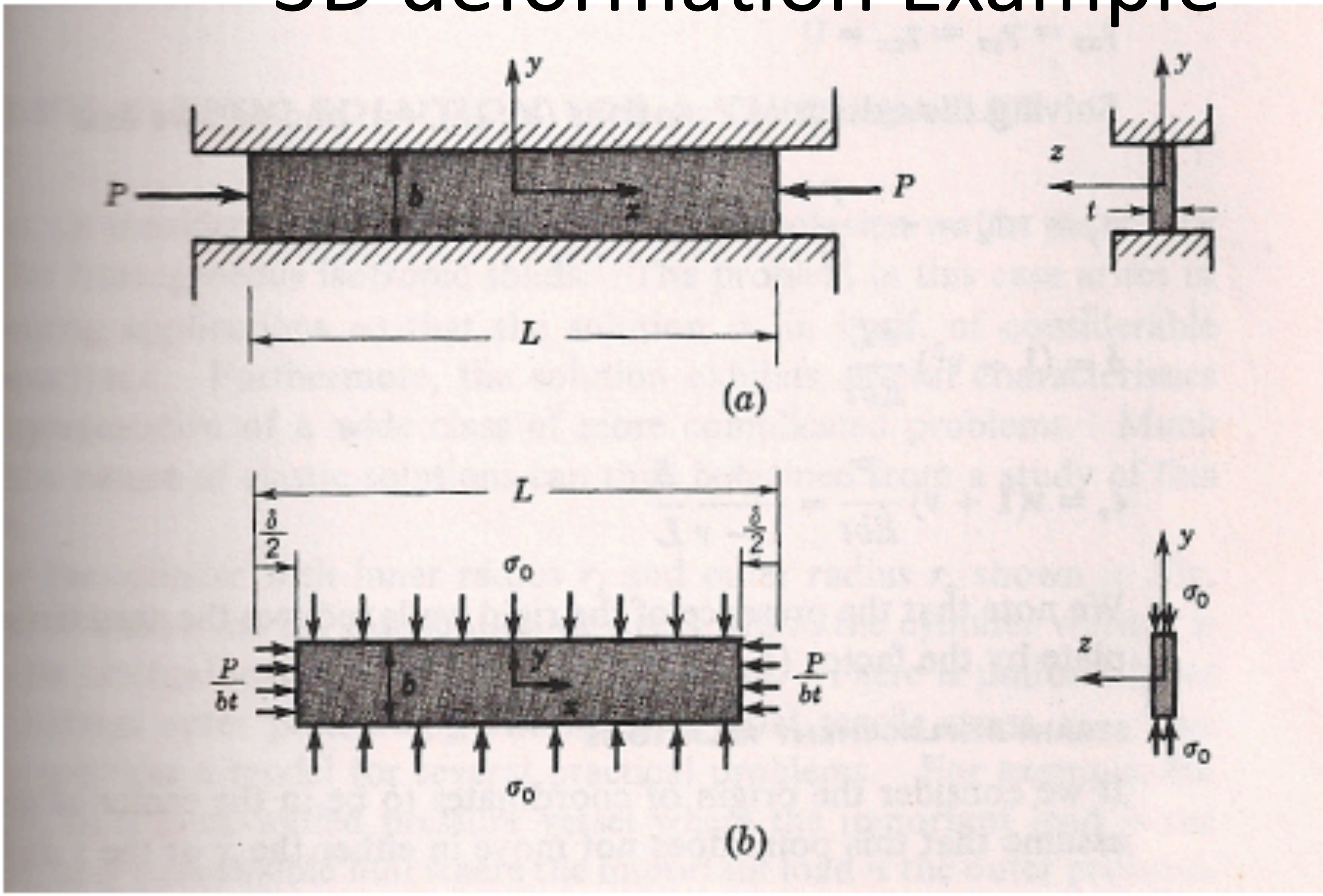
$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

Negative Poisson Ratio Material

http://www.youtube.com/watch?v=nDuR9hHIpZM

3D deformation Example



- What is the deflection in direction parallel to P ?

Sign up for lab

- sign up sheets posted outside BH 604 (near the hw drop off box)
- groups of upto 6

Shearing Strain

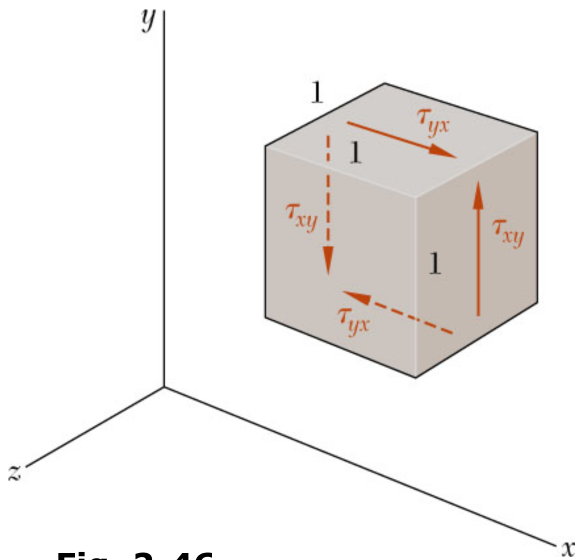


Fig. 2-46

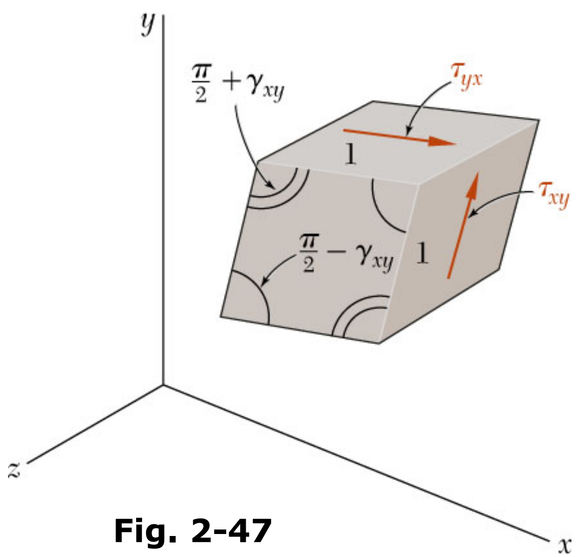


Fig. 2-47

A cubic element subjected to a shear stress will deform into a rhomboid. The corresponding *shear* strain is quantified in terms of the change in angle between the sides,

$$\tau_{xy} = f\left(\gamma_{xy}\right)$$

A plot of shear stress vs. shear strain is similar to the previous plots of normal stress vs. normal strain except that the strength values are approximately half. For small strains,

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

where *G* is the modulus of rigidity or shear modulus.

3D stress-strain temperature relations

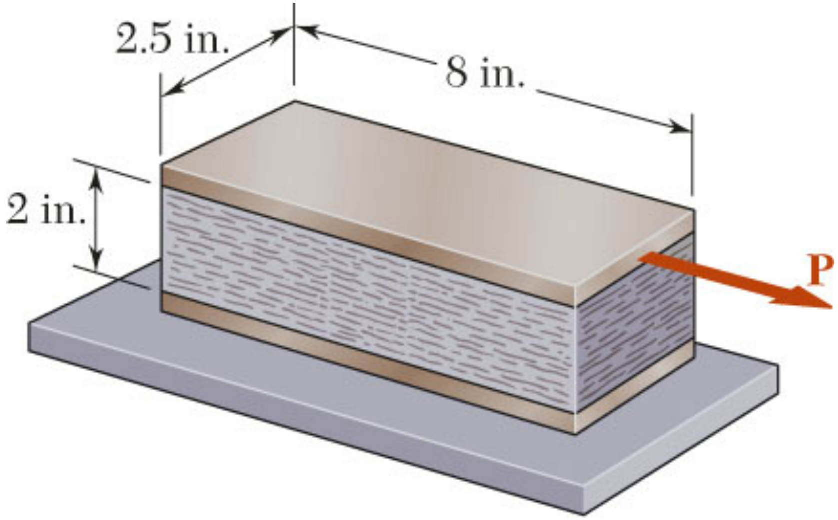
$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} + \alpha\Delta T$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E} + \alpha\Delta T$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E} + \alpha\Delta T$$

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{zx} = G\gamma_{zx}$$

Example 2.10



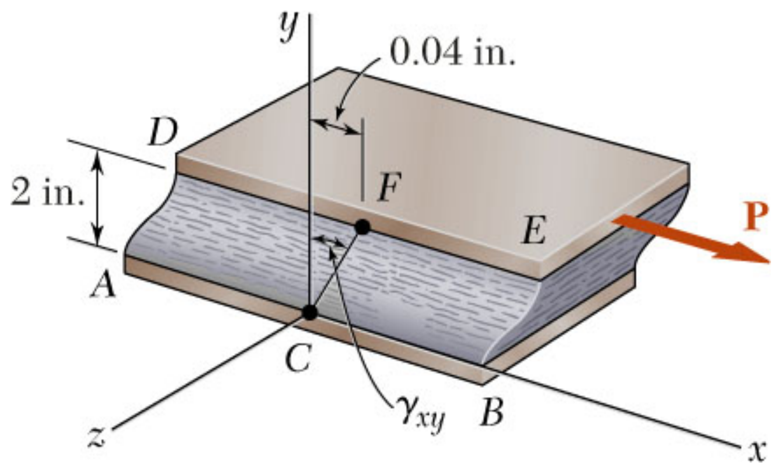
A rectangular block of material with modulus of rigidity G = 90 ksi is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force P. Knowing that the upper plate moves through 0.04 in. under the action of the force, determine a) the average shearing strain in the material, and b) the force P exerted on the plate.

SOLUTION:

Determine the average angular deformation or shearing strain of the block.

Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.

Use the definition of shearing stress to find the force P.



Determine the average angular deformation or shearing strain of the block.

$$\gamma_{xy} \approx \tan \gamma_{xy} = \frac{0.04\,\text{in.}}{2\,\text{in.}} \qquad \gamma_{xy} = 0.020\,\text{rad}$$

Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.

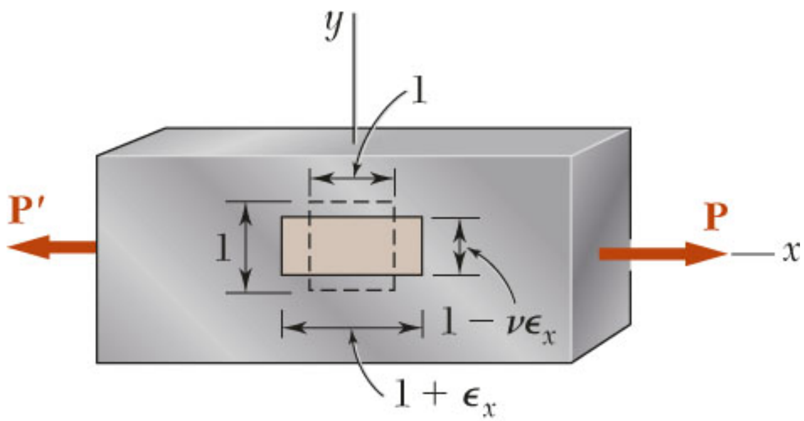
$$\tau_{xy} = G\gamma_{xy} = \left(90 \times 10^3\,\text{psi}\right)\left(0.020\,\text{rad}\right) = 1800\,\text{psi}$$

Use the definition of shearing stress to find the force *P*.

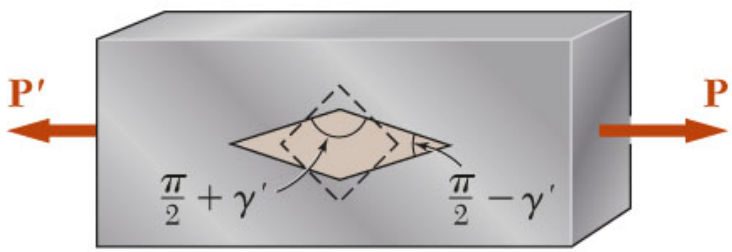
$$P = \tau_{xy} A = \left(1800\,\text{psi}\right)\left(8\,\text{in.}\right)\left(2.5\,\text{in.}\right) = 36 \times 10^3\,\text{lb}$$

$$\boxed{P = 36.0\,\text{kips}}$$

Relation Among *E*, ν, and *G*



(a)



(b)

An axially loaded slender bar will elongate in the axial direction and contract in the transverse directions.

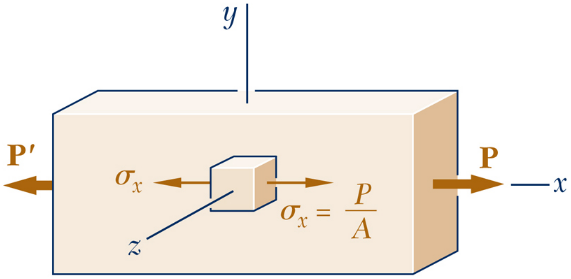
An initially cubic element oriented as in top figure will deform into a rectangular parallelepiped. The axial load produces a normal strain.

If the cubic element is oriented as in the bottom figure, it will deform into a rhombus. Axial load also results in a shear strain.

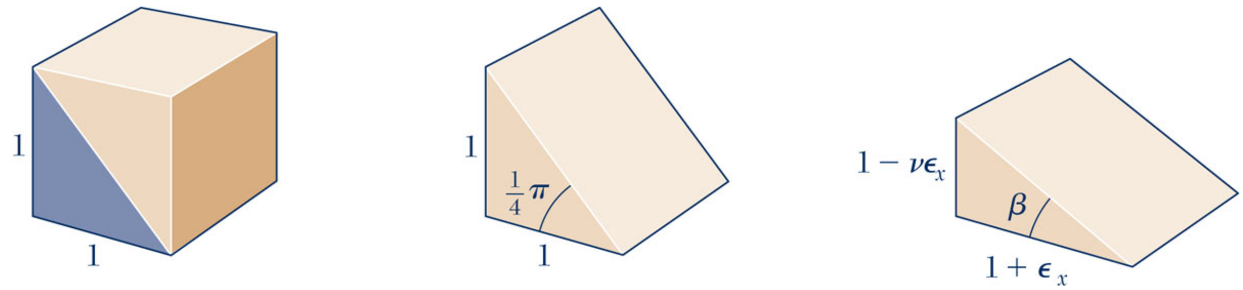
Components of normal and shear strain are related,

$$\frac{E}{2G} = (1 + \nu)$$

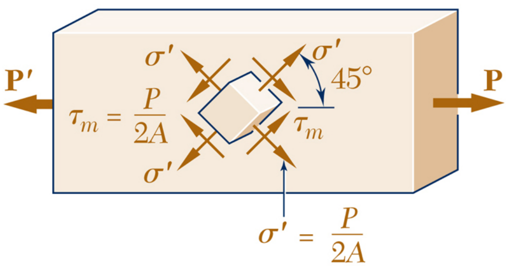
Relation Among E, ν, and G



(a)



(a) (b) (c)



(b)

$$\gamma_m = (1+\nu)\varepsilon_x \quad \Rightarrow \frac{\tau_m}{G} = (1+\nu)\frac{\sigma_x}{E} \quad \Rightarrow \frac{E}{G} = (1+\nu)\frac{\sigma_x}{\tau_m}$$

$$\frac{E}{2G} = (1+\nu)$$

E and ν are the only two independent elastic constants for isotropic materials

Read Sec. 2.15