

HW10_Sol

SOLUTION 10.4

Let δ be the deflection of point C .

Using free body AC and

$$+\curvearrowright \sum M_C = 0: -\frac{1}{3}LR_A + P\delta = 0 \quad R_A = \frac{3P\delta}{L}$$

Using free body BC and

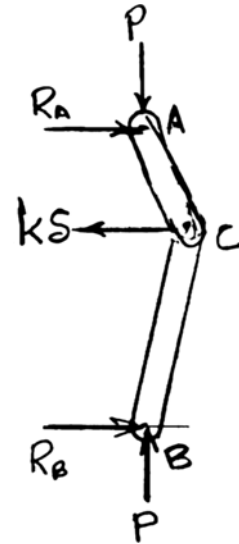
$$+\curvearrowright \sum M_C = 0: \frac{2}{3}LR_B - P\delta = 0 \quad R_B = \frac{3P\delta}{2L}$$

Using both free bodies together,

$$\pm \rightarrow \sum F_x = 0: R_A + R_B - k\delta = 0$$

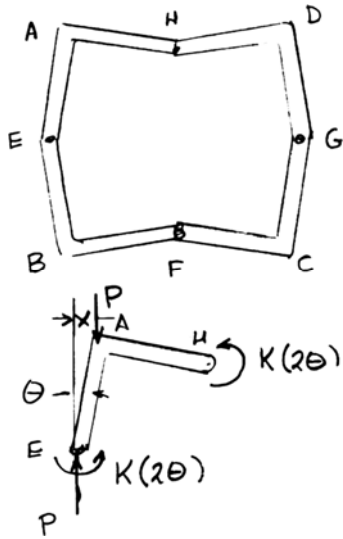
$$\frac{3P\delta}{L} + \frac{3P\delta}{2L} - k\delta = 0$$

$$\left(\frac{9P}{2L} - k \right) \delta = 0$$



$$P_{cr} = \frac{2kL}{9} \quad \blacktriangleleft$$

SOLUTION 10.8



Let θ be the rotation of each L-shaped member.

Angle change across each torsional spring is 2θ .

$$x = \frac{1}{2}L \sin \theta \approx \frac{1}{2}L\theta$$

$$\Sigma M_E = 0:$$

$$K(2\theta) + K(2\theta) - Px = 0$$

$$P_{cr} = \frac{4K\theta}{x}$$

$$P_{cr} = \frac{8K}{L} \blacktriangleleft$$

SOLUTION 10.13

Arrangement (a).

$$I_a = \frac{1}{12}d^4$$

$$P_{cr,a} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 Ed^4}{12 L_e^2}$$

Arrangement (b).

$$I_{\min} = I_y = \frac{1}{12}\left(\frac{d}{3}\right)(d^3) + \frac{1}{12}(d)\left(\frac{d}{3}\right)^3 + \frac{1}{12}\left(\frac{d}{3}\right)(d^3) = \frac{19}{324}d^4$$

$$P_{cr,b} = \frac{\pi^2 EI}{L_e^2} = \frac{19\pi^2 Ed^4}{324 L_e^2}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = \frac{1}{12} \cdot \frac{324}{19} = \frac{27}{19}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = 1.421 \blacktriangleleft$$

SOLUTION 10.20

Joint B: From force triangle,

$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$F_{AB} = 3.1079 \text{ kN (Comp)}$$

$$F_{BC} = 2.5152 \text{ kN (Comp)}$$

Member AB:

$$I_{AB} = \frac{\pi \left(\frac{d}{2}\right)^4}{4} = \frac{\pi \left(\frac{18}{2}\right)^4}{4} = 5.153 \times 10^3 \text{ mm}^4$$

$$= 5.153 \times 10^{-9} \text{ m}^4$$

$$F_{AB,cr} = \frac{\pi^2 EI_{AB}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9) (5.153 \times 10^{-9})}{(1.2)^2}$$

$$= 7.0636 \times 10^3 \text{ N} = 7.0636 \text{ kN}$$

$$F.S. = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

Member BC:

$$I_{BC} = \frac{\pi \left(\frac{d}{2}\right)^4}{4} = \frac{\pi \left(\frac{22}{2}\right)^4}{4}$$

$$= 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$$

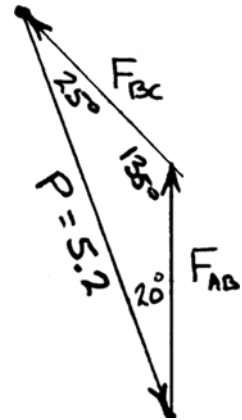
$$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$$

$$F_{BC,cr} = \frac{\pi^2 EI_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-9})}{2.88}$$

$$= 7.8813 \times 10^3 \text{ N} = 7.8813 \text{ kN}$$

$$F.S. = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest $F.S.$ governs.



$F.S. = 2.27$ ◀

SOLUTION 10.25

$$\text{W10} \times 22: I_x = 118 \text{ in}^4$$

$$I_y = 11.4 \text{ in}^4$$

$$P = 15 \times 10^3 \text{ lb}$$

$$P_{\text{cr}} = (F.S.)P = (2.2)(15 \times 10^3) = 33 \times 10^3 \text{ lb}$$

Buckling in xz -plane.

$$L_e = 0.7L$$

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(0.7L)^2} \quad L = \frac{\pi}{0.7} \sqrt{\frac{EI_y}{P_{\text{cr}}}}$$

$$L = \frac{\pi}{0.7} \sqrt{\frac{(29 \times 10^6)(11.4)}{33 \times 10^3}} = 449.2 \text{ in.}$$

Buckling in yz -plane.

$$L_e = 2L$$

$$P_{\text{cr}} = \frac{\pi^2 EI_x}{(2L)^2}$$

$$L = \frac{\pi}{2} \sqrt{\frac{EI_x}{P_{\text{cr}}}} = \frac{\pi}{2} \sqrt{\frac{(29 \times 10^6)(118)}{33 \times 10^3}} = 505.8 \text{ in.}$$

Smaller value for L governs. $L = 449.2 \text{ in.}$

$L = 37.4 \text{ ft} \blacktriangleleft$