HW9_Sol

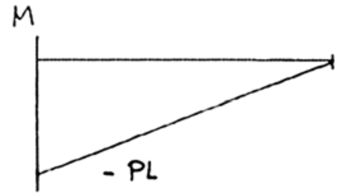
SOLUTION 11.63

Bending moment: $M = -Pv$

<u>Over AB</u>:

$$U_{AB} = \int_{L/2}^{L} \frac{M^2}{4EI} dv = \frac{P^2}{4EI} \int_{L/2}^{L} v^2 dv$$

$$= \frac{P^2}{12EI} \left[L^3 - \left(\frac{L}{2} \right)^3 \right] = \frac{7}{96} \frac{P^2 L^3}{EI}$$



<u>Over BC</u>:

$$U_{BC} = \int_{0}^{L/2} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_{0}^{L/2} v^2 dv$$

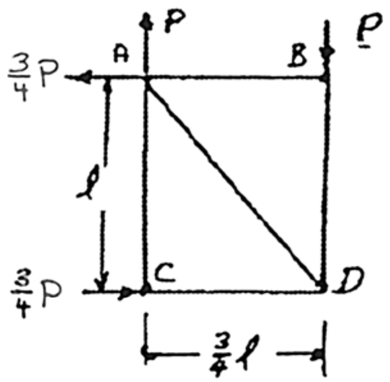
$$= \frac{1}{48} \frac{P^2 L^3}{EI}$$

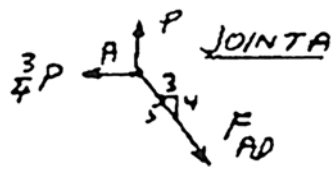
<u>Total</u>:

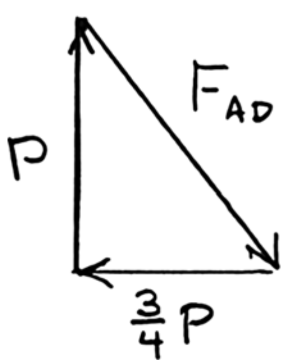
$$U = U_{AB} + U_{BC} = \frac{3}{32} \frac{P^2 L^3}{EI}$$

$$\frac{1}{2} P\delta_C = 0 \qquad \delta_C = \frac{2U}{P} \qquad \delta_C = \frac{3PL^3}{16EI} \downarrow \blacktriangleleft$$

SOLUTION 11.71







Reactions: $C = \frac{3}{4}P \rightarrow, \quad A_x = \frac{3}{4}P \leftarrow, \quad A_y = P \uparrow$

Members *AB* and *AC* are zero force members.

$$F_{BD} = -P$$

$$F_{CD} = \frac{3}{4}P$$

Equilibrium of joint *A*.

Using the force triangle,

$$F_{AD}^2 = P^2 + \left(\frac{3}{4}P\right)^2 = \frac{25}{16}P^2$$

$$F_{AD} = \frac{5}{4}P \quad \text{(tension)}$$

Strain energy: $U = \sum \frac{F_i^2 L_i}{2EA}$

	F_i	L_i	$F_i^2 L_i$
AD	$+\frac{5}{4}P$	$\frac{5}{4}\ell$	$1.9531P^2\ell$
BD	$-P$	ℓ	$P^2\ell$
CD	$-\frac{3}{4}P$	$\frac{3}{4}\ell$	$0.4219P^2\ell$

$$\sum = 3.375P^2\ell$$

$$U = \frac{3.375P^2\ell}{2EA}$$

$$\frac{1}{2}Py_B = U \qquad y_B = \frac{2U}{P}$$

Deflection at *B*. $y_B = 3.375\frac{P\ell}{EA}$ ◀

SOLUTION 11.92

Units: Forces in kips; lengths in ft.

$$E = 29 \times 10^3 \text{ ksi} \qquad I = 291 \text{ in}^4$$

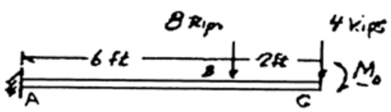
$$EI = (29 \times 10^3)(291) = 8.439 \times 10^6 \text{ kip} \cdot \text{in}^2 = 58.604 \times 10^3 \text{ kip} \cdot \text{ft}^2$$

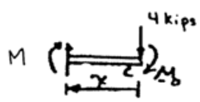
Add clockwise couple M_0 at end C.

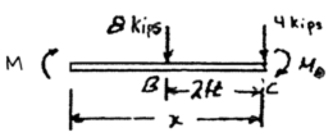
Strain energy: $$U = \int \frac{M^2}{2EI} dx = \frac{1}{2EI} \int M^2 dx$$

Slope at C (formula). $$\theta_C = \frac{\partial U}{\partial M_0} = \frac{1}{EI} \int M \frac{\partial M}{\partial M_0} dx$$

Bending moment diagram.







Portion BC: $(0 \le x \le 2 \text{ ft})$

$$M = -4x - M_0$$

$$\frac{\partial M}{\partial M_0} = -1; \qquad \text{Set } M_0 = 0.$$

$$\int_0^2 M \frac{\partial M}{\partial M_0} dx = \int_0^2 (-4x)(-1) dx = \frac{4}{2} x^2 \Big|_0^2 = 8 \text{ kip} \cdot \text{ft}^2$$

Portion AB: $(2 \text{ ft} \le x \le 8 \text{ ft})$

$$M = -8(x-2) - 4x - M_0$$

$$\frac{\partial M}{\partial M_0} = -1; \qquad \text{Set } M_0 = 0.$$

$$\int_2^8 M \frac{\partial M}{\partial M_0} dx = \int_2^8 [-8(x-2-4x](-1) dx$$

$$= \frac{8}{2}(x-2)^2 \Big|_2^8 + \frac{4}{2} x^2 \Big|_2^8 = (4)(6)^2 + (2)(64-4) = 264 \text{ kip} \cdot \text{ft}^2$$

Slope at end C. (calculated)

$$\theta_C = \frac{8 + 264}{EI} = \frac{272}{58.604 \times 10^3} \qquad \theta_C = 4.64 \times 10^{-3} \text{ rad} \; \blacktriangleleft$$

SOLUTION 11.93

SOLUTION

<u>Units</u>: Forces in kips; lengths in ft.

$$E = 29 \times 10^3 \text{ ksi} \quad I = 291 \text{ in}^4$$

$$EI = (29 \times 10^3)(291) = 8.439 \times 10^6 \text{kip} \cdot \text{in}^2 = 58.604 \times 10^3 \text{ kip} \cdot \text{ft}^2$$

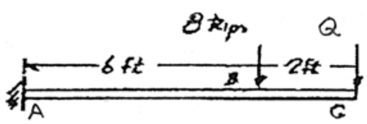
Let Q be the force at end C. It is later set equal to 4 kips.

<u>Strain energy:</u> $U = \int \frac{M^2}{2EI} dx = \frac{1}{2EI} \int M^2 dx$

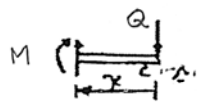
<u>Deflection at C</u> (formula).

$$\delta_C = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int M \frac{\partial M}{\partial Q} dx$$

<u>Bending moment diagram.</u>



<u>Portion BC</u>: $(0 \le x \le 2 \text{ ft})$

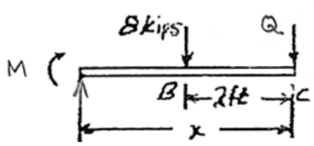


$$M = -Qx$$

$$\frac{\partial M}{\partial Q} = -x \qquad \text{Set } Q = 4 \text{ kips.}$$

$$\int_0^2 M \frac{\partial M}{\partial M_o} dx = \int_0^2 (-4x)(-x) dx$$

$$= \frac{4}{3} x^3 \Big|_0^2 = 10.67 \text{ kip} \cdot \text{ft}^3$$



<u>Portion AB</u>: $(2 \text{ ft} \le x \le 8 \text{ ft})$

$$M = -8(x-2) - Qx$$

$$\frac{\partial M}{\partial M_0} = -x \qquad \text{Set} \quad Q = 4 \text{ kips.}$$

$$\int_2^6 M \frac{\partial M}{\partial Q} dx = \int_2^8 [-8(x-2) - 4x](-x) dx$$

$$= 8\int_2^8 (x^2 - 2x) dx + 4\int_2^8 x^2 dx$$

$$= 8\left[\frac{x^3}{3}\Bigg|_2^8 - 16\frac{x^2}{2}\Bigg|_2^8 + 4\frac{x^3}{3}\Bigg|_2^8 \right]$$

$$= \frac{4096}{3} - \frac{64}{3} - \frac{1024}{2} + \frac{64}{2} + \frac{2048}{3} - \frac{32}{3}$$

$$= 1536 \text{ kip} \cdot \text{ft}^3$$

Deflection at *C*. (calculated)

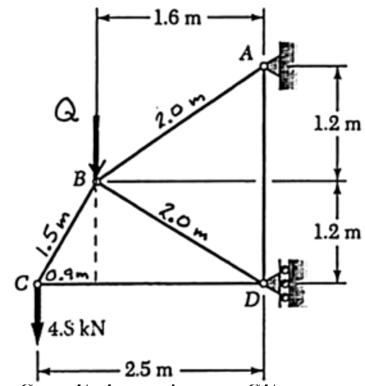
$$\delta_C = \frac{1536 + 10.67}{EI} = \frac{1546.67}{58.604 \times 10^3} = 0.0264 \text{ ft}$$

$$\delta_C = 0.317 \text{ in.} \downarrow \blacktriangleleft$$

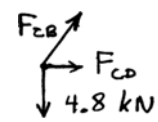
SOLUTION 11.101

Find the length of each member as shown.

Add vertical force Q at joint B.



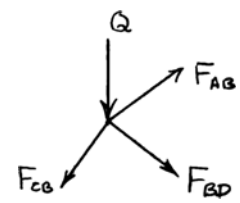
$$\delta_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\Sigma\frac{F^2L}{2EA} = \frac{1}{EA}\Sigma F\frac{\partial F}{\partial Q}L$$



Joint C:

$$+\uparrow\Sigma F_y = 0: \quad \frac{4}{5}F_{CB} - 4.8 = 0 \quad F_{CB} = 6.0 \text{ kN}$$

$$+\longrightarrow\Sigma F_x = 0: \quad \frac{3}{5}F_{CB} + F_{CD} = 0 \quad F_{CD} = -3.6 \text{ kN}$$



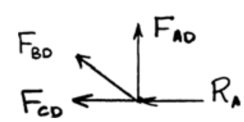
Joint B:

$$+\longrightarrow\Sigma F_x = 0: \quad \frac{4}{5}F_{AB} + \frac{4}{5}F_{BD} - 3.6 = 0$$

$$+\uparrow\Sigma F_y = 0: \quad \frac{3}{5}F_{AB} - \frac{3}{5}F_{BD} - 4.8 - Q = 0$$

Solving simultaneously,

$$F_{AB} = 6.25 + 0.8333Q \quad \text{kN}$$
$$F_{BD} = -1.75 - 0.8333Q \quad \text{kN}$$



Joint D:

$$+\uparrow\Sigma F_y = 0: \quad \frac{3}{5}F_{BD} + F_{AD} = 0$$

$$F_{AD} = -\frac{3}{5}F_{BD} = 1.05 + 0.5Q$$

Member	$F(10^3\,\text{N})$	$\partial F/\partial Q$	$L\,(\text{m})$	with $Q = 0$ $F(\partial F/\partial Q)L\ (10^3\,\text{N}\cdot\text{m})$
AB	$6.25 + 0.8333Q$	0.8333	2.0	10.4167
AD	$1.05 + 0.5Q$	0.5	2.4	1.26
BD	$-1.75 - 0.8333Q$	-0.8333	2.0	2.9167
BC	6.0	0	1.5	0
CD	-3.6	0	2.5	0
Σ				14.593

$$\delta_B = \frac{1}{EA}\Sigma F(\partial F/\partial Q)L$$

$$= \frac{14.593\times10^3}{(200\times10^9)(500\times10^{-6})}$$

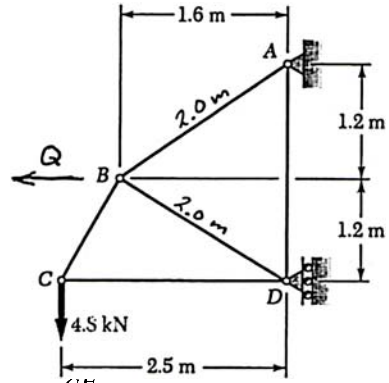
$$= 145.9\times10^{-6}\ \text{m}$$

$$\delta_B = 0.1459\ \text{mm}\downarrow \blacktriangleleft$$

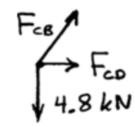
SOLUTION 11.102

Find the length of each member as shown.

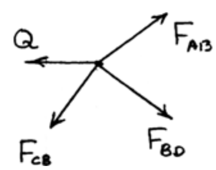
Add horizontal force Q at joint B.



$$\delta_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q}\Sigma \frac{F^2 L}{2EA} = \frac{1}{EA}\Sigma F \frac{\partial F}{\partial Q} L$$



Joint C: $\quad +\uparrow \Sigma F_y = 0: \quad \frac{4}{5}F_{CB} - 4.8 = 0 \qquad F_{CB} = 6.0 \text{ kN}$



Joint B: $\quad +\rightarrow \Sigma F_x = 0: \quad \frac{3}{5}F_{CB} + F_{CD} = 0 \qquad F_{CD} = -3.6 \text{ kN}$

$$+\rightarrow \Sigma F_x = 0: \quad \frac{4}{5}F_{AB} + \frac{4}{5}F_{BD} - 3.6 - Q = 0$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{AB} - \frac{3}{5}F_{BD} - 4.8 = 0$$

Solving simultaneously,

$$F_{AB} = 6.25 + 0.625Q \text{ kN}$$
$$F_{BD} = -1.75 + 0.625Q \text{ kN}$$

Joint D: $\quad +\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} + F_{AD} = 0$

$$F_{AD} = -\frac{3}{5}F_{BD} = 1.05 - 0.375Q$$

Member	$F\,(10^3\,\text{N})$	$\partial F/\partial Q$	$L\,(\text{m})$	$F(\partial F/\partial Q)L$ $(10^3\,\text{N}\cdot\text{m})$
AB	$6.25 + 0.625Q$	0.625	2.0	7.8125
AD	$1.05 + 0.375Q$	–0.375	2.4	–0.9450
BD	$-1.75 + 0.625Q$	0.625	2.0	–2.1875
BC	6.0	0	1.5	0
CD	–3.6	0	2.5	0
Σ				4.680

$$\delta_B = \frac{1}{EA}\Sigma F(\partial F/\partial Q)L$$

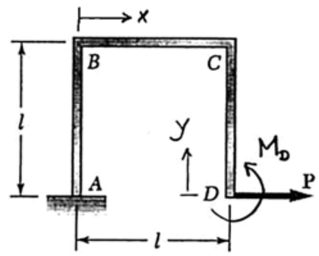
$$= \frac{4.680\times 10^3}{(200\times 10^9)(500\times 10^{-6})}$$

$$= 46.8\times 10^{-6}\,\text{m}$$

$\delta_B = 0.0468\ \text{mm} \leftarrow$ ◀

SOLUTION 11.106

Add couple M_D at point D.



Reactions at A: $\quad R_{Ay} = 0, \quad R_{Ax} = P \leftarrow, \quad M_A = M_0$ ↺

<u>Member *AB*</u>: $\quad M = M_A + R_A y = M_D + Py \qquad \dfrac{\partial M}{\partial P} = y, \qquad \dfrac{\partial M}{\partial M_D} = 1$

$$U_{AB} = \int_0^l \frac{M^2}{2EI}\,dy$$

Set $M_D = 0$.

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^l M\,\frac{\partial M}{\partial P}\,dy = \frac{1}{EI}\int_0^l (Py)\,y\,dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{AB}}{\partial M_0} = \frac{1}{EI}\int_0^l M\,\frac{\partial M}{\partial M_0}\,dy = \frac{1}{EI}\int_0^l (Py)(1)\,dy = \frac{Pl^2}{2EI}$$

<u>Member *BC*</u>: $\quad M = M_A + R_A l = M_D + Pl \qquad \dfrac{\partial M}{\partial P} = l, \qquad \dfrac{\partial M}{\partial M_D} = 1$

$$U_{BC} = \int_0^l \frac{M^2}{2EI}\,dx$$

Set $M_D = 0$.

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^l M\,\frac{\partial M}{\partial P}\,dx = \frac{1}{EI}\int_0^l (Pl)(l)\,dx = \frac{Pl^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial M_D} = \frac{1}{EI}\int_0^l M\,\frac{\partial M}{\partial M_D}\,dx = \frac{1}{EI}\int_0^l (Pl)(1)\,dx = \frac{Pl^2}{EI}$$

<u>Member *CD*</u>: $\quad M = M_D + Py \qquad \dfrac{\partial M}{\partial P} = y \qquad \dfrac{\partial M}{\partial M_D} = 1$

$$U_{CD} = \int_0^l \frac{M^2}{2EI}\,dy$$

Set $M_D = 0$.

$$\frac{\partial U_{CD}}{\partial P} = \frac{1}{EI}\int_0^l M\,\frac{\partial M}{\partial P}\,dy = \frac{1}{EI}\int_0^l (Py)(y)\,dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{CD}}{\partial M_D} = \frac{1}{EI}\int_0^l M\,\frac{\partial M}{\partial M_D}\,dy = \frac{1}{EI}\int_0^l (Py)(1)\,dy = \frac{Pl^2}{2EI}$$

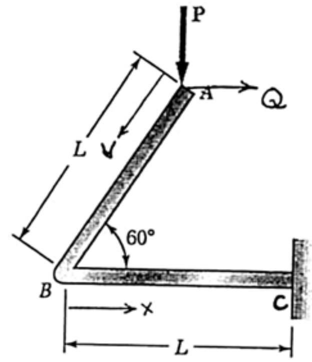
(*a*) <u>Horizontal deflection of point *D*.</u>

$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P} = \left(\frac{1}{3} + 1 + \frac{1}{3}\right)\frac{Pl^3}{EI} \qquad \delta_P = \frac{5Pl^3}{3EI} \rightarrow \blacktriangleleft$$

(*b*) <u>Slope at point *D*.</u>

$$\theta_D = \frac{\partial U_{AB}}{\partial M_D} + \frac{\partial U_{BC}}{\partial M_D} + \frac{\partial U_{CD}}{\partial M_D} = \left(\frac{1}{2} + 1 + \frac{1}{2}\right)\frac{Pl^2}{EI} \qquad \theta_D = \frac{2Pl^2}{EI} \curvearrowright \blacktriangleleft$$

SOLUTION 11.108



Add horizontal force Q at point A.

Over AB:

$$M = \frac{1}{2}Pv + \frac{\sqrt{3}}{2}Qv$$

$$\frac{\partial M}{\partial P} = \frac{1}{2}v \qquad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}v$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI}dx$$

Set $Q = 0.$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}dv = \frac{1}{EI}\int_0^L \left(\frac{1}{2}Pv\right)\left(\frac{1}{2}v\right)dv$$

$$= \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}dv = \frac{1}{EI}\int_0^L \left(\frac{1}{2}Pv\right)\frac{\sqrt{3}}{2}dv = \frac{\sqrt{3}}{12}\frac{PL^3}{EI}$$

Over BC:

$$M = -P\left(x - \frac{L}{2}\right) + \frac{\sqrt{3}}{2}QL, \qquad \frac{\partial M}{\partial P} = -\left(x - \frac{L}{2}\right), \qquad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}L$$

$$U_{BC} = \int_0^L \frac{M^2}{2EI}dx$$

Set $Q = 0.$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}dx = \frac{1}{EI}\int_0^L P\left(x - \frac{L}{2}\right)^2 dx = \frac{P}{3EI}\left(x - \frac{L}{2}\right)^3\Bigg|_0^L = \frac{1}{12}\frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}dx = \frac{-1}{EI}\int_0^L P\left(x - \frac{L}{2}\right)\left(\frac{\sqrt{3}}{2}\right)L\,dx = -\frac{\sqrt{3}P}{4EI}\left(x - \frac{L}{2}\right)^2\Bigg|_0^L = 0$$

(a) Vertical deflection of point A.

$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} \qquad\qquad \delta_P = \frac{PL^3}{6EI}\downarrow \blacktriangleleft$$

(b) Horizontal deflection of point A.

$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{\sqrt{3}}{12}\frac{PL^3}{EI} \qquad\qquad \delta_Q = 0.1443\frac{PL^3}{EI}\rightarrow \blacktriangleleft$$