HW2_Sol

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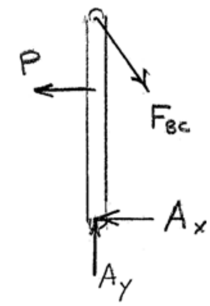
SOLUTION 2.13

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar *AB* as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$$

$$P = 0.9509F_{BC}$$

Considering allowable stress: $\sigma = 190 \times 10^6 \text{ Pa}$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

Considering allowable elongation: $\delta = 6 \times 10^{-3} \text{ m}$

$$\delta = \frac{F_{BC}L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3 \text{ N}$

$$P = 0.9509F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N}$$

$P = 1.988$ kN ◀

SOLUTION 2.22

$$\delta_{BD} = 1.6 \times 10^{-3}\text{m}, \quad A_{BD} = 1920\ \text{mm}^2 = 1920 \times 10^{-6}\text{m}^2$$

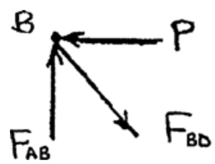
$$L_{BD} = \sqrt{5^2 + 6^2} = 7.810\ \text{m}, \quad E_{BD} = 200 \times 10^9\text{Pa}$$

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}}$$

$$F_{BD} = \frac{E_{BD}A_{BD}\delta_{BD}}{L_{BD}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81}$$

$$= 78.67 \times 10^3\text{N}$$

Use joint B as a free body. $\overset{+}{\longrightarrow} \Sigma F_x = 0$:



$$\frac{5}{7.810}F_{BD} - P = 0$$

$$P = \frac{5}{7.810}F_{BD} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

$$= 50.4 \times 10^3\text{N}$$

$P = 50.4$ kN ◀

SOLUTION 2.41

<u>*A* to *C*</u>: $E = 200 \times 10^9\ \text{Pa}$

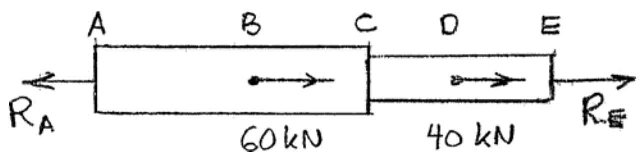
$$A = \frac{\pi}{4}(40)^2 = 1.25664 \times 10^3\ \text{mm}^2 = 1.25664 \times 10^{-3}\ \text{m}^2$$

$$EA = 251.327 \times 10^6\ \text{N}$$

<u>*C* to *E*</u>: $E = 105 \times 10^9\ \text{Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86 \times 10^{-6}\ \text{m}^2$$

$$EA = 74.220 \times 10^6\ \text{N}$$



<u>*A* to *B*</u>: $P = R_A$

$$L = 180\ \text{mm} = 0.180\ \text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327 \times 10^6}$$

$$= 716.20 \times 10^{-12} R_A$$

<u>*B* to *C*</u>: $P = R_A - 60 \times 10^3$

$$L = 120\ \text{mm} = 0.120\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.120)}{251.327 \times 10^6}$$

$$= 447.47 \times 10^{-12} R_A - 26.848 \times 10^{-6}$$

<u>*C* to *D*</u>: $P = R_A - 60 \times 10^3$

$$L = 100\ \text{mm} = 0.100\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60 \times 10^3)(0.100)}{74.220 \times 10^6}$$

$$= 1.34735 \times 10^{-9} R_A - 80.841 \times 10^{-6}$$

$\underline{D \text{ to } E}$: $P = R_A - 100 \times 10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100 \times 10^3)(0.100)}{74.220 \times 10^6}$$

$$= 1.34735 \times 10^{-9} R_A - 134.735 \times 10^{-6}$$

$\underline{A \text{ to } E}$: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.85837 \times 10^{-9} R_A - 242.424 \times 10^{-6} = 0 \quad R_A = 62.831 \times 10^3 \text{ N}$ $R_A = 62.8 \text{ kN} \leftarrow$ ◀

$R_E = R_A - 100 \times 10^3 = 62.8 \times 10^3 - 100 \times 10^3 = -37.2 \times 10^3 \text{ N}$ $R_E = 37.2 \text{ kN} \leftarrow$ ◀

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367 \times 10^{-9} R_A - 26.848 \times 10^{-6}$

$$= (1.16369 \times 10^{-9})(62.831 \times 10^3) - 26.848 \times 10^{-6}$$

$$= 46.3 \times 10^{-6} \text{ m}$$

$\delta_C = 46.3 \ \mu\text{m} \rightarrow$ ◀

SOLUTION 2.43

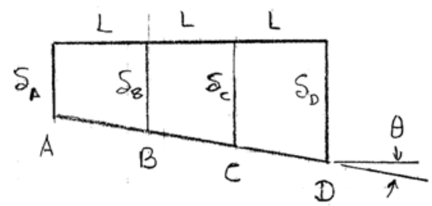
Deformations Let θ be the rotation of bar *ABCD* and δ_A, δ_B, δ_C and δ_D be the deformations of wires *A*, *B*, *C*, and *D*.

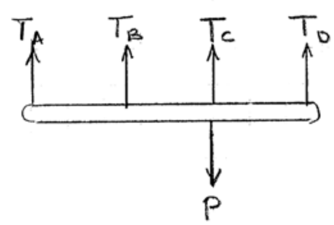
From geometry,
$$\theta = \frac{\delta_B - \delta_A}{L}$$

$$\delta_B = \delta_A + L\theta$$

$$\delta_C = \delta_A + 2L\theta = 2\delta_B - \delta_A \qquad (1)$$

$$\delta_D = \delta_A + 3L\theta = 3\delta_B - 2\delta_A \qquad (2)$$





Since all wires are identical, the forces in the wires are proportional to the deformations.

$$T_C = 2T_B - T_A \qquad (1')$$

$$T_D = 3T_B - 2T_A \qquad (2')$$

Use bar *ABCD* as a free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -2LT_A - LT_B + LT_D = 0 \qquad (3)$$

$$+\uparrow \Sigma F_y = 0: \quad T_A + T_B + T_C + T_D - P = 0 \qquad (4)$$

Substituting (2′) into (3) and dividing by *L*,

$$-4T_A + 2T_B = 0 \qquad T_B = 2T_A \qquad (3')$$

Substituting (1′), (2′), and (3′) into (4),

$$T_A + 2T_A + 3T_A + 4T_A - P = 0 \qquad 10T_A = P$$

$$T_A = \frac{1}{10}P \quad \blacktriangleleft$$

$$T_B = 2T_A = (2)\left(\frac{1}{10}\right)P \qquad T_B = \frac{1}{5}P \quad \blacktriangleleft$$

$$T_C = (2)\left(\frac{1}{5}P\right) - \left(\frac{1}{10}P\right) \qquad T_C = \frac{3}{10}P \quad \blacktriangleleft$$

$$T_D = (3)\left(\frac{1}{5}P\right) - (2)\left(\frac{1}{10}P\right) \qquad T_D = \frac{2}{5}P \quad \blacktriangleleft$$

SOLUTION 2.47

$$A_s = 6\frac{\pi}{4}d^2 = 6\frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 3.6079 \text{ in}^2$$

$$A_c = 10^2 - A_s = 10^2 - 3.6079 = 96.392 \text{ in}^2$$

Let $P_c =$ tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c.

Strains: $\varepsilon_s = -\frac{P_c}{E_s A_s} + \alpha_s(\Delta T) \qquad \varepsilon_c = \frac{P_c}{E_c A_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s \qquad \frac{P_c}{E_c A_c} + \alpha_c(\Delta T) = -\frac{P_c}{E_s A_s} + \alpha_s(\Delta T)$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right) P_c = (\alpha_s - \alpha_c)(\Delta T)$$

$$\left[\frac{1}{(3.6\times10^6)(96.392)} + \frac{1}{(29\times10^6)(3.6079)}\right] P_c = (1.0\times10^{-6})(65)$$

$$P_c = 5.2254\times10^3 \text{ lb}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{5.2254\times10^3}{96.392} = 54.210 \text{ psi} \qquad \sigma_c = 54.2 \text{ psi}$$

$$\sigma_s = -\frac{P_c}{A_s} = -\frac{5.2254\times10^3}{3.6079} = -1448.32 \text{ psi} \qquad \sigma_s = -1.448 \text{ ksi}$$

SOLUTION 2.60

$$\Delta T = 140 - 20 = 120\ °\text{C}$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)\\ &= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)\\ &= 1.347\times10^{-3}\,\text{m}\end{aligned}$$

Shortening due to P to meet constraint:

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\,\text{m}$$

$$\begin{aligned}\delta_P &= \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P\\ &= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P\\ &= 3.6447\times10^{-9}\,P\end{aligned}$$

Equating,

$$3.6447\times10^{-9}\ P = 0.847\times10^{-3}$$

$$P = 232.39\times10^3\,\text{N}$$

(*a*) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\,\text{Pa}$ $\qquad\sigma_a = -116.2\ \text{MPa}$ ◀

(*b*) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_aA_a}$

$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\,\text{m}$$

$\delta_a = 0.363\ \text{mm}$ ◀