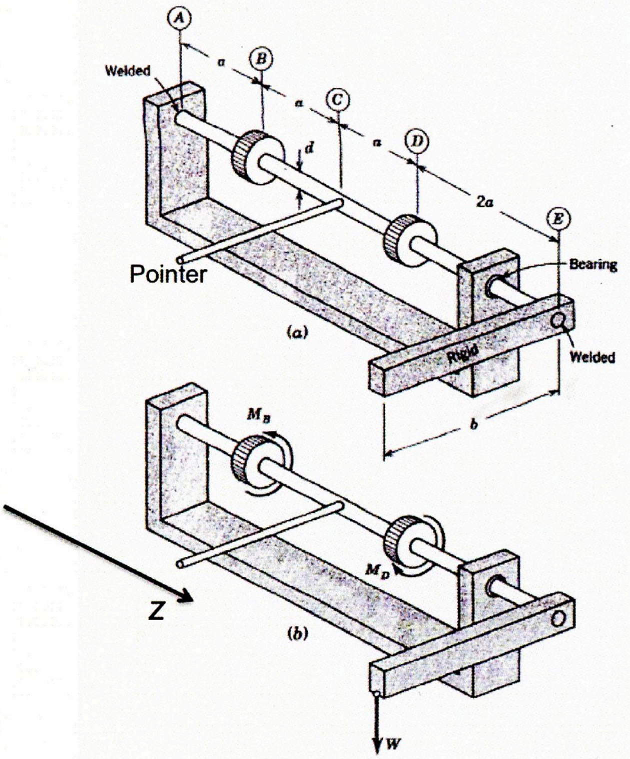
>>>>>> **Name:** <<<<<<

Problem 1:

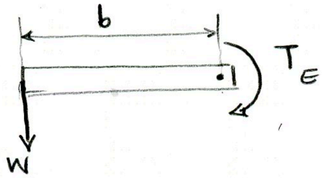
A circular shaft of a weighing machine with length *5a*, diameter *d* and shear modulus *G* is welded to a support at *A* and has a rigid arm of length *b*, welded to its other end, *E*. A pointer is fastened to the shaft at *C*. When the shaft is unloaded, the pointer and the rigid arm are horizontal, as shown in (a). A weight *W* is now hung on the end of the rigid arm and, at the same time, moments M_D and M_B are applied to the point *D* and *B*, respectively.



a) (10 Points) What is the reaction torque exerted by the support on the shaft at the point *A* ? Mark the sense clearly (+*z* or –*z*). Your answer will depend on M_D, M_B, W and *b*.

$\Sigma M_E = 0 \Rightarrow$ $\boxed{T_E = W.b}$ (2)

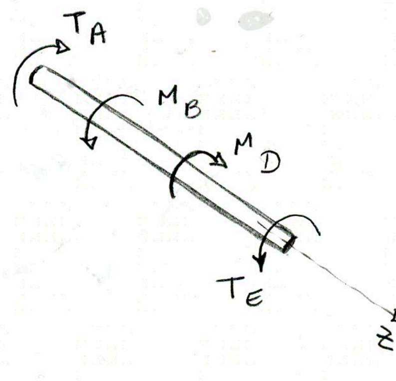
(2.5)



$\Sigma M_z^A = 0 \Rightarrow$

$-T_A + M_B - M + T_E = 0$

(2.5)



$\Rightarrow \boxed{T_A = Wb + M_B - M_D}$ (3)

T_A is along $-z$ direction

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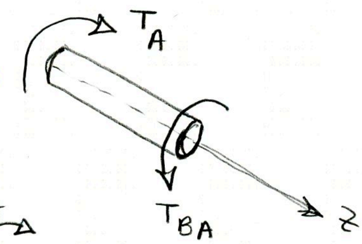
b) (15 Points) What are the torques in the shafts *AB* and *BC* (indicate the sense of the torques) ? Your answer will depend on M_D, M_B, W and b.

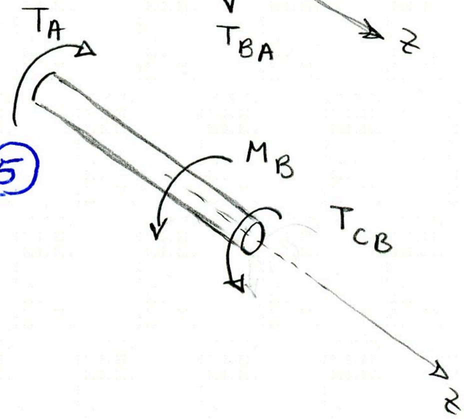
$$T_{BA} = T_A = wb + M_B - M_D$$ (2.5)

(5)



$$T_{CB} = T_A - M_B = wb - M_D$$ (2.5)

(5)



c) (25 Points) Based on your answer in b), what is the angle of rotation of the pointer ? Your answer will depend on M_D, M_B, W, b, a and d. Also, clearly mark the sense of rotation.

$$\phi_C = \phi_{C/B} + \phi_B$$

$$\phi_B = \phi_{B/A} + \phi_A \; (\phi_A \to 0) \Rightarrow \phi_B = \phi_{B/A}$$

$$\Rightarrow \phi_C = \phi_{C/B} + \phi_{B/A}$$ (5)

(10) $$\phi_{C/B} = \frac{T_{CB} \cdot a}{GJ} = \frac{(wb - M_D)a}{GJ}$$

(10) $$\phi_{B/A} = \frac{T_{BA}\, a}{GJ} = \frac{(wb + M_B - M_D)a}{GJ}$$

$$\Rightarrow \phi_C = \frac{(2wb + M_B - 2M_D)a}{GJ}$$

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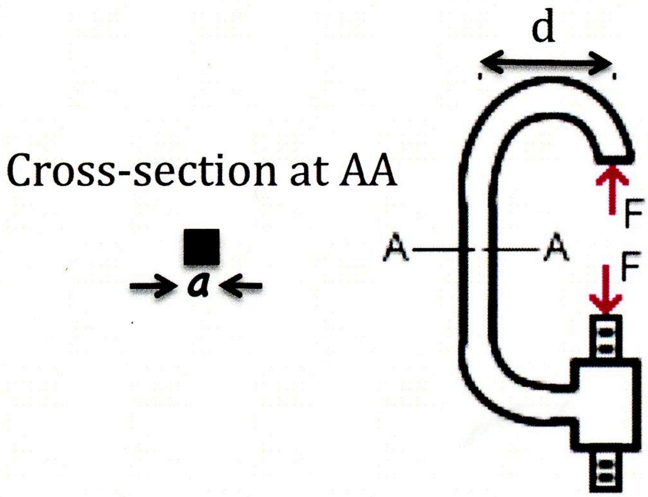
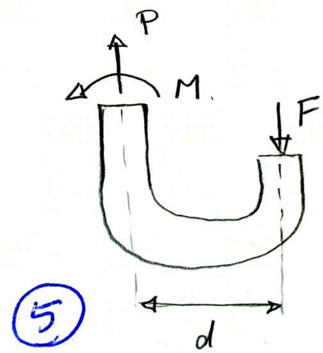
Problem 2:

(25 Points) The c-clamp shown below has a solid square cross-section of side length, *a*. When it is adjusted the force acting on each of the grips is *F*. What is the maximum stress in plane A-A? Your answer will depend on *a, F* and *d*.

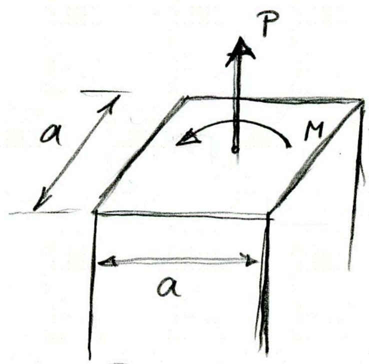




FBD

$$\Sigma M_{A-A} = 0 \Rightarrow \boxed{M = Fd}$$

$$\Sigma F_y = 0 \Rightarrow \boxed{P = F}$$



Maximum stress would be :

$$\sigma_{max} = \frac{P}{A} + \frac{M \cdot c}{I} \quad (10)$$

where (5) $\boxed{c = \frac{a}{2}, \quad A = a^2, \quad I = \frac{1}{12} a^3 \cdot a = \frac{1}{12} a^4}$

$$\Rightarrow \sigma_{max} = \frac{F}{a^2} + \frac{Fd(a/2)}{\frac{1}{12} a^4} \quad (5)$$

$$\Rightarrow \boxed{\sigma_{max} = \frac{F}{a^2} + \frac{6Fd}{a^3}}$$

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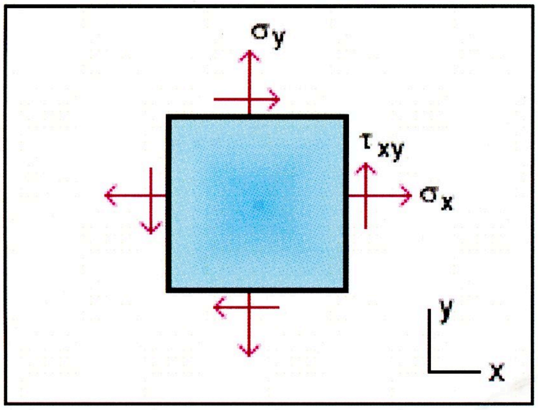
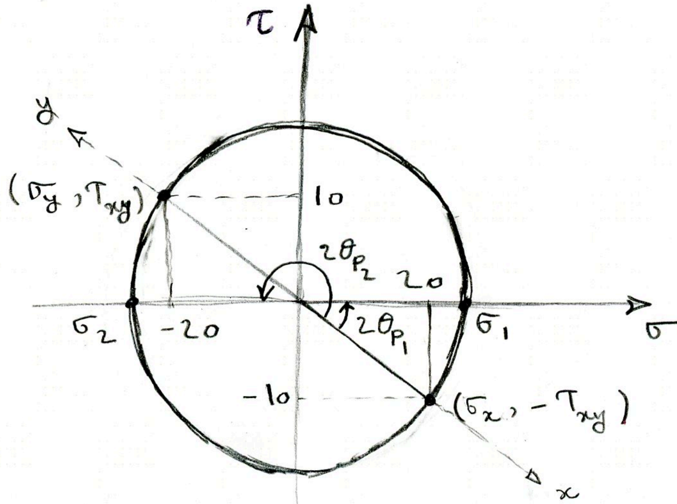
Problem 3:

(25 Points) The general stress element shown is loaded such that $\sigma_x = 20$ MPa, $\sigma_y = -20$ MPa, $\tau_{xy} = 10$ MPa. What are the principal stresses and what are orientation of the principal axes with respect to the x-axis?





(7.5) $$\sigma_1 = \frac{\sigma_x + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 + \sqrt{\left(\frac{40}{2}\right)^2 + 10^2} \Rightarrow \boxed{\sigma_1 = \sqrt{500}\ \text{MPa}}$$

(7.5) $$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 - \sqrt{\left(\frac{40}{2}\right)^2 + 10^2} \Rightarrow \boxed{\sigma_2 = \sqrt{500}\ \text{MPa}}$$

(5) $$\tan 2\theta_{p_1} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{20}{40} = \frac{1}{2} \Rightarrow \boxed{\theta_{p_1} = \frac{1}{2}\arctan\left(\frac{1}{2}\right)}$$

(5) $$2\theta_{p_2} = 2\theta_{p_1} + 180^\circ \Rightarrow \boxed{\theta_{p_2} = \frac{1}{2}\arctan\left(\frac{1}{2}\right) + 90^\circ}$$