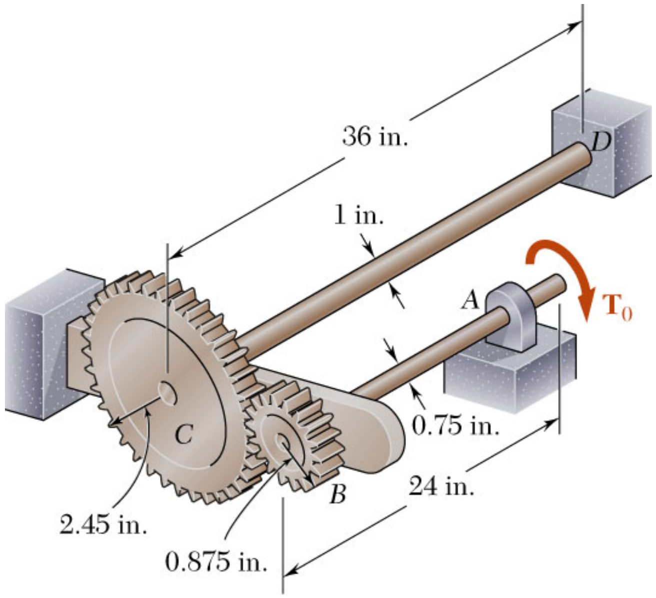
The exam is open book, open notes and home work solutions.

You can also bring a 2-page formula sheet.

There are 2 problems (multiple parts).
-- Lots of partial credit will be given for demonstrating understanding of concepts.
-- Remember to draw free body diagrams

If you have permission to take the exam in a separate room, please come to Barus and Holley 633 at 8:30 AM.

Sample Problem 3.4



Two solid steel shafts are connected by gears. Knowing that for each shaft G = 11.2 x 10^6 psi and that the allowable shearing stress is 8 ksi, determine (*a*) the largest torque T_0 that may be applied to the end of shaft *AB*, (*b*) the corresponding angle through which end *A* of shaft *AB* rotates.

SOLUTION:

Apply a static equilibrium analysis on the two shafts to find a relationship between T_{CD} and T_0.

Apply a kinematic analysis to relate the angular rotations of the gears.

Find the maximum allowable torque on each shaft – choose the smallest.

Find the corresponding angle of twist for each shaft and the net angular rotation of end *A.*

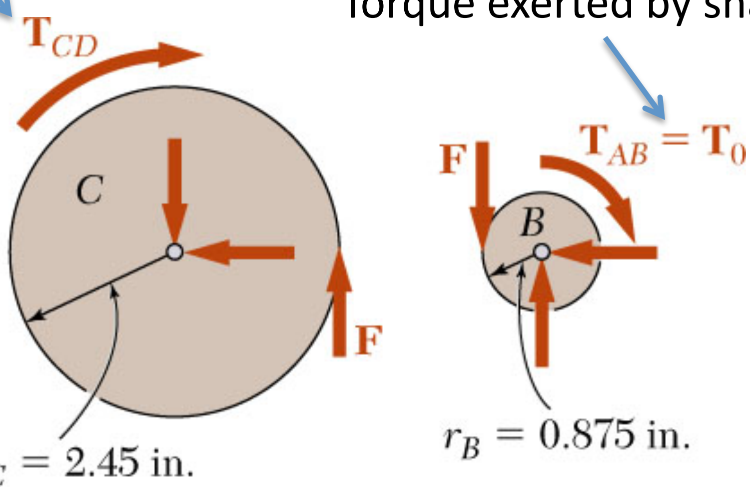
Sample Problem 3.4

SOLUTION:

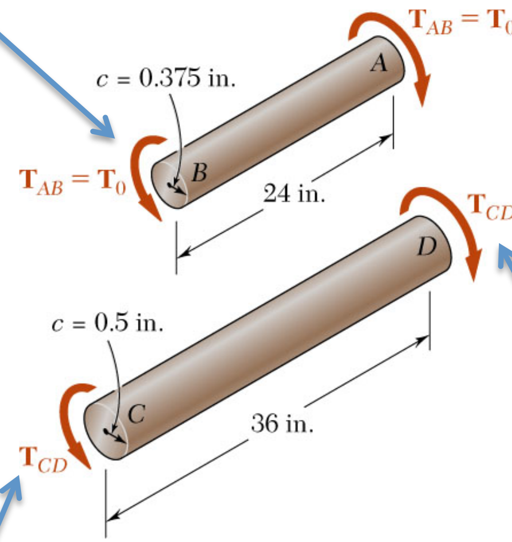
Apply a static equilibrium analysis on the two shafts to find a relationship between T_{CD} and T_0 .

Torque exerted by shaft CD

Torque exerted by shaft BC



Torque exerted by the gear B



Torque exerted by the wall

Torque exerted by gear C

$$\sum M_B = 0 = F(0.875\,\text{in.}) - T_0$$

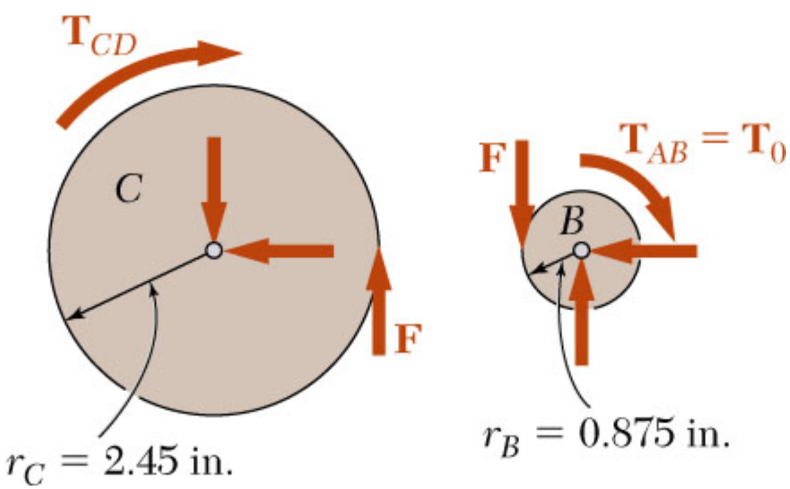
$$\sum M_C = 0 = F(2.45\,\text{in.}) - T_{CD}$$

$$T_{CD} = 2.8T_0$$

Sample Problem 3.4

SOLUTION:

Apply a static equilibrium analysis on the two shafts to find a relationship between T_{CD} and T_0 .

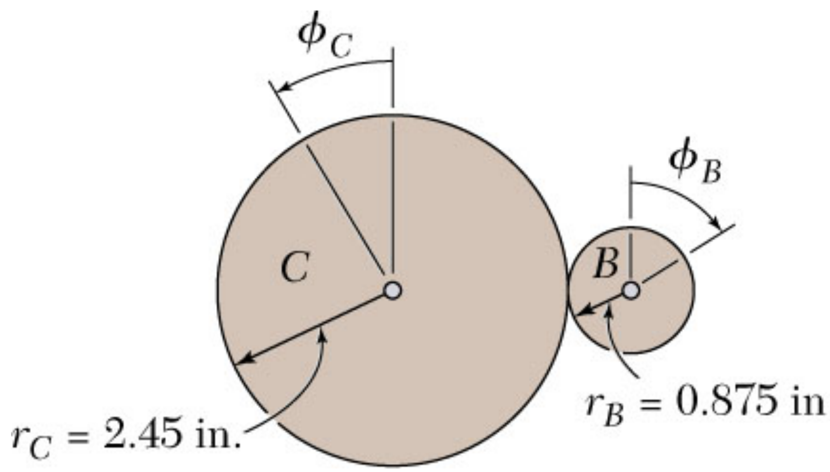


$$\sum M_B = 0 = F(0.875\,\text{in.}) - T_0$$

$$\sum M_C = 0 = F(2.45\,\text{in.}) - T_{CD}$$

$$T_{CD} = 2.8T_0$$

Apply a kinematic analysis to relate the angular rotations of the gears.



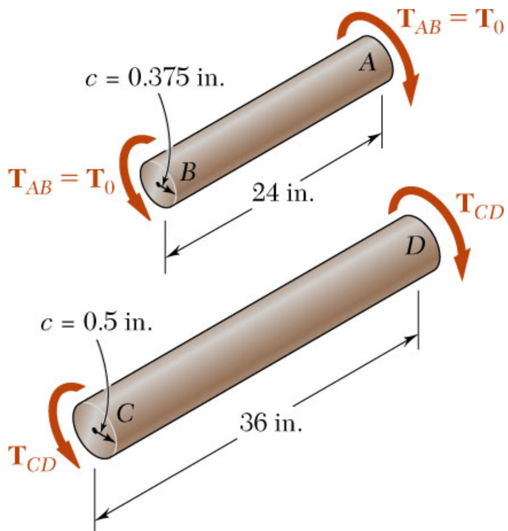
$$r_B\phi_B = r_C\phi_C$$

$$\phi_B = \frac{r_C}{r_B}\phi_C = \frac{2.45\,\text{in.}}{0.875\,\text{in.}}\phi_C$$

$$\phi_B = 2.8\phi_C$$

Sample Problem 3.4

Find the T_0 for the maximum allowable torque on each shaft – choose the smallest.



$$\tau_{max} = \frac{T_{AB}c}{J_{AB}} \qquad 8000\,psi = \frac{T_0(0.375\,\text{in.})}{\frac{\pi}{2}(0.375\,\text{in.})^4}$$

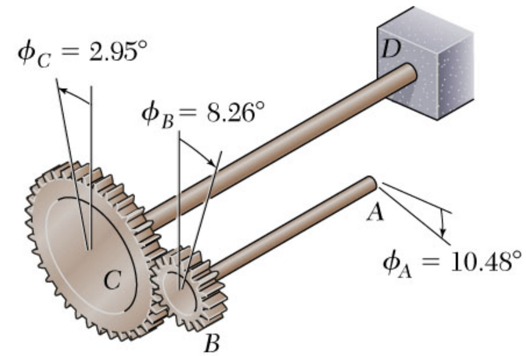
$$T_0 = 663\,\text{lb}\cdot\text{in.}$$

$$\tau_{max} = \frac{T_{CD}c}{J_{CD}} \qquad 8000\,psi = \frac{2.8T_0(0.5\,\text{in.})}{\frac{\pi}{2}(0.5\,\text{in.})^4}$$

$$T_0 = 561\,\text{lb}\cdot\text{in.}$$

$$\boxed{T_0 = 561\,\text{lb}\cdot\text{in}}$$

Find the corresponding angle of twist for each shaft and the net angular rotation of end *A*.



$$\phi_{C/D} = \frac{T_{CD}L}{J_{CD}G} = \frac{2.8(561\,\text{lb}\cdot\text{in.})(24\,\text{in.})}{\frac{\pi}{2}(0.5\,\text{in.})^4(11.2\times10^6\,\text{psi})}$$

$$= 0.514\,\text{rad} = 2.95^\circ$$

$$\phi_B = 2.8\phi_C = 2.8(2.95^\circ) = 8.26^\circ$$

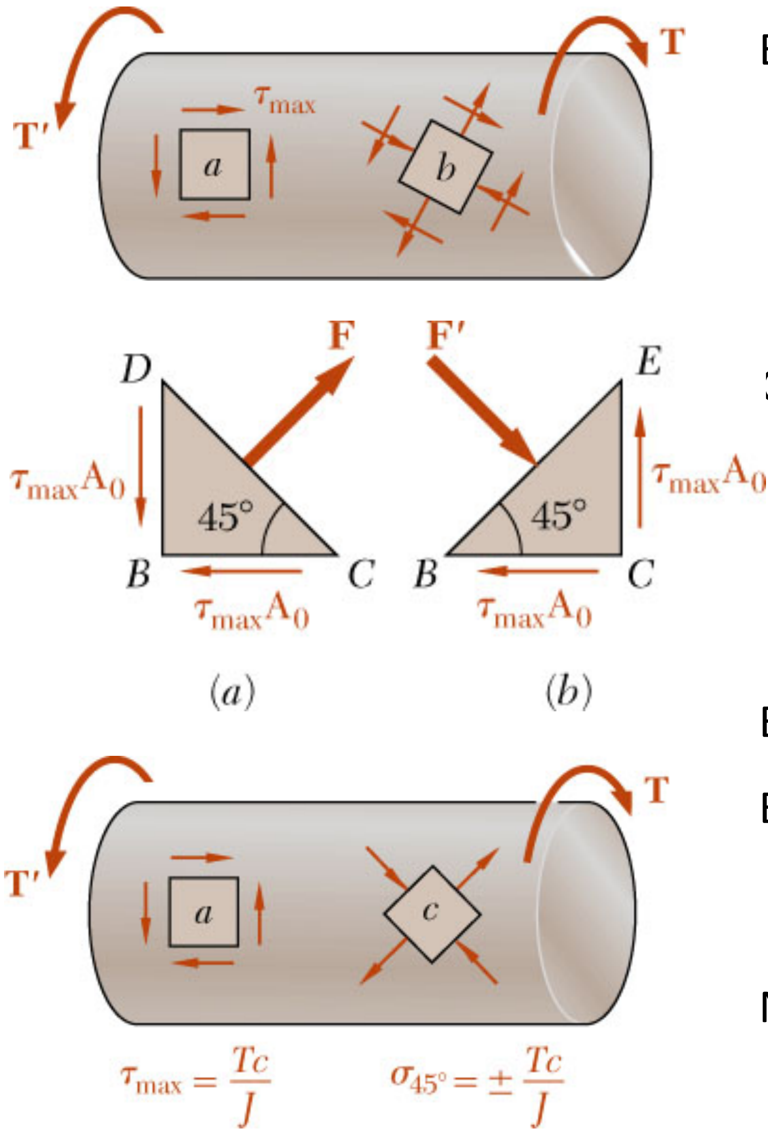
$$\phi_{A/B} = \frac{T_{AB}L}{J_{AB}G} = \frac{(561\,\text{lb}\cdot\text{in.})(24\,\text{in.})}{\frac{\pi}{2}(0.375\,\text{in.})^4(11.2\times10^6\,\text{psi})}$$

$$= 0.387\,\text{rad} = 2.22^\circ$$

$$\phi_A = \phi_B + \phi_{A/B} = 8.26^\circ + 2.22^\circ$$

$$\boxed{\phi_A = 10.48^\circ}$$

Normal Stresses



Elements with faces parallel and perpendicular to the shaft axis are subjected to shear stresses only. Normal stresses, shearing stresses or a combination of both may be found for other orientations.

Consider an element at 45° to the shaft axis,

$$F = 2(\tau_{max} A_0)\cos 45° = \tau_{max} A_0 \sqrt{2}$$

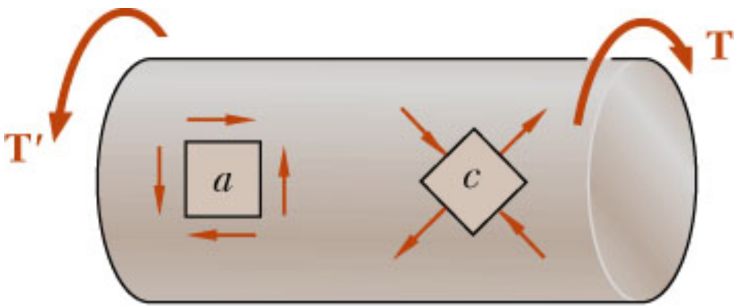
$$\sigma_{45°} = \frac{F}{A} = \frac{\tau_{max} A_0 \sqrt{2}}{A_0 \sqrt{2}} = \tau_{max}$$

Element *a* is in pure shear.

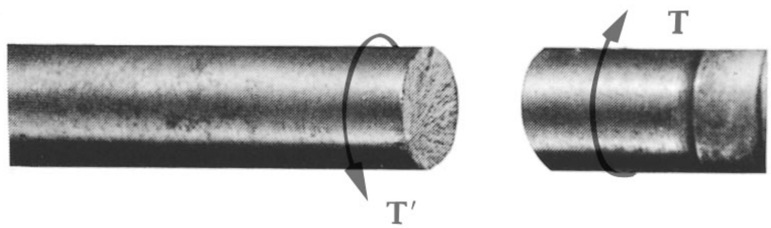
Element *c* is subjected to a tensile stress on two faces and compressive stress on the other two.

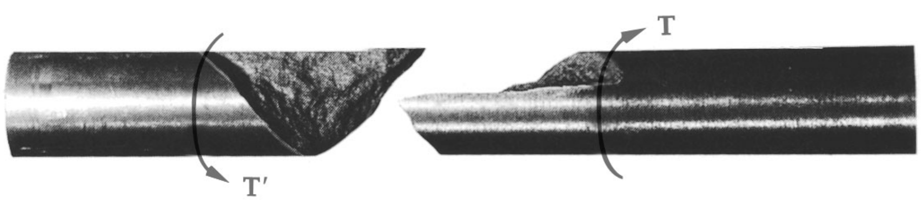
Note that all stresses for elements *a* and *c* have the same magnitude

Torsional Failure Modes



$$\tau_{max} = \frac{Tc}{J} \qquad \sigma_{45^\circ} = \pm \frac{Tc}{J}$$





Ductile materials generally fail in shear. Brittle materials are weaker in tension than shear.

When subjected to torsion, a ductile specimen breaks along a plane of maximum shear, i.e., a plane perpendicular to the shaft axis.

When subjected to torsion, a brittle specimen breaks along planes perpendicular to the direction in which tension is a maximum, i.e., along surfaces at 45° to the shaft axis.