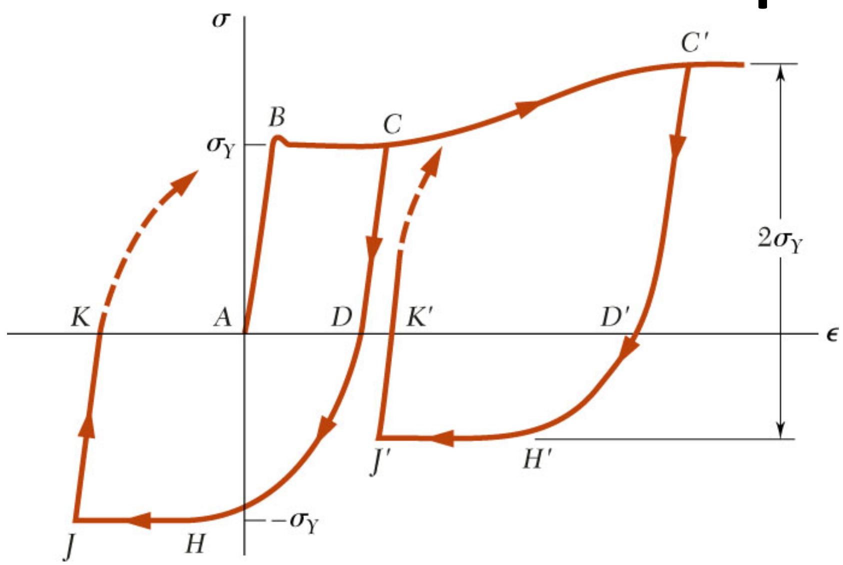
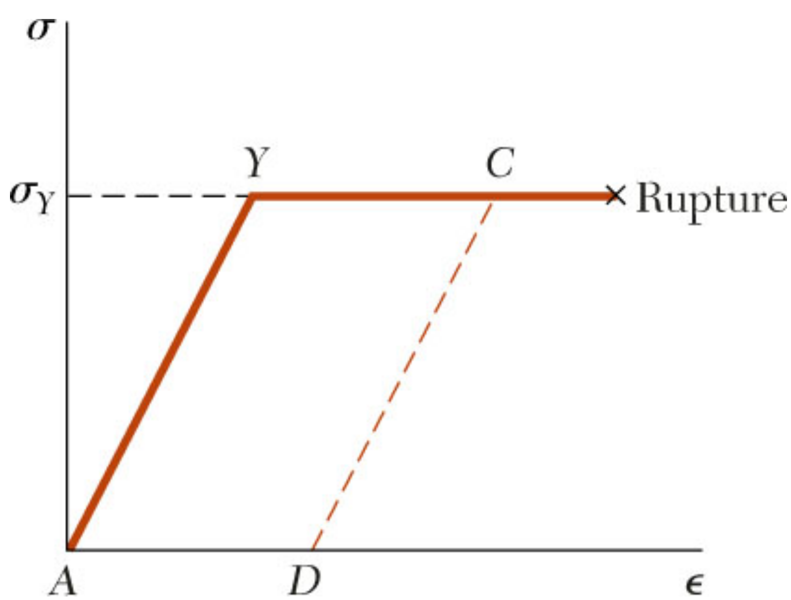
Elastoplastic Materials



- Previous analyses based on assumption of linear stress-strain relationship, i.e., stresses below the yield stress
- Assumption is good for brittle material which rupture without yielding
- If the yield stress of ductile materials is exceeded, then plastic deformations occur



- Analysis of plastic deformations is simplified by assuming an idealized *elastoplastic material*
- Deformations of an elastoplastic material are divided into elastic and plastic ranges
- Permanent deformations result from loading beyond the yield stress

Residual Stresses

When a single structural element is loaded uniformly beyond its yield stress and then unloaded, it is permanently deformed but all stresses disappear. This is not the general result.

Residual stresses will remain in a structure after loading and unloading if

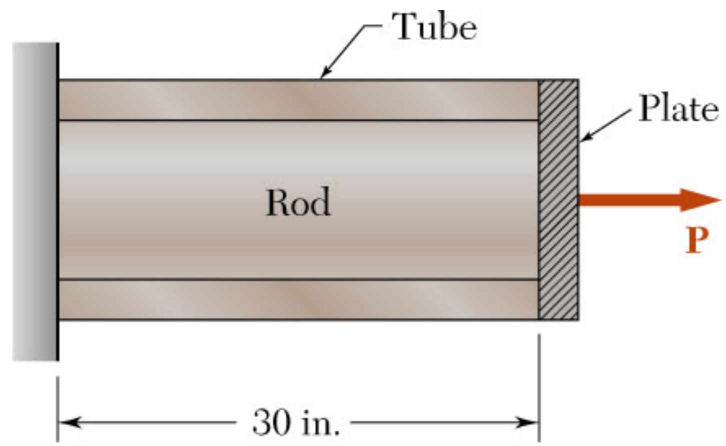
- only part of the structure undergoes plastic deformation
- different parts of the structure undergo different plastic deformations

Residual stresses also result from the uneven heating or cooling of structures or structural elements

Example 2.14, 2.15, 2.16

A cylindrical rod is placed inside a tube of the same length. The ends of the rod and tube are attached to a rigid support on one side and a rigid plate on the other. The load on the rod-tube assembly is increased from zero to 5.7 kips and decreased back to zero.

a) draw a load-deflection diagram for the rod-tube assembly
b) determine the maximum elongation
c) determine the permanent set
d) calculate the residual stresses in the rod and tube.

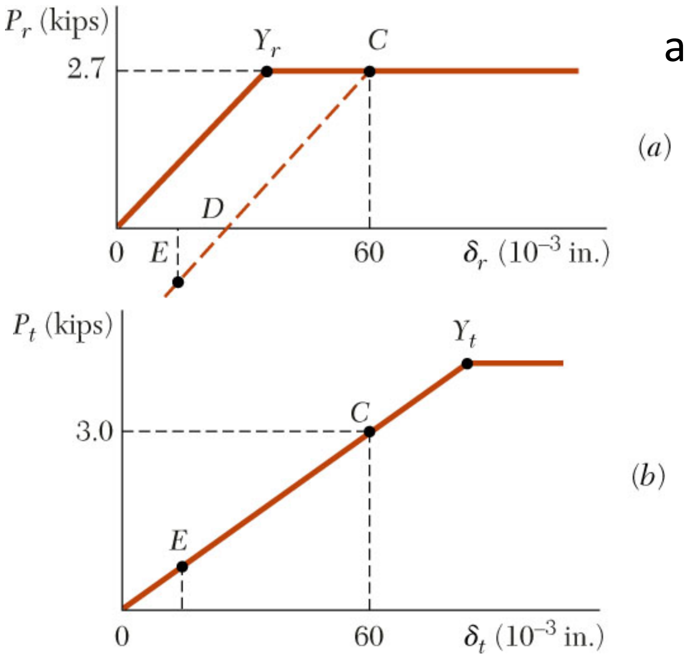


$A_r = 0.075\,\text{in.}^2$ $\quad$ $A_t = 0.100\,\text{in.}^2$

$E_r = 30\times10^6\,\text{psi}$ $\quad$ $E_t = 15\times10^6\,\text{psi}$

$(\sigma_r)_Y = 36\,\text{ksi}$ $\quad$ $(\sigma_t)_Y = 45\,\text{ksi}$

Example 2.14, 2.15, 2.16



a) Draw a load-deflection diagram for the rod-tube assembly

$$(P_r)_Y = (\sigma_r)_Y A_r = (36\,\text{ksi})(0.075\,\text{in}^2) = 2.7\,\text{kips}$$

$$(\delta_r)_Y = (\varepsilon_r)_Y L = \frac{(\sigma_r)_Y}{E_r} L = \frac{36\times10^3\,\text{psi}}{30\times10^6\,\text{psi}}(30\,\text{in.})$$

$$= 36\times10^{-3}\,\text{in.}$$

$$(P_t)_Y = (\sigma_t)_Y A_t = (45\,\text{ksi})(0.100\,\text{in}^2) = 4.5\,\text{kips}$$

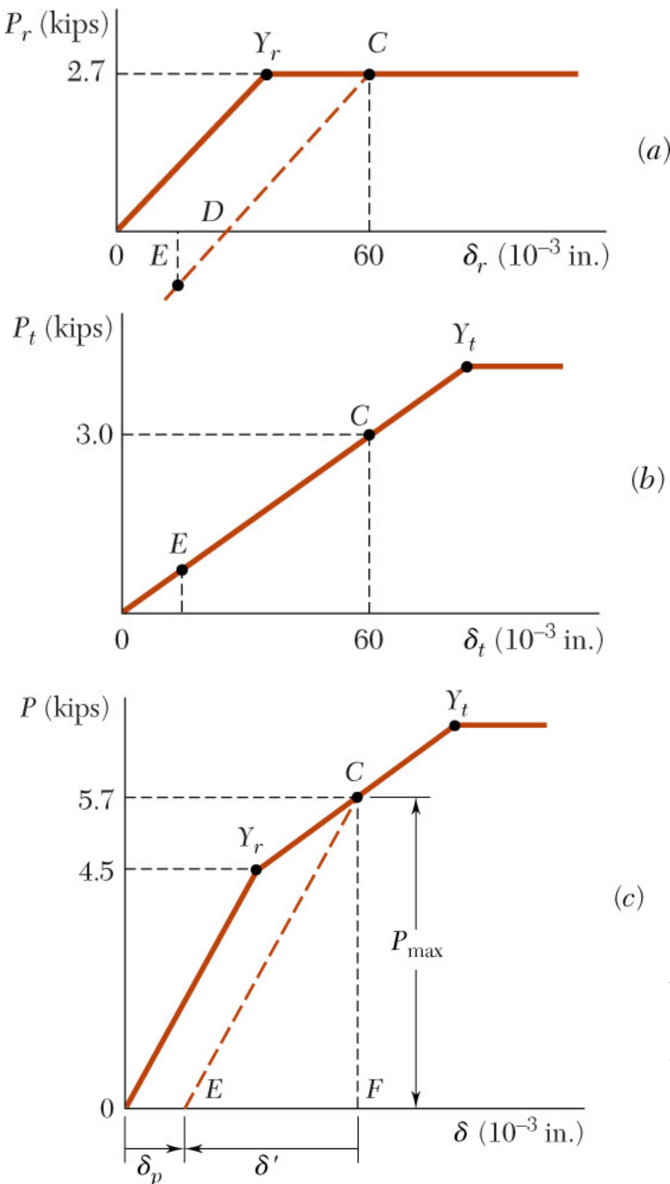
$$(\delta_t)_Y = (\varepsilon_t)_Y L = \frac{(\sigma_t)_Y}{E_t} L = \frac{45\times10^3\,\text{psi}}{15\times10^6\,\text{psi}}(30\,\text{in.})$$

$$= 90\times10^{-3}\,\text{in.}$$

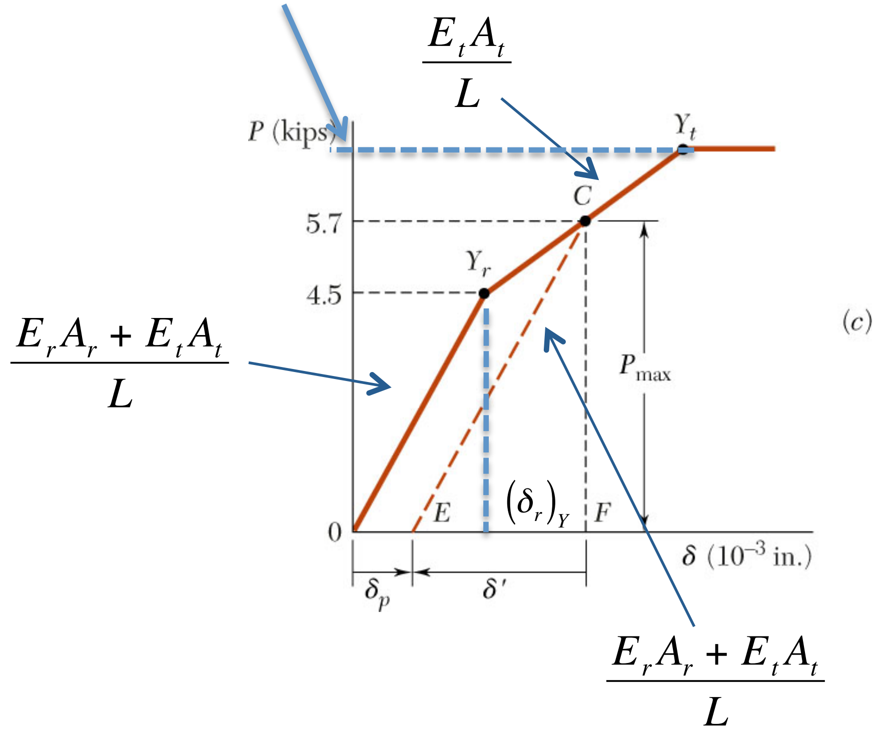
$$P = P_r + P_t$$

$$\delta = \delta_r = \delta_t$$

Example 2.14, 2.15, 2.16



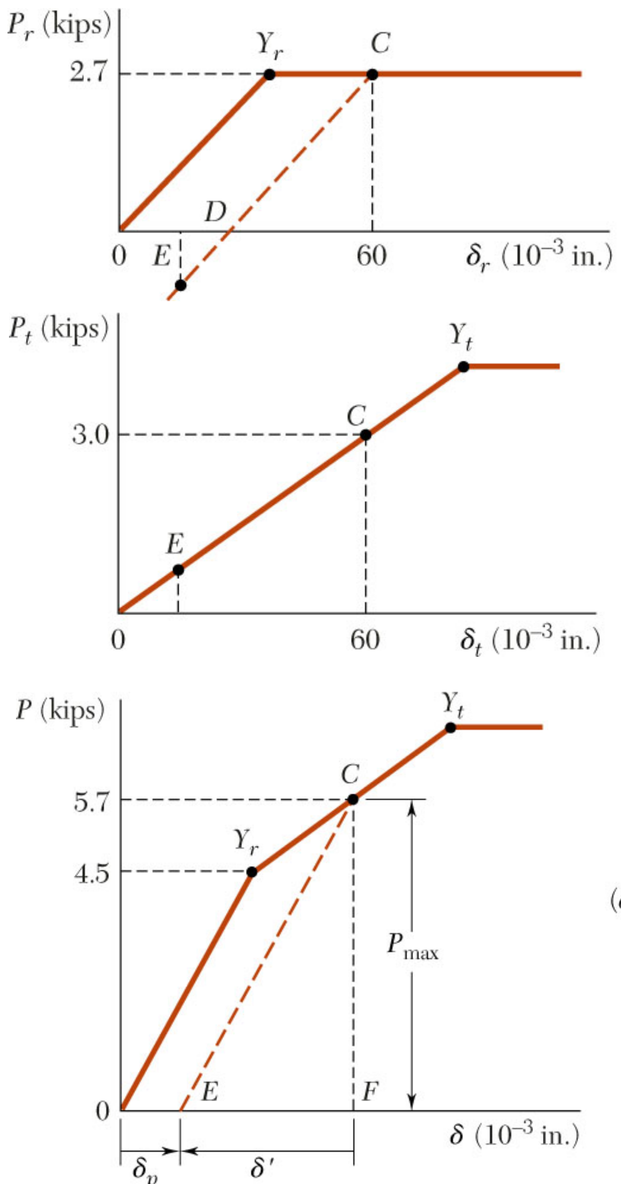
$$\left(P_r\right)_Y + \left(P_t\right)_Y = 7.2\ kips$$



$$P = P_r + P_t$$

$$\delta = \delta_r = \delta_t$$

Example 2.14, 2.15, 2.16



a load of P = 5.7 kips, the rod has reached the plastic range while the tube is still in the elastic range

(a)

$$P_r = (P_r)_Y = 2.7\,\text{kips}$$

$$P_t = P - P_r = (5.7 - 2.7)\,\text{kips} = 3.0\,\text{kips}$$

$$\sigma_t = \frac{P_t}{A_t} = \frac{3.0\,\text{kips}}{0.1\,\text{in}^2} = 30\,\text{ksi}$$

(b)

$$\delta_t = \varepsilon_t L = \frac{\sigma_t}{E_t} L = \frac{30\times10^3\,\text{psi}}{15\times10^6\,\text{psi}} 30\,\text{in.}$$

$$\boxed{\delta_{max} = \delta_t = 60\times10^{-3}\,\text{in.}}$$

The rod-tube assembly unloads along a line parallel to $0Y_r$

$$m = \frac{4.5\,\text{kips}}{36\times10^{-3}\,\text{in.}} = 125\,\text{kips/in.} = \text{slope}$$

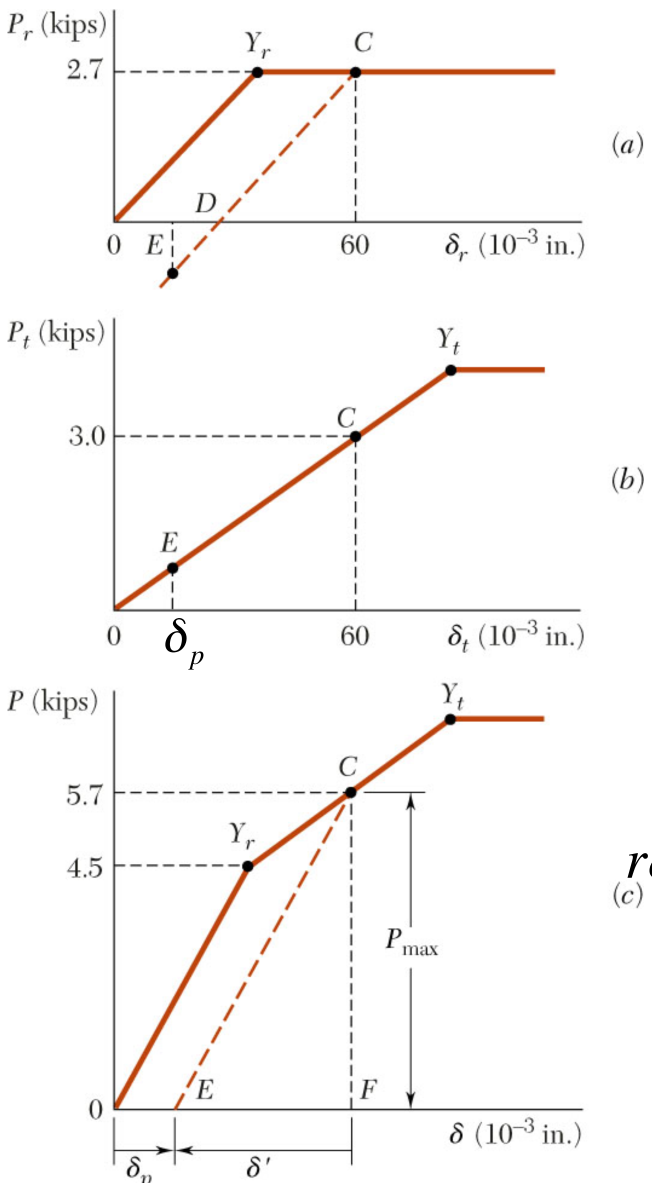
(c)

$$\delta' = -\frac{P_{max}}{m} = -\frac{5.7\,\text{kips}}{125\,\text{kips/in.}} = -45.6\times10^{-3}\,\text{in.}$$

$$\delta_p = \delta_{max} + \delta' = (60 - 45.6)\times10^{-3}\,\text{in.}$$

$$\boxed{\delta_p = 14.4\times10^{-3}\,\text{in.}}$$

Example 2.14, 2.15, 2.16



What are the residual stresses in the rod and tube?

$$when \quad \delta = \delta_p \quad P_t = \delta_p \frac{E_t A_t}{L} = 0.72\ kips$$

$$residual\ stress\ in\ the\ tube: \quad \frac{P_t}{A_t} = \frac{0.72\ kips}{0.1\ in^2} = 7.2 ksi$$

$$P_r = -P_t = -0.72\ kips$$

$$residual\ stress\ in\ the\ rod: \quad \frac{P_r}{A_r} = \frac{-0.72\ kips}{0.075\ in^2} = -9.6 ksi$$

First Mid-term will be held on Friday Oct 14 (in class)

Material to be covered – Chap 1 and 2