**On adhesion enhancement due to concave surface geometries**

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**Abstract**

Recent experimental and theoretical work in literature showed that several biological attachment devices are concave in shape and that such a shape enhances adhesion strength. In this paper, an analytical model is developed for the pull-off forces required to separate a rigid axisymmetric concave punch from an elastic half-space with the goal of understanding how concave shapes seen in both biological and man-made adhesive systems can outperform other geometries. Significant enhancement in adhesion strength compared to that of a flat punch was predicted for both spherical and elliptical concave punch profiles. Increasing the degree of the concavity initially increases the resulting pull-off force, but for higher depths the contact interface will separate before pull-off forces higher than those for the flat punch are achieved. Experiments performed with machined aluminum elliptical concave punches on gelatin qualitatively verified the trends predicted by the analytical model, and the maximum pull-off forces measured are within 80% of the predicted optimum pull-off force. These findings emphasize the benefits of concave geometries for designing interfaces which enhance adhesion through geometry alone.

**Keywords:** adhesion enhancement, concave shapes, biological attachment.
1. Introduction

A diverse variety of structures and shapes are seen in biological attachment systems. As illustrated by Spolenak et al. [1], natural adhesive structures can terminate in spherical, conical, toroidal, flat, or concave ends. Concave geometries have been shown numerically [2] and experimentally [3] to significantly enhance adhesion in the presence of capillary forces, and experiments performed on fly seta using atomic force microscopy indicated that the adhesion pull-off forces were highest in their wet concave centers [4]. Interestingly, analytical models for dry van der Waals adhesion have also indicated that concave geometries are favorable for adhesive binding [5,6]. Researchers have fabricated geometric structures designed to mimic the range of attachment geometries seen in nature [7], and have found that the concave shapes can outperform the adhesion strengths of other geometries when the applied compressive preload is high enough to achieve good initial contact. Such results indicate that geometry plays a significant role in the strength of dry adhesion generated by short-range van der Waals forces and prompt further study of the adhesive mechanics of concave geometries. The role of surface topography and the associated contact instabilities in increasing adhesion strength and toughness was illustrated by Guduru [8] and Guduru & Bull [9] theoretically and experimentally. In the context of concave shapes, of particular interest is the work of Gao and Yao [5], who showed that the “optimal” shape for maximizing van der Waals adhesion is a concave geometry described by an elliptic integral. Such shapes were investigated in more detail analytically by Yao and Gao [10] and atomistically by Buehler et al. [11]. Figure 1 summarizes the main results of Gao and coworkers. As originally presented in Gao and Yao [5], a numerical Lennard-Jones model showed that the pull-off force for a flat-tipped fiber converges to the optimal pull-off force as the radius of the fiber decreases, indicating that size reduction is a natural mechanism to allow contacts to achieve their optimal adhesion. However, the results of Gao and Yao (2004) also indicate that significant adhesion enhancement is possible by shape optimization for larger radii fibers or more compliant materials, as highlighted in Fig. 1. Understanding such enhancement could provide insights into the superior performance of concave geometries seen in the adhesion studies of del Campo et al. [12] and provide further insights into the benefits of the concave structures seen in nature.

The objective of this paper is to investigate geometric adhesion enhancements by developing a contact mechanics model to estimate the dry adhesion of general concave shapes.
The model is shown to predict significant adhesion enhancement for a range of concave geometries, including those with spherical and elliptic integral profiles. (The latter will henceforth be referred to as “elliptical” profiles.) The predicted pull-off force is seen to increase initially with the depth of the concavity but decrease for higher depths, leaving an intermediate concavity depth where the optimal pull-off force can be approached. To verify the analytical model, adhesion experiments are performed with elliptical concave aluminum punches on gelatin. The experimental data qualitatively confirms the trends predicted by the model, and the measured pull-off forces come within 80% of the predicted optimum pull-off force. These findings highlight the role of geometry in enhancing dry adhesion and suggest that concave geometries hold promise for the efficient design of new adhesive systems.

The rest of the paper is organized as follows. In Section 2, an analytical model is presented for the adhesion pull-off force of a rigid axisymmetric concave indenter from an elastic half-space. In Section 3, experimental results are presented for the pull-off forces of a series of elliptical concave punches which verify the qualitative trends predicted by the analytical model and demonstrate that significant adhesion enhancement is possible via geometric optimization. Finally, the conclusions drawn from the analytical model and experiments are summarized in Section 4.

2. A model for the adhesive contact of axisymmetric concave punch geometries

2.1 Contact model

To investigate possible adhesion enhancements arising from concave punch geometries, the boundary value problem of a rigid axisymmetric concave punch with radius $b$ indenting an elastic half-space with elastic modulus $E$ and Poisson’s ratio $\nu$ is formulated. The concave profile is assumed to be continuously differentiable for $r \leq b$, with maximum depth $h$ occurring at radial coordinate $r = 0$. The goal of this analysis is to obtain relationships between the rigid indenter displacement $d$, contact pressure distribution $p(r)$, and applied load $P$. The geometry of this problem is illustrated in Fig. 2.

Assuming the concave punch is initially subjected to sufficiently high loading such that intimate contact is achieved with the surface for $r < b$, for frictionless contact the boundary value problem becomes equivalent to prescribing an axisymmetric normal displacement on the surface of an elastic half-space. Following a classical solution method for such problems outlined in
Barber [13], the harmonic potential representation of Green and Zerna [14] can be used to express the prescribed surface displacement beneath the punch, \( u(r) \), as

\[
\int_0^r \frac{g_1(t)dt}{\sqrt{r^2 - t^2}} = -\frac{1}{2} E^* u(r), \quad 0 \leq r \leq b, \tag{1}
\]

where the reduced modulus \( E^* \) is defined as, \( E^* = E/(1-\nu^2) \). Equation (1) is an Abel integral equation for the unknown function \( g_1(t) \), which is given by

\[
g_1(s) = \frac{E^*}{\pi} \frac{d}{ds} \int_0^s \frac{ru(r)dr}{\sqrt{s^2 - r^2}}, \quad 0 \leq s \leq b \tag{2}
\]

in terms of the known surface displacement \( u(r) \). As discussed by Barber [13], the solution for \( g_1(s) \) can be used to obtain expressions for the contact pressure,

\[
p(r) = -(\frac{\sigma_{zz}}{E})_{z=0} = \frac{1}{r} \frac{d}{dr} \int_r^b \frac{t g_1(t)dt}{\sqrt{t^2 - r^2}}, \quad 0 \leq r \leq b \tag{3}
\]

and the total indenting force,

\[
P = -2\pi \int_0^b g_1(t)dt, \tag{4}
\]

where the sign convention is such that \( P > 0 \) for compressive loads.

First consider the case of a spherical concave punch, where the prescribed surface displacement \( u(r) \) can be expressed in terms of the rigid displacement \( d \) of the punch into the half-space as

\[
u_{sph}(r) = (d - h) + h \left( \frac{r}{b} \right)^2, \tag{5}
\]

with the spherical surface approximated by a paraboloid. Substituting Eq.(5) into Eqs. (2)–(4) results in the solutions

\[
g_{1,sph}(s) = -\frac{E^*}{\pi} \left[ d + h \left( \frac{2s^2}{b^2} - 1 \right) \right] \tag{6}
\]
\[ p_{sph}(r) = \frac{E^*}{\pi} \left[ d + h \left( \frac{4r^2}{b^2} - 3 \right) \right] \]  
(7)

\[ p_{sph} = 2bE^*(d - h/3) \]  
(8)

Equation (8) matches the spherical concave punch solution given by Barber [15] and Shibuya [16], obtained by considering the limiting case of a concave punch in partial contact with a half space. Note that if \( h = 0 \), the above expressions for displacement, contact pressure and applied load reduce to the classical results for a flat punch given by Kendall [17]:

\[ u_{flat}(r) = d \]  
(9)

\[ p_{flat}(r) = \frac{E^*d}{\pi\sqrt{b^2 - r^2}} \]  
(10)

\[ P_{flat}(r) = 2bE^*d \]  
(11)

Now consider a concave punch with prescribed surface displacement \( u(r) \) expressed as

\[ u(r) = (d - h) + \frac{h}{\frac{4}{\pi} - 2} \left[ \frac{4}{\pi} E\left(\frac{r}{b}\right) - 2 \right] \]  
(12)

where \( E(\cdot) \) is the complete elliptic integral of the second kind. This geometry will be referred to as "elliptical" for convenience. [Note that \( u(0) = d - h \) since \( E(0) = \pi/2 \) and \( u(b) = d \) since \( E(1) = 1 \).] Gao and Yao [5] found that an elliptical concave profile with maximum depth \( h \) given by

\[ h = \frac{\sigma_{th}b}{E^*} \left( 2 - \frac{4}{\pi} \right) \]  
(13)

would optimize the punch geometry such that the tensile contact stress sustained by the interface for all \( r \leq b \) was equal to the theoretical strength of adhesion at the moment of separation, i.e., \( p(r) = -\sigma_{th} \), resulting in an optimal pull-off force \( P_{opt} = -\pi b^2 \sigma_{th} \). For a general non-optimal geometry, substituting Eq. (13) into Eqs. (2)–(4) obtains the solutions
Equations (7)–(8) and (15)–(16) allow the investigation of pull-off forces for spherical and elliptical concave geometries. In general, interface separation will initiate from either the periphery of contact \((r = b)\) or from the center of contact \((r = 0)\). Both cases are examined in the following sections.

### 2.2 Punch separation initiating from periphery of contact

For shallow concave profiles, interface separation will initiate from the periphery of contact at \(r = b\), just as it does for a flat punch. Maugis and Barquins [18] showed that a Griffith fracture criterion can be used to describe JKR [19] adhesive contact, defining the stress intensity factor

\[
K_I = \lim_{r \to b} \sqrt{2\pi(b - r)} p(r) \tag{17}
\]

and setting the strain energy release rate \(G\) equal to the work of adhesion \(w\) of the interface at equilibrium, or

\[
G = \frac{K_I^2}{2E^*} = w. \tag{18}
\]

Typically, Eq. (18) is used to determine the size of the contact radius for a given rigid indenter displacement \(d\). In this case, however, a solution is desired for the displacement \(d\) for which the stress intensity factor becomes large enough to drive interface failure at the fixed contact radius \(b\). For the spherical concave punch, Eq. (17) gives the stress intensity factor

\[
g_{1,\text{ell}} = -\frac{E^*}{\pi} \left[ d - \frac{h}{\pi - 2} \left( \pi \sqrt{1 - \frac{s^2}{b^2}} - 2 \right) \right] \tag{14}
\]

\[
p_{\text{ell}}(r) = -\frac{E^*}{\pi} \left[ \frac{1}{\sqrt{b^2 - r^2}} \left( d + \frac{2h}{\pi - 2} \right) - \frac{\pi^2 h}{2b(\pi - 2)} \right] \tag{15}
\]

\[
P_{\text{ell}} = 2bE^* \left[ d - \frac{h(\pi^2 - 8)}{4(\pi - 2)} \right] \tag{16}
\]
\[ K_{L,sph} = \frac{E^*}{\sqrt{\pi b}} (d + h), \quad (19) \]

and hence the equilibrium condition of Eq. (18) at \( r = b \) determines the displacement \( \hat{d} \) at separation,

\[ \hat{d}_{sph}^G = -\frac{2\pi bw}{E^*} - h, \quad (20) \]

where the negative root has been taken so that the flat punch displacement from Kendall [17] will be obtained when \( h \to 0 \). Substituting this into Eq. (8), the spherical pull-off force \( \hat{P}_{sph}^G \) is found to be

\[ \hat{P}_{sph}^G = -\sqrt{8\pi E^*wb^3} - \frac{8}{3} bE^*h. \quad (21) \]

Note that setting \( h = 0 \) in the above expression recovers the classical solution for the pull-off force of a flat punch, \( \hat{P}_{flat} = -\sqrt{8\pi E^*wb^3} \) [17]. The second term in the expression for \( \hat{P}_{sph}^G \) increases the magnitude of the tensile pull-off force for any depth of concavity \( h \). Thus, the Griffith criterion for interface separation at the periphery predicts that any spherical concave geometry will have a higher pull-off force than a flat punch.

Repeating the above procedure for an elliptical concave punch yields the stress intensity factor

\[ K_{L,ell} = \frac{E^*}{\sqrt{\pi b}} \left( d + \frac{2h}{\pi - 2} \right), \quad (22) \]

which gives the following solution for displacement when substituted into Eq. (18) with \( r = b \):

\[ \hat{d}_{ell}^G = -\frac{2\pi bw}{E^*} - \frac{2h}{\pi - 2}. \quad (23) \]
Substituting this into Eq. (16), the elliptical pull-off force \( \hat{P}_{ell} \) is found to be

\[
\hat{P}_{ell} = -\sqrt{8\pi E^* w b^3} - \frac{\pi^2 b E^* h}{2(\pi - 2)}.
\]  

As before, the second term in the expression for \( \hat{P}_{ell} \) increases the magnitude of the tensile pull-off force for any depth of concavity \( h \). The magnitude of this term is approximately two times greater than the corresponding term in the expression for \( \hat{P}_{sph} \), indicating that an elliptical geometry is more effective at enhancing pull-off forces than a spherical geometry.

### 2.3 Punch separation initiating from center of contact

As the maximum depth of concavity \( h \) increases, it becomes more likely that interface separation will initiate from the center of contact (\( r = 0 \)) than at the periphery. In other words, the contact pressure at \( r = 0 \) will reach a critical value, assumed here to equal the theoretical tensile strength of adhesion (i.e., \( p(0) = -\sigma_{th} \)), before the stress intensity factor at the periphery is large enough to initiate separation. This condition for \( p(0) \) allows the corresponding displacement \( \hat{d} \) and load \( \hat{P} \) at separation to be determined, although fracture mechanics requires the presence of a flaw and an energy release rate criterion to propose interface separation. Hence, the strength criterion used here should be viewed in the spirit of the Gao and Yao [5] approach to determine the optimal profiles.

For the spherical concave geometry, Eq. (7) gives the displacement corresponding to \( p(0) = -\sigma_{th} \) as

\[
\hat{d}_{sph}^c = -\frac{\sigma_{th} \pi b}{E^*} + 3h.
\]  

Substituting into Eq. (8) yields the resulting pull-off force

\[
\hat{P}_{sph}^c = -2\pi b^2 \sigma_{th} + \frac{16}{3} b E^* h.
\]  

Similarly, for the elliptical concave geometry, Eq. (15) gives the displacement corresponding
to $p(0) = -\sigma_{th}$ as

$$d_{sph}^c = -\frac{\sigma_{th}\pi b}{E^*} + \frac{\pi + 2}{2} h. \tag{27}$$

Substituting into Eq. (16) yields the resulting pull-off force

$$\hat{p}_{ell}^c = -2\pi b^2 \sigma_{th} + \frac{\pi^2 b^2 E^* h}{2(\pi - 2)} \tag{28}$$

The reduction in pull-off force given by the rightmost term in Eq. (26) is 1.2 times greater than the corresponding reduction in pull-off force given by the rightmost term in Eq. (28), indicating once more that elliptical profiles are more effective at amplifying adhesion pull-off forces than spherical profiles. Note that when $h$ is the optimal depth derived by Gao and Yao [5], substituting Eq. (13) into Eq. (28) results in $\hat{p}_{ell} = \hat{p}_{opt} = \pi b^2 \sigma_{th}$, as expected.

### 2.4 Dimensionless parameterization of results

Normalizing the expressions for pull-off forces with respect to that of a flat punch of the same radius $b$ gives a direct measurement of the adhesion enhancement due to a concave geometry. The optimal pull-off force can be normalized as

$$\frac{\hat{p}_{opt}}{\hat{p}_{flat}} = \frac{-\pi b^2 \sigma_{th}}{-\sqrt{8\pi E^* w b^3}} = \left(\frac{\pi \sigma_{th}^2 b}{\sqrt{8 E^* w}}\right) \tag{29}$$

which is plotted in Fig. 1. From Eq. (29) the dimensionless parameters $\sigma_{th}/E^*$ and $\sigma_{th} b/w$ can be identified. These parameters, along with the normalized depth of concavity $h/b$, allow for the normalization of the pull-off force expressions for the spherical and elliptical concave punch geometries as follows:
With the above equations, the pull-off forces for an elliptical geometry can be compared to those for a spherical geometry for the same $h/b$, $\sigma_{th}/E^*$, and $\sigma_{th}b/w$. If $\frac{\bar{P}}{\bar{P}_{flat}} > 1$, the concave geometry is predicted to enhance adhesion. Figure 3a shows the predicted normalized pull-off forces for the case $\sigma_{th}/E^* = 0.2$ and $\sigma_{th}b/w = 115$, where $\frac{\bar{P}}{\bar{P}_{flat}} = 3$. For this combination of parameters, the spherical pull-off forces are below the elliptical pull-off forces for all $h/b$, and fall short of reaching the maximum possible pull-off force $\bar{P}_{opt}$. On the other hand, for the elliptical geometry, there is a range of $h/b$ where $\bar{P}_{opt}$ is reached; in fact, this is found to be true for any combination of $\sigma_{th}/E^*$ and $\sigma_{th}b/w$. This apparent non-uniqueness of the optimal geometry is discussed in more detail below. Figure 3b shows the predicted normalized pull-off forces for the case $\sigma_{th}/E^* = 0.2$ and $\sigma_{th}b/w = 29$, where $\frac{\bar{P}}{\bar{P}_{flat}} = 1.5$. Here, the spherical geometry also achieves $\bar{P}_{opt}$ due to the smaller value of $\sigma_{th}b/w$. Again, $\bar{P}_{sph} \leq \bar{P}_{ell}$ for all $h/b$.

The individual effects of $\sigma_{th}/E^*$ and $\sigma_{th}b/w$ on the predicted pull-off forces are now investigated, focusing on the elliptical geometry. The effect of holding $\sigma_{th}b/w$ constant while changing $\sigma_{th}/E^*$ is illustrated in Fig. 4a. Here, a higher $\sigma_{th}/E^*$ results in a higher $\bar{P}_{opt}$ and $\bar{P}^C$ but a lower $\bar{P}^G$ for a given $h/b$. Alternatively, the effect of holding $\sigma_{th}/E^*$constant while changing $\sigma_{th}b/w$ is illustrated in Fig. 4b. Here, a higher $\sigma_{th}b/w$ results in a higher $\bar{P}_{opt}$, $\bar{P}^C$ and $\bar{P}^G$ for a given $h/b$. Similar effects are seen for the spherical geometry as well.
From this analytical model, it appears that the optimal shape of the concave punch for maximizing adhesive pull-off force is non-unique. Figures 3 and 4 illustrate that there are ranges of $h/b$ for both elliptical and spherical geometries for which $\hat{P}_{\text{opt}}$ is achieved, rather than a unique $h/b$. However, this non-uniqueness is an artifact of the Griffith model, which allows for singular tractions on the interface. An atomistic or cohesive model which limits the interface traction to $\sigma_{th}$ would show the elliptical pull-off force gradually approaching $\hat{P}_{\text{opt}}$ as $h/b$ increases and attaining the optimal value at a unique value of $h/b$, as in the atomistic results of Buehler et al. [11]. This optimal value of $h/b$ occurs when the predicted elliptical pull-off force for interface separation initiating from the center of the contact area is identical to the optimal pull-off force, i.e., when $h/b$ is given by Eq. (13). The spherical pull-off force is always less than the elliptical pull-off force for a given $h/b$, and therefore the spherical geometry remains suboptimal. Thus, the Griffith fracture criterion results should be interpreted as upper bounds to the true pull-off force for these concave geometries, which is sufficient for the present purposes of obtaining an estimate of adhesion enhancement. To better understand the limitations of this model, adhesion experiments were performed on gelatin, as described below.

3 Experimental results for concave punch geometries

3.1 Experimental setup and procedure

To verify the analytical model presented in Section 2, pull-off experiments were performed on soft gelatin blocks. The low elastic modulus of gelatin allowed for these experiments to be performed using macroscale punches with precision-machined elliptical concave profiles.

To prepare the gelatin blocks, granular gelatin (MP Biomedicals, Solon, OH) was dissolved in water at 60°C, at a ratio of 1:8 by weight. In order to minimize dehydration of gelatin during the experiments, glycerol was added to the solution. The final solution contained 10wt% glycerol, 10wt% gelatin and 80wt% water. To obtain a gelatin block with a smooth and flat surface suitable for the contact experiments, the solution was then poured into a mold which had a flat glass plate on the bottom, and refrigerated at 4°C for 24 hours. When removed from the mold, the top surface of the gelatin block inherited the flatness and smoothness of the glass plate. The RMS surface roughness of the glass plate and the gelatin block were measured with a white
light interferometer (Zygo NewView 5000) and found to be less than 10 nm. The size of the gelatin blocks was approximately 10×10×7 cm. The gelatin blocks were allowed to equilibrate to room temperature for 4 hours before testing.

Aluminum punches with radius \( b = 12.7 \text{ mm} \) were used in these experiments and modeled as rigid in comparison to the gelatin. A series of 17 elliptical concave punches with the profile of Eq. (12) were machined with a CNC lathe for \( 0 \leq h/b \leq 0.2 \). A convex punch with spherical radius of 97 mm was also machined for the purposes of determining the material properties of the gelatin. For each of the concave profiles, a single small hole (0.5 mm diameter) was drilled in the center of the punch to vent trapped air and allow full contact to be possible for all \( r < b \). Although the machined punch surfaces were not particularly smooth, hand polishing may have distorted the machined profiles and thus was not performed. The RMS roughness of the aluminum surface was measured with a white light interferometer and found to be 350 nm. This roughness was uniform for all punches.

The details of the experimental setup are illustrated in Fig. 5. For each experiment, an aluminum punch was attached to a high precision miniature load cell (Honeywell Sensotec; load range \( \pm 10 \text{ N} \), force resolution 10 mN). The applied compressive and adhesive forces measured during the course of the experiments were of the order of 0.1–1 N. The load cell was attached to the fixed crosshead of an electro-mechanical universal testing machine (Instron 5800), which had a displacement resolution of 1 \( \mu \text{m} \). The gelatin block was placed on a transparent tilt stage which was used to align the gelatin surface normal to the punch axis. Beneath the tilt stage, a 45° mirror was used to view the contact area and measure the contact radius during the experiment. In order to minimize the loading rate effects caused by the viscoelastic nature of the gelatin [20], the crosshead velocity was kept constant at 3 mm/min in all experiments. During the experiment, the load cell output was amplified and recorded, along with the crosshead displacement, by the Instron data acquisition system at a sampling rate of 5 Hz.

### 3.2 Measurement of elastic modulus and work of adhesion

To determine the reduced elastic modulus \( E^* \) and work of adhesion \( w \) of the gelatin blocks, an indentation and pull-off cycle was performed using the convex spherical punch with radius \( R = 97 \text{ mm} \). Digital images of the contact area were captured every two seconds with a Nikon D80 digital camera and the resulting data were fit to the JKR adhesion theory [19] using
the method of Chaudhury et al. [21], who rearranged the JKR relation between load $P$ and contact radius $a$ as

$$\frac{a^{3/2}}{R} = \frac{1}{K} \frac{P}{a^{3/2}} + \sqrt{\frac{6\pi w}{K}} \quad (34)$$

where the bulk modulus $K = 4E^*/3$. Thus, a linear relationship is obtained between the variables $P/a^{3/2}$ and $a^{3/2}/R$, with slope $1/K$ and intercept $\sqrt{6\pi w/K}$. Data collected during both approach and withdrawal from the gelatin blocks is shown in Fig. 6. Because of adhesion hysteresis, the measured work of adhesion during approach ($w = 0.04 \text{ J/m}^2$), is much lower than that measured during withdrawal ($w = 0.2 \text{ J/m}^2$). (The effect of adhesion hysteresis is discussed in more detail by Chen et al. [22], Choi et al. [23], She et al. [24], and Silberzan et al. [25]) Since the pull-off forces required for separation during withdrawal from the gelatin samples are of interest, $w = 0.2 \text{ J/m}^2$ is used. The reduced moduli fit from the approach and withdrawal data are virtually identical; for consistency with the work of adhesion, the modulus fit from withdrawal ($E^* = 17.6 \text{ kPa}$) is used.

### 3.3 Experimental results and discussion

For each concave aluminum punch, an approach-withdrawal cycle was performed on gelatin and the maximum tensile force sustained before separation during withdrawal was recorded as the pull-off force. Three tests were performed for each profile, resulting in three data points for each value of $h/b$ tested; these data points were then normalized by the average experimental value of $\bar{P}_{flat}$. These results are shown in Fig. 7. Pull-off point is the peak in the force-displacement plots, which are similar to those shown by Guduru and Bull [9]. The force-displacement The pull-off force data qualitatively matches the trends predicted by the analytical model for increasing $h/b$. Three distinct regions are seen: for low $h/b$, separation initiates at the periphery (as observed during the experiment using the 45° mirror); for high $h/b$, separation initiates at the center of the contact area; and for intermediate $h/b$, the initial separation location is seen to be highly sensitive to the alignment of the punch with the gelatin surface, and can
fluctuate between tests. For this intermediate range $h/b$, the pull-off force is seen to remain relatively constant.

To quantitatively compare these results to the analytical model for the elliptical concave punch, the dimensionless parameters $\sigma_{th}/E^*$ and $\sigma_{th}b/w$ must be determined. The values of $E^*$ and $w$ were determined independently using the JKR adhesion test described in Section 3.2, and $b$ is the fixed radius of the aluminum punches. The value of $\sigma_{th}$, however, cannot be determined independently and must be fit to the measured pull-off force data. Figure 7 shows the analytical results for two choices of $\sigma_{th}$ which best fit the data: $\sigma_{th}/E^* = 0.18$ (dashed-dot line) and $\sigma_{th}/E^* = 0.19$ (dashed line). The Griffith criterion result $P_{opt}/P_{flat}$ remains unchanged for both values of $\sigma_{th}/E^*$ because $E^*$, $w$, and $b$ remain fixed; as discussed in Section 3.1, it overpredicts the measured pull-off force due to integration of physically unrealistic singular tractions. Both the optimal pull-off force $P_{opt}/P_{flat}$ and center-separation pull-off force $P_{opt}/P_{flat}$ increase for the higher value of $\sigma_{th}/E^* = 0.19$. The experimental results are 75-80% of the value of $P_{opt}/P_{flat}$ predicted by the model for these values of $\sigma_{th}/E^*$. It is likely that the presence of the small central air hole introduces a local flaw into the contact area which reduces the maximum tensile force sustainable before separation in the experiments. It is also likely that the surface roughness of the punch interferes with obtaining a good fit for the strength of adhesion $\sigma_{th}$; thus, the fitted value of $\sigma_{th}$ should not be thought of as a molecular-level intrinsic quality, but rather an effective strength of adhesion modulated by the surface roughness, similar to those obtained from the contact experiments performed on a micrometer-scale asperity with nanometer-scale roughness by Li and Kim [26]. Even with the above limitations, however, the analytical model captures the trends in pull-off behavior of the elliptical concave punches with increasing $h/b$ well.

The experimental data supports the analytical predictions of significant adhesion enhancement being possible simply due to a concave geometry. The results show that a three-fold enhancement in pull-off force over that of a flat punch is seen for a range of $h/b$, indicating that enhanced adhesion strength is not tied to a unique geometry. Achieving the optimal pull-off force, however, would require a flaw-free interface and a precise geometry tied to a unique $h/b$. Thus, concave adhesive systems seen in nature may significantly enhance adhesion but likely do not “optimize” it.
Rather than focusing on adhesion enhancement through shape optimization, Gao and Yao [5] used their results to emphasize the concept of achieving robust, flaw-tolerant adhesion through size reduction, and noted that the critical fiber size at which a flat geometry has the same pull-off force as the optimal concave geometry is given by

\[ b_{cr} = \frac{8 E^* w}{\pi \sigma_{th}^2}. \] (35)

For the ‘dry’ van der Waals adhesion seen in geckos, Gao and Yao [5] estimated \( w = 10 \text{ mJ/m}^2, \sigma_{th} = 20 \text{ MPa}, E^* = 1 \text{ GPa}, \) and hence \( b_{cr} \approx 64 \text{ nm}, \) indicating that the small size of the gecko fibrillar spatula (\( b \approx 100 \text{ nm} \)) may be the result of natural selection to ensure a shape-insensitive optimum fracture strength. In the present case, the experimentally determined values of \( w = 200 \text{ mJ/m}^2, \sigma_{th} \approx 3.2 \text{ kPa}, \) and \( E^* = 17.6 \text{ kPa} \) indicate \( b_{cr} = 0.9 \text{ mm} \) for the gelatin-aluminum system. Below this critical size, the pull-off forces for the flat and concave shapes should converge to the same value; above this critical size, adhesion enhancement is predicted to be possible through concave geometries. The details of this predicted transition from shape-sensitive to shape-insensitive adhesion with decreasing contact size could be investigated with contact experiments on gelatin similar to those described here. Gelatin is much more compliant than the keratin found in gecko attachment systems, but other biological adhesive systems have an elastic modulus similar to that of gelatin, including the adhesive pads of the bush cricket Tettigonia viridissima studied by Gorb et al. [27], for which \( E^* = 50 \text{ kPa}. \)

**2.4 Summary**

In this paper, an analytical model was developed for the pull-off forces required to separate a rigid axisymmetric concave punch from an elastic half-space with the goal of understanding how concave shapes seen in both biological and man-made adhesive systems can outperform other geometries. Significant enhancement in adhesion strength compared to that of a flat punch was predicted for both spherical and elliptical concave punch profiles. Increasing the depth of the concavity initially increases the resulting pull-off force, but for higher depths the contact interface will separate before higher pull-off forces than those for the flat punch are achieved. Experimental results performed with machined aluminum elliptical concave punches
on gelatin qualitatively verified the trends predicted by the analytical model, and the maximum pull-off forces measured came within 80% of the predicted optimum pull-off force. These findings emphasize the benefits of concave geometries for designing interfaces which enhance adhesion through geometry alone. In the analysis and experiments presented in this paper, the indenting punch is rigid and the indented solid is elastic. In case of biological attachment, the situation is usually reversed, where the insect foot is more deformable than the surface on which it walks. Such a situation was considered by Buehler et al. [11], who employed atomistic simulations to study the effect of concave shapes when the indenter is deformable and the substrate is rigid. They showed that optimal adhesion can be realized even in the case of an elastic punch with a concave end indenting a rigid surface, which demonstrates the robustness of the concave shapes in enhancing adhesion strength.

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References


Figure Captions

Figure 1: A comparison of the predicted pull-off forces for a flat fiber to those with the “optimal” geometry found by Gao and Yao (2004) for a fiber radius $b$, reduced elastic modulus $E^*$, work of adhesion $w$, theoretical strength of adhesion $\sigma_{th}$. For smaller $b$, the numerical results (square dots) for a flat-tipped fiber converge to the analytical solution for the optimal pull-off force, indicating that size reduction is a mechanism for achieving optimal adhesion. For lower $E^*$ or larger $b$, significant adhesion enhancement is possible via shape optimization.

Figure 2: Geometry of the concave punch contact problem. An axisymmetric rigid indenter with radius $b$ and maximum depth of concavity $h$ is pressed into an elastic half-space with load $P$, such that the displacement under the indenter is described by $u(r)$ with maximum displacement $d$.

Figure 3: Comparison of pull-off force predictions for spherical and elliptical concave geometries. (a) For higher $\sigma_{th}/b/w$, the elliptical geometry achieves the optimal pull-off force but the spherical geometry falls short for all $h/b$. (b) For lower $\sigma_{th}b/w$, the spherical geometry is also predicted to achieve the optimal pull-off force. Physically realizable portions of the plots are shaded.

Figure 4: Effects of the dimensionless parameters $\sigma_{th}/E^*$ and $\sigma_{th} b/w$ on the pull-off force for the concave elliptical geometry. (a) Changing $\sigma_{th}/E^*$ while holding $\sigma_{th}b/w$ constant. (b) Changing $\sigma_{th}b/w$ while holding $\sigma_{th}/E^*$ constant. Physically realizable portions of the plots are shaded.

Figure 5: Schematic of the experimental setup. An aluminum concave punch is pressed into full contact and then pulled off a gelatin block, with forces $P$ and displacements $d$ continuously measured by an Instron mechanical testing machine. A tilt stage provides for alignment of the top gelatin surface normal to the punch, and a mirror allows for observation of the contact area.
Figure 6: Fitting of $E^*$ and $w$ to data from an approach and withdrawal cycle for the 97 mm convex spherical punch, using Eq. (34) to represent the JKR relationship between load $P$ and contact radius $a$. The work of adhesion is higher during the withdrawal cycle due to adhesion hysteresis.

Figure 7: Experimental results for pull-off forces of aluminum concave punches from gelatin. For low $h/b$, separation initiated from the periphery, whereas for high $h/b$, separation initiated from the center of the punch. The values of $\sigma_{th}/E^*$ shown are best fits to the data. The highest pull-off forces measured are approximately three times greater than the pull-off force measured for the flat punch.
Figure 1

- Optimal shape
- Adhesion enhancement by shape optimization
- Flaw tolerance by size reduction
- Flat punch-Griffith criterion
- Flat punch-numerical results (Gao and Yao, 2004)

The graph shows the pull-off force ratio $\hat{P}/P_{\text{flat}}$ as a function of $(E^*w/\sigma_{th}^2b)^{1/2}$. The optimal shape and adhesion enhancement are indicated by curves, while the flaw tolerance by size reduction and flat punch-Griffith criterion are shown by dashed lines.
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6
Figure 7