

Time Travel and Consistency Constraints

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The possibility of time travel, as permitted in General Relativity, is responsible for constraining physical fields beyond what laws of nature would otherwise require. In the special case where time travel is limited to a single object returning to the past and interacting with itself, consistency constraints can be avoided if the dynamics is continuous and the object's state space satisfies a certain topological requirement: that all null-homotopic mappings from the state-space to itself have some fixed point. Where consistency constraints do exist, no new physics is needed to enforce them. One needs only to accept certain global topological constraints as laws, something that is reasonable in any case.

1. Introduction. The possibility of time travel into the past typically imposes constraints on what one can do, constraints that merely reflect the impossibility of an inconsistent physical evolution. In preventing inconsistencies, the constraints have remarkable effects, permitting violations of thermodynamic regularities and even quantum mechanical statistics. While potentially dramatic, consistency constraints nevertheless demand no new physics. If they are laws at all, they are laws derivable from either the dynamics alone or the dynamics together with some conditions on the global topology of the universe. At most, the possibility of time travel motivates us to accept certain kinds of global boundary conditions as laws of nature.

2. Preliminary Distinctions. In the General Theory of Relativity (GTR), spacetime is represented by a differentiable manifold M with a metric g . The physical state is represented by various fields over M , like scalar or vector fields, and these fields contribute to the stress-energy tensor field T , that in turn, is related to the spacetime's metric structure by Einstein's field equations. In GTR, the histories of ordinary particles are represented by time-like or null curves in spacetime. With the right M , g there can be

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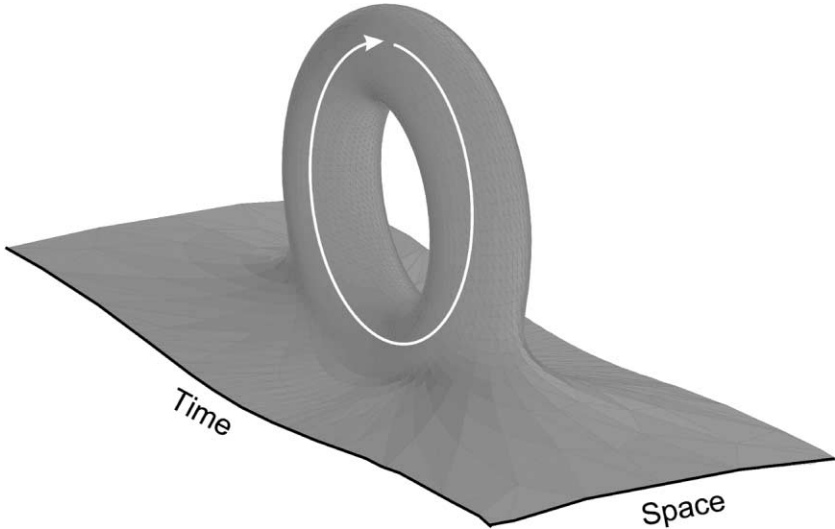


Figure 1

closed time-like curves, which is the relativistic version of a path for possible time travel. There are two ways to get closed time-like curves. One is for the local physical fields to evolve according to the dynamics in such a way as to generate a topological structure like the jug handle pictured in Figure 1. If such a local topological feature imposes consistency constraints, we can call them dynamical.

Another way to get closed time-like curves is to have a special global spacetime topology. In GTR, the field equations determine the local metrical structure of spacetime but leave open its global topology. The same matter content can appear in two different spacetimes, one with constraints and one without. The spacetime of special relativity, with topology \mathbb{R}^4 and the Minkowski metric, has no closed time-like curves, but Figure 2 illustrates a similar spacetime with closed time-like curves due to its $\mathbb{R}^3 \times S^1$ topology (pictured as $\mathbb{R}^1 \times S^1$). Consistency constraints arising from global topologies like this can be called nondynamical.

An example of how nondynamical consistency constraints arise is to take the $\mathbb{R}^3 \times S^1$ spacetime and place a separated pair of mutually attracting, sticky test particles initially at rest (Figure 3). As the state evolves, the particle paths wrap around the cylinder until they intersect the initial state in between the initial particle positions, contradicting the assumption that there were initially only two particles. To clarify the nature of such contradictions, we distinguish two kinds of possibility. A state is locally possible if it is compatible with all the laws of nature. A state is globally possible if that state can be extended to some lawful physical state

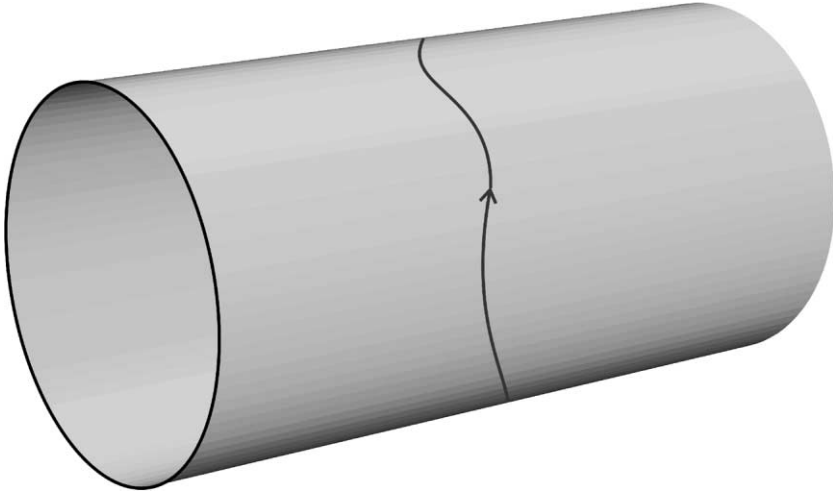


Figure 2

covering the entire universe. Thus, the initial $t = 0$ state is locally possible, but not globally possible. It follows by definition that if a state is globally possible, it is locally possible, but Figure 3 illustrates a counterexample to the converse.

3. What's the Problem with Time Travel? The usual stock of time-travel paradoxes begs us to distinguish between global and local possibility. In stories where the protagonist time travels into the past to kill himself as a youngster, it is purportedly paradoxical how the time traveler can kill himself. Yet, the resolution of the paradox is well known. The time traveler can kill his earlier self in the local sense because given only the local physics, the time traveler is no less capable of murder for his time traveling: he has his trusty sword, the requisite malice, etc. It is impossible for him, though, to kill himself in the global sense because there is no possible world (barring resurrection) where he dies as youth and then later journeys back through time. Conflicting intuitions about the abilities of the time traveler arise only by equivocating between local and global possibility. The global/local distinction makes the world safe for time travel.

John Earman (1995) contests this resolution of the time-travel conundrum. A reasonable complaint can be made, he claims, that the notion of physical possibility is equivalent to compatibility with the laws of nature and hence is properly identified with local possibility. This leaves global possibility as a “label not backed by any explanatory power” (1995, 175). In response, Earman suggests we work to reduce global possibility to local possibility by seeking new (local) laws that will enforce the consistency

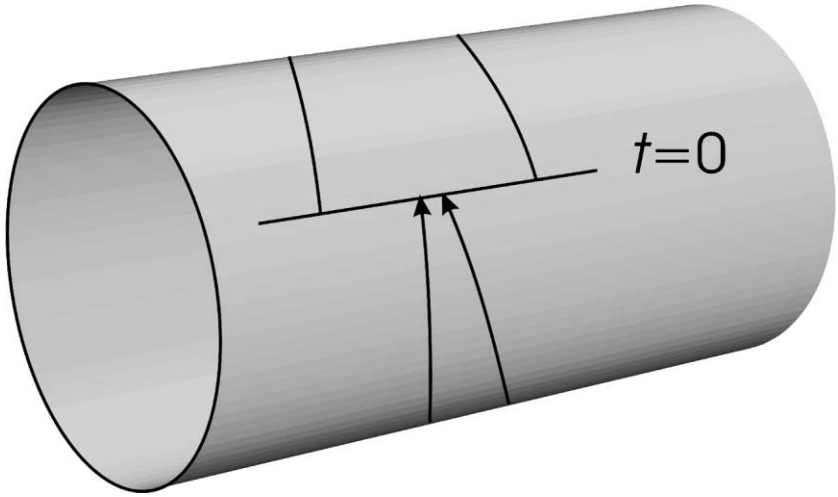


Figure 3

constraints even though he is pessimistic about the prospects for success and is not confident that the original complaint is justified. Earman's suspicions about his complaint are justified. The complaint is not backed by a substantive argument, and its prima facie plausibility is undermined with a closer examination of the concept of global possibility.

In distinguishing local from global in the context of GTR, care needs to be taken to respect the proper relationship between matter content and geometry. Because the physical fields are coupled to the metric, different local configurations of matter will, in general, necessitate a different M, g . Strictly speaking, we cannot fix an M, g and then ask what consistency constraints are implied by various matter configurations. Rather, a local state has to be defined as some physical fields on some local part of a manifold together with a compatible metric. But since the appropriate part of a manifold is a manifold in its own right, a locally possible state is ipso facto a globally possible state. At this point, the global/local distinction threatens to crumble and along with it, our explanation of the time-travel paradoxes. In response, to give the global/local distinction some teeth, one restricts consideration to maximal spacetimes. A maximal spacetime is essentially one where the manifold (with metric) cannot be extended further. It is defined as a spacetime where there is no M', g' with the same dimension and number of connected components as M into which M, g can be isometrically imbedded. A globally possible state is now defined as any state that is a submanifold (with fields) of a lawful, maximal spacetime. Under this more precise definition, one cannot take any old snippet of a

manifold, slap some law-abiding fields on it, and call it a globally possible state; it must have a consistent maximal extension. (This definition of global possibility would be nearly worthless if the actual world did not count as globally possible. Fortunately, we have no evidence that our spacetime is submaximal. So far as these things can be known, our spacetime has no edges or holes except possibly where they are unavoidable, for example, in black holes. In what follows, I assume it is true with no further justification offered.)

Maximality ought to be taken not only as true but as a law of nature. First, maximality is a very simple claim that is extremely informative. That it is simply formulated is clear enough; that it is informative is justified by its ruling out spacetimes that have abrupt edges and by the kinds of consistency constraints it imposes. Under some best systems' account of law, this in itself is a significant reason to think that it might be derivable from the best set of axioms describing physical facts. Even under other accounts of laws, simplicity and informativeness can count as significant evidence of its being a law.

Second, it plays an important role in grounding counterfactuals. For example, when evaluating whether the match would have lit, had it been struck, one should not be considering possible worlds where the match was struck and then the universe immediately ends. Taking maximality as a law protects *in general* against such deviant evaluations if we do the reasonable thing and take the relevant possibilities under consideration to be possibilities where the actual laws of nature hold (at least over the relevant physical evolution).

One reason that boundary conditions are not usually thought of as laws is that in scientific practice, they are usually not laws. For most systems a scientist bothers to consider, the boundary conditions are contingent because they are just the physical state at the boundary of whatever interests the investigator. The universe's boundary conditions have no such contingency to them, but one still might think that because there is only one global boundary condition, we have no compelling reason to say that this particular boundary condition is accidental or necessary. Yet, regardless of whether one should count all facts about the actual boundary conditions as laws, there remains some reason to think that the particular boundary condition of interest—the universe's maximality—is a law. Unlike facts like "a neutrino is present at such-and-such a location on this edge of spacetime," maximality plays a role in routine scientific induction. It is a precondition of the limiting case of the principles (1) that the future will resemble the past and (2) that the laws here are the laws way over there. Our as-yet-successful induction that the physics laws of today are the physical laws of tomorrow and that the physics here is the physics over there requires that there be a tomorrow and an over there. The

maximality of spacetime implies that, barring troublesome dynamics, there will be a tomorrow and an over there.

Once we take maximality as a law, the global/local distinction used to explain time-travel paradoxes is quite natural. When one considers the possibilities open to the time traveler, one can try to hold all the laws fixed, or one can be more liberal, keeping the dynamical laws but forgoing boundary condition laws like maximality. Distinct senses of physical possibility thereby earn their keep. We need not follow Earman's suggestion to seek a univocal concept of physical possibility.

The only problem with the time travel paradox, then, is as follows. Because we usually suppose that our spacetime is maximal, we can expect the local data in our world to be globally possible. This means, if we look hard enough, we might find conditions on the local data that are stricter than what we would otherwise admit. These consistency constraints will be significantly different in structure from other kinds of constraints. Gauss's law, for example, constrains the electric field to respect the location of charge sources so that the flux through any closed surface equals the amount of charge that surface encloses. This reduces the space of possible states to some subsurface. Consistency constraints, however, effectively ask one to calculate all possible dynamical evolutions from a given state and if an inconsistency arises in every evolution, the state is impossible; otherwise it is possible. The result is that the space of states allowed by consistency constraints can be expected in general to be not a simple subsurface, but some much messier space. The possibility of time travel, then, imposes unfamiliar constraints on local data.

4. Topological Constraints on Time Travel. There are many ways to explore the conditions under which consistency constraints arise (see Earman, 1995 for a review). One approach comes from an insightful idea from Wheeler and Feynman (1949) who recognized that under continuous dynamics, strong constraints on the physics exist that can ensure consistent solutions to the dynamical equations. Maudlin (1990; see also Arntzenius and Maudlin 2000) applied this idea by investigating conditions under which the continuity of the dynamics could free us from the scourge of consistency constraints. The kinds of time travel Maudlin considers are equivalent to situations where an object starts towards a wormhole, interacts with a future stage of itself, travels through the wormhole back in time, and then interacts with its younger self. This can be modeled with the spacetime shown in Figure 4 where the spatial squares U and V are identified topologically. One assumes here that the kind of interaction that occurs between the younger and older versions of the object that we throw in the wormhole is never enough to destroy the younger object or to make it miss the wormhole. Set aside concern about the impossibility of an object

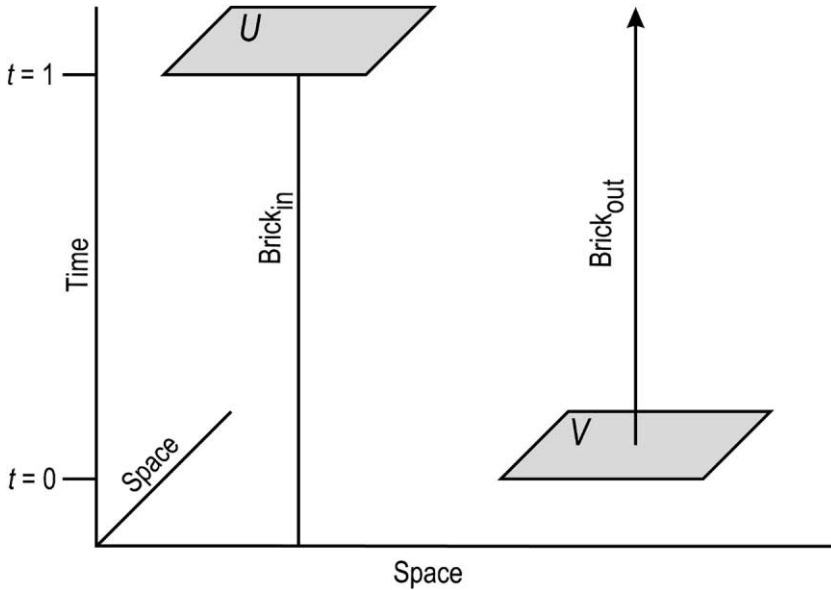


Figure 4

coming out of a wormhole to prevent its younger self from going into the wormhole and focus instead on some space of internal states of the object to determine under what conditions these evolve in an inconsistent way. Further assume that the dynamics governing the interaction of the objects is to be continuous. Otherwise, the dynamics is unconstrained. One is free to consider any interaction one wishes by including any number of additional (non time-traveling) objects and even by defining arbitrary interaction forces, no matter how unrealistic.

For an example of a situation that *prima facie* appears to cause trouble, let the single object that goes through the wormhole be an ordinary brick. Let “brick_{in}” denote the brick during its stage shown on the left, i.e., before it reaches the wormhole and let “brick_{out}” be the name for its later stage. Set the interaction between brick_{in} and brick_{out} so that a thermometer measures the temperature, T , of brick_{out} and then some machinery heats or cools brick_{in} according to the following rule: If $T > 300\text{K}$, cool brick_{in} to less than 200K . If $T \leq 300\text{K}$, heat brick_{in} to over 400K . If perfectly implemented, this would create a consistency constraint that would prevent anyone from throwing a brick into the wormhole. This can be seen in Figure 5, where the solid curve represents the temperature of brick_{in} after it has undergone the heating or cooling interaction and just before it enters the wormhole as a function of brick_{out}’s temperature. Since the wormhole does not affect the temperature, brick_{in} must have the same temperature as

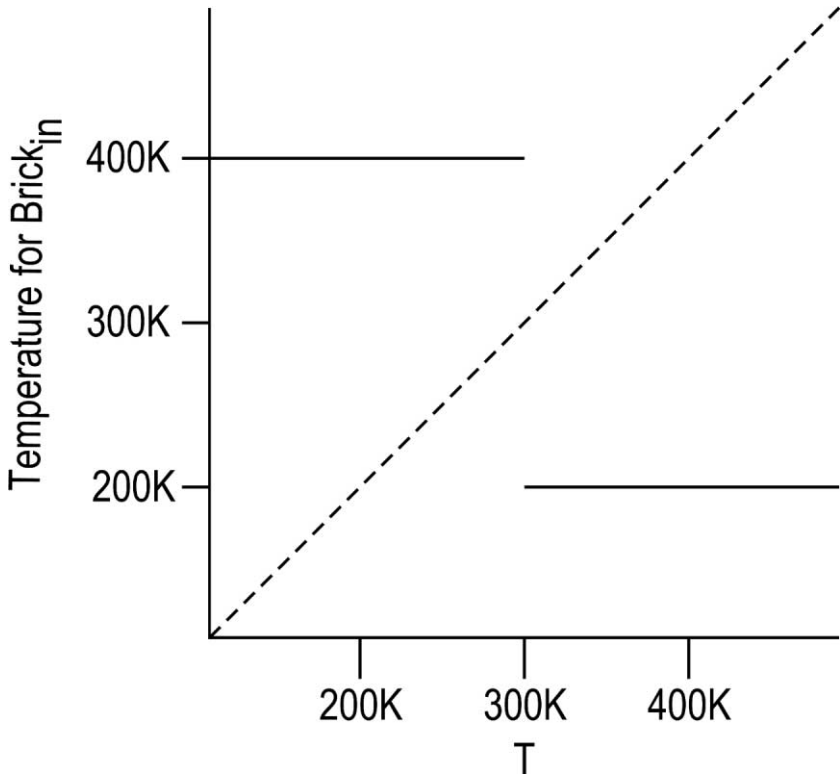


Figure 5

brick_{out}, shown as a diagonal on the graph. That the curves do not intersect indicates that there is no consistent solution to the dynamical equations. If perfectly implemented, this rule would prevent anyone from throwing a brick into the wormhole because there is no consistent global extension for any such state.

In reality, any implementation of the heating and cooling procedure described above can only approximate the given rule with some continuous graph (Figure 6) where the curve intersects the diagonal. Because any curve representing a continuous evolution that approximates the original rule will intersect the diagonal, there will always be a globally consistent solution. Any attempted implementation of such a setup will produce a consistent evolution: Any brick thrown at the wormhole will be heated or cooled until its temperature reaches the temperature at the intersection point on the graph.

While the continuity condition spares us from having that one rule imposing consistency constraints, there are more troublesome rules. Suppose

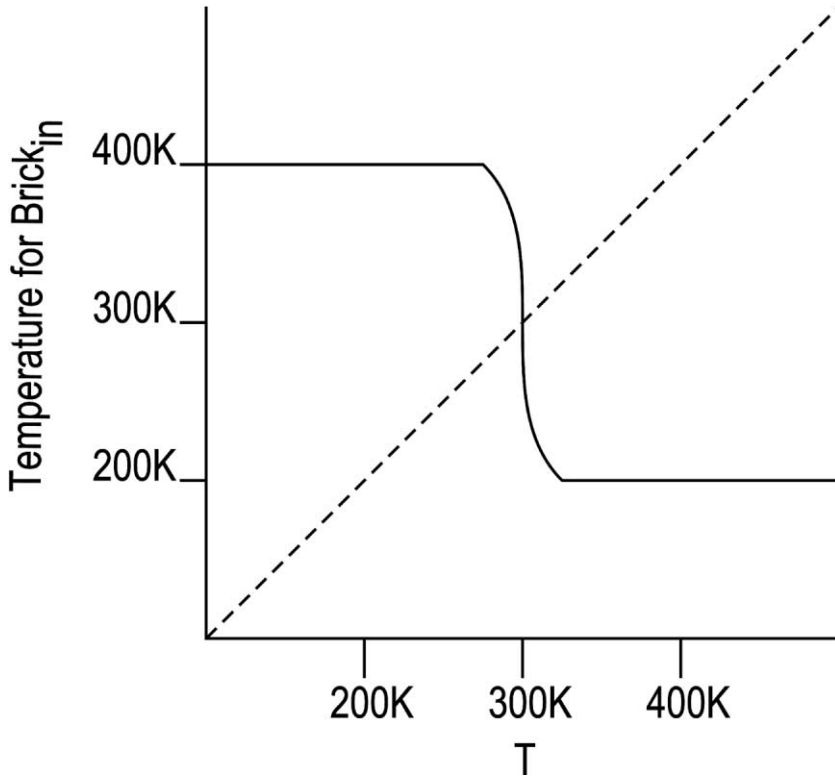


Figure 6

we set up the mechanism to heat or cool the incoming brick until it reaches $T/2$, half the temperature of $\text{brick}_{\text{out}}$. There are two cases to consider. If the possible temperatures of the brick range over the half-closed interval $[0, \infty)$, then from the graph, we can see that there is a consistent solution at $T = 0$. But if the temperature ranges only over the open interval $(0, \infty)$, then the $T = 0$ solution is not possible. Indeed, no solution is possible. Thus, if the brick is incapable of being cooled to $T = 0$, it will be impossible to throw the brick into such a wormhole setup. This presents the curious circumstance where the existence of consistency constraints on initial conditions depends critically on whether the state space of a potential time traveler is an open set.

Before addressing this curiosity, we can try to generalize the arguments above to find the precise criteria that determine whether consistency constraints exist. In general, the state space of an object is given by some topological space, Y . The dynamics that affect the evolution of both the incoming and outgoing objects can depend on the state of the object as it is

incoming, the state of the object as it outgoing, and a bunch of external factors. Whatever the external factors are, we know that their effect on the object is continuous over time, so we can bundle all these effects together and say that they affect the joint state of the incoming and outgoing stages of the object continuously. So our configuration space is the product space $Y \times Y$. The goal is to determine, for any initial state of the incoming object that we choose to fix at time $t = 0$, whether that initial state has a permissible dynamic continuation for all possible continuous dynamics it may experience. If some initial state plus dynamic environment—including rules about how objects evolve as well as what external factors are present—precludes a consistent evolution, then we have a consistency constraint.

Start by picking an arbitrary point in Y , corresponding to picking an arbitrary but fixed initial state a for the object that is being thrown in the wormhole. This constrains the state of the joint system at $t = 0$ to the surface (a, Y) . The continuity restriction on the dynamics means that one is restricted to performing a (continuous) deformation of this surface in the larger space $Y \times Y$. To see this, consider the special case where $Y = S^1$, the topology of a circle (Figure 7). The configuration space in such a case is

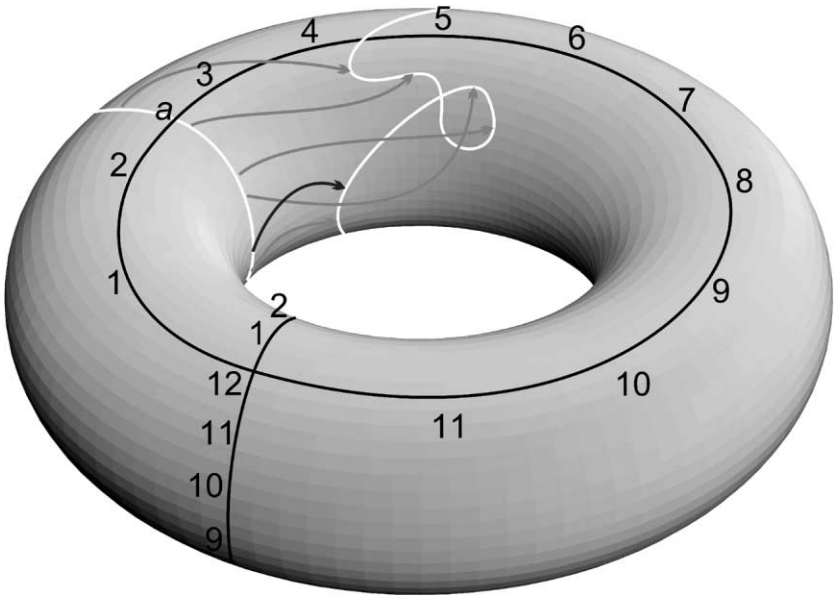


Figure 7. The two coordinates axes are labeled like a clock. The two lighter shaded loops around the torus are the possible initial states and possible final states. The arrows mark the evolution for sample states. The darker arrow shows a permissible evolution from (3.9, a) to (3.9, c), because it fits the $U = V$ condition whereas the lighter arrows signify impermissible evolutions.

the torus, $S^1 \times S^1$, and the initial state of the joint system is a circle on this torus. There are two continuity constraints at work. First, the dynamics does not allow any state to hop discontinuously to a new state as time passes continuously. This means that each point of the initial circle, considered individually, will traverse a path on the torus continuously over time. Second, the dynamics preserves continuity in a set of states. From the example in Figure 6 we observed that the dynamics we see in nature never breaks apart continuous states (like the temperatures around $T = 300\text{K}$) into disjoint sets. We always have as possibilities a continuous set of intermediate states between any two states, at least locally. This condition implies that we can never break apart the loop as it evolves over time. These two constraints combine to make the set of possible states equivalent to the set of possible deformations of the circle.

The function describing how a curve continuously deforms over the (time) interval $[0, 1]$ is called a homotopy, and closed curves that can be continuously deformed into one another are said to be homotopically equivalent or homotopic, for short. In looking for consistency constraints, we seek loops that are homotopic to the initial loop, but which fail the $U=V$ condition imposed by the wormhole topology. The $U=V$ condition means that what goes into the wormhole equals what comes out. This means that there is some $b \in S^1$, such that the system at $t = 0$ is in state (a, b) and at $t = 1$ is in state (b, c) , where a is our fixed initial state, and c is a state of the outgoing object after it has become irrelevant. The important question is whether there is some point in the loop whose initial second coordinate is equal to its final first coordinate. The trick to seeing how this condition is met is to interpret the loop in $S^1 \times S^1$ as a mapping from the second coordinate space to the first coordinate space. Seen this way, the initial state of the joint system is a constant map because it projects every point in the second coordinate space, the loop, down to the single point a in the first coordinate space. The $U=V$ condition becomes the condition that the map has some fixed point. We will find consistency constraints just in case we find a fixed-point free mapping that is homotopic to a constant map, i.e., is null-homotopic. A general parameter space Y will avoid the need for consistency constraints iff all of Y 's null-homotopic mappings into Y have a fixed point.

The open question for topologists is, "What spaces have this property?" Given the simplicity of the problem's formulation, it may be surprising that so little recent work has contributed directly to our understanding of these spaces. The classic work of Lefschetz (1937) is of tremendous importance. We can apply his famous fixed point theorem to show that for compact polyhedra, at least, null-homotopic maps always have fixed points. To see how his theorem applies, note that a polyhedron is defined to be a space homeomorphic to a simplicial complex, which is just a bunch of points, line segments, triangles, tetrahedra, and their higher dimensional analogs,

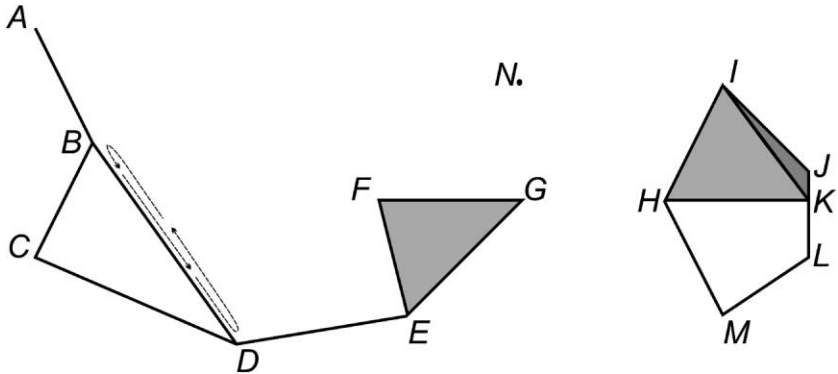


Figure 8. A Simplicial Complex.

unioned together in a reasonable way. For example, line segments only connect with each other at their endpoints and only connect to triangles by fitting perfectly along an edge or by touching an endpoint to a vertex (Figure 8). The Lefschetz number for a mapping $f: Y \rightarrow Y$, is defined by $L(f) = \sum_{n=0} (-1)^n \text{tr}(f_{*n})$ where the f_{*n} represent homomorphisms induced by f on the n th homology group, H_n . L is the same for any two maps that are homotopic and the Lefschetz fixed point theorem says that if $L \neq 0$ for a compact polyhedron, then f has a fixed point. So all we have to prove is that for null-homotopic maps, $L \neq 0$.

A simplistic picture of what these f_{*n} s are doing is that they are giving us information about the failure of n -dimensional surfaces to be contractible. For the $n = 0$ case, one considers how 0-dimensional surfaces (vertices) contract. Trivially, they contract to points, so f_{*0} is the identity homomorphism and its trace is the dimension of f_{*0} . For $n = 1$, one considers whether closed paths can contract to points. In general, paths will not always contract. For example, a loop around the line segments BC , CD , DB cannot contract because there is no triangular face spanning the interior of BC , CD , DB , but a loop around ΔEFG can contract. If there is a failure of contractibility for any loop, we get nontrivial matrices for f_{*1} . Luckily, because we are concerned only with null-homotopic maps, we only need to consider the failure of contractibility for loops that are homotopic to constant loops, like the dashed line in Figure 8. By definition, these loops can always contract to a point regardless of the structure of the simplicial complex. This implies that for all $n > 0$, f_{*n} is a null map, represented by a matrix full of zeros. This means $\text{tr}(f_{*n}) = 0$. Thus, we can ignore all the higher terms in the Lefschetz formula. Thus, $L(f) = \text{dim}(f_{*0}) > 0$ which guarantees a fixed point for compact, polyhedral spaces.

It would be too good to be true if the spaces that avoid consistency constraints turned out to be simply the compact spaces. As usual, it *is* too good to be true, as we know from Vladimir Okhezin's (1995) generalization of the fixed-point theorem to some noncompact polyhedra. Specifically, he shows that null-homotopic mappings are guaranteed to have a fixed point if and only if the polyhedron is rayless. A rayless polyhedron is homeomorphic to a simplicial complex where no path along the line segments goes out to infinity. An example that highlights the generalization is the "fan of many polygons." To construct it, label a three-dimensional Euclidean space with cylindrical coordinates, start at the origin, and draw three connected line segments of fixed size to form the edges of a (hollow) triangle lying completely in the half-plane at $\theta = K$ (Figure 9). Then trace the edge of a square in the same way in $\theta = K/2$, a pentagon edge in $\theta = K/3$, a hexagon edge in $\theta = K/4$, etc. It contains no rays that go out to infinity because any path that goes away from the origin must go along the edge of some polygon, and traveling around a polygon always leads you back to the origin. So by Okhezin's proof, it has a fixed point for null-homotopic

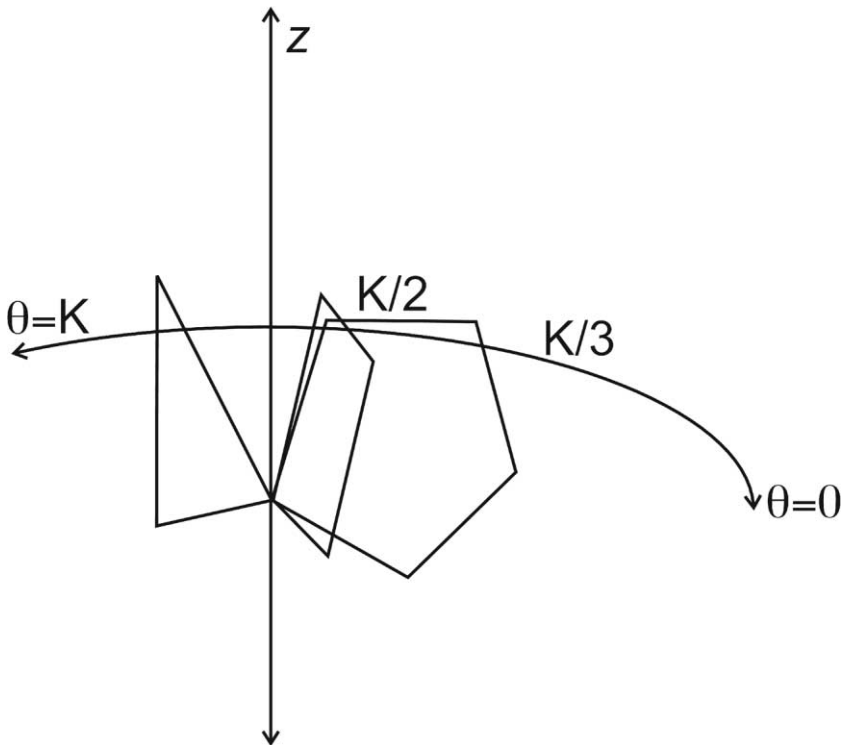


Figure 9

mappings. Yet, the “fan of many polygons” is not compact because the sizes of the polygons grow unbounded as $\theta \rightarrow 0$.

While there are certainly more details to explore in finding other spaces where null-homotopic mappings have fixed-points,¹ one will likely struggle to find a realistic example from physics where the fine distinction between rayless and compact will find application. The puzzling fact from the standpoint of physics is that something like compactness matters at all. If the state space is the closed interval $[0, 1]$ we have a guaranteed fixed point. Remove a point to create the half-open interval $(0, 1]$ and the guarantee is gone. So the existence of consistency constraints depends on the exact range of the state space down to the last point. Yet, in many cases one will be hard pressed to find any empirical grounds for distinguishing between a compact and noncompact state space: Can the needle on that voltmeter go down to some precise minimum voltage level, or can it instead only go arbitrarily close to the lower bound? While for some system it might be possible that there is a matter of fact about the state-space topology, the smart money has it that the problem is generated by the artificial idealizations placed on the particular model of time travel being considered here. The time traveling object by presumption remains intact and travels through the wormhole no matter what, but if the relevant state of the object supervenes on the positions and momenta of particles or on the magnitude of some fields or similar variables, then the state space of the object extends into regions where the edge of the wormhole comes into play. For example, one might naïvely say that the state space for a magnetic compass has the topology S^1 , but a real compass needle has more degrees of freedom because it can be moved up and down and can be bent as well. Moreover, the matter comprising the compass has the dynamical freedom to break the compass into many small bits, leaving it with no state at all as a compass. These observations cast suspicion on whether there can be any real object with a rayless polyhedral state-space. It appears that—so far as the special case considered here is concerned—consistency constraints cannot be dismissed.

5. Are Consistency Constraints Laws of Nature? If the court of physics has ordered consistency constraints to be imposed on our world, topological arguments have so far failed to vacate that order. It remains then, to determine the legal status of these consistency constraints.

Any argument that the constraints themselves are stand-alone laws must deal with the extraordinary messiness of the constraints. In an $R^1 \times S^1$

1. For example, Okhezin characterizes some nonpolyhedral spaces as well: Any metric absolute neighborhood retract will have a fixed point for its null-homotopic mappings if and only if it is approximatively rayless.

spacetime, for example, we can place a pair of particles under a dynamics of classical gravitational attraction with perfectly elastic collisions. The particles will have a characteristic collision frequency that is a function of their initial separation distance and relative speed. Only if the temporal circumference is an integral multiple of that frequency is that state allowed. Simply adding a third particle complicates the consistency condition tremendously, as now there is no general analytic solution to the equations of motion. So even for a three-particle system under an extremely simple dynamics, the consistency condition is far more complicated than just restricting the manifold to $\mathbb{R}^1 \times S^1$.

The fact that consistency constraints are deducible from the dynamical laws that we already accept conjoined with simple topological conditions gives us good reason to dismiss the need for special laws to rule out CTC-promoted inconsistencies. If a local state has data that must evolve inconsistently around a CTC, the local dynamical laws conjoined with the data are sufficient to entail a contradiction. Thus, the dynamical consistency constraints are consequences of the dynamical laws alone. Nondynamical consistency constraints are deducible from other laws as well if only the global topological constraint that creates them is taken to be a law. To count the global topology as a law, one can mobilize the arguments that justified taking maximality as a law of nature. The global topology is presumably simple and certainly highly informative, and because it implies maximality, it has the previous arguments working in its favor. Even if one wants to be skeptical about whether the actual topology is nonaccidental, its broad strength in supporting counterfactuals, its simplicity, and its informativeness are sufficient to allow it to serve at least as a kind of second-class citizen in the society of natural laws. In this role, it is superior to special chronology protecting local laws, because it implies more about the physical world in a more concise way.

In the end we should not become apoplectic at the thought of having laws that permit closed time-like curves. We do not need any new physics to describe what happens around closed time-like curves, and we do not need laws to prevent time travelers from fiddling with history. We need only admit the global topology as a law, something that would be reasonable even without closed time-like curves.

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