

Logic—Sample Test A2 Answers

NAME \_\_\_\_\_

1. Define 'counterexample'. (10 points)

A counterexample is a possible situation where the premises of an argument are all true and the conclusion is false.

2. Define 'valid'. (20 points)

An argument is valid if and only if it has no counterexamples.

Identify the following arguments properly. If the argument is valid, mark it with a V. If it is invalid mark it with an I. (These problems are worth 2 points each)

3. Horses never eat Bermuda grass.

Goats never eat Bermuda grass.

Goats and horses never eat the same kind of grass.

Which is it: valid or invalid? INVALID

(counterexample: Horses and goats eat St. Augustine grass.)

4. All things that have motors require oil.

All Chevrolet vehicles require oil.

Thus, all Chevrolet vehicles have motors.

Which is it: valid or invalid? INVALID

(counterexample: Chevrolet makes a bicycle that requires oil.)

5. It looks like a duck.

It sounds like a duck.

It feels like a duck.

Thus, it is a duck.

Which is it: valid or invalid? INVALID

(counterexample: It's a very well crafted duck decoy.)

6. Porpoises are not dangerous to humans.

It is not the case that both sharks and porpoises are dangerous to humans.

Thus, sharks are dangerous to humans.

Which is it: valid or invalid? INVALID

(counterexample: Neither are dangerous to humans.)

7. Every white flower has a pleasant fragrance.

Thus, if a flower lacks a pleasant fragrance, it isn't white.

Which is it: valid or invalid? VALID

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Translate the following sentences into the language of sentential logic using the abbreviations given to you. (These problems are worth 2 points each.)

$G$  = "Gordon went to the bank."

$H$  = "Gordon stayed home."

$I$  = "Gordon repaired the motorcycle."

$M$  = "Max went to the bank."

$N$  = "Max stayed home."

$O$  = "Max repaired the motorcycle."

8. Max stayed home and repaired the motorcycle.

$N \& O$

9. If Gordon went to the bank, then Max stayed home.

$G \supset N$

10. If Max did not repair the motorcycle, Gordon did not go to the bank.

$\sim O \supset \sim G$

11. If Max stayed home and repaired the motorcycle, then Gordon stayed home too.

$(N \& O) \supset H$

12. Unless Gordon repaired the motorcycle, Max and Gordon both stayed home.

$\sim I \supset (H \& N)$

13. Gordon did not repair the motorcycle if Max did.

$O \supset \sim I$

14. The motorcycle was repaired by either Max or Gordon.

$I \vee O$

15. Gordon went to the bank only if Max repaired the motorcycle.

$G \supset O$

16. Max and Gordon both went to the bank unless Gordon did not repair the motorcycle.

$\sim \sim I \supset (M \& G)$

17. Max and Gordon did not both go to the bank.

$\sim(M \& G)$

18. Neither Max nor Gordon stayed home.

$\sim(H \vee N)$  which is equivalent to  $(\sim H \& \sim N)$

19. Max and Gordon repaired the motorcycle.

$I \& O$

20. Either Gordon or Max went to the bank, and whichever one of them did not go to the bank stayed home.

$(G \& N) \vee (M \& H)$

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Construct truth tables to test whether these arguments are valid or invalid. *In the case of an invalid argument, indicate the row or rows that show that the argument is invalid by one of them.* (These problems are worth 4 points each.)

$$\begin{array}{l} 21. \quad A \vee B \\ \quad \sim B \\ \hline \quad \sim A \supset B \end{array}$$

A	B	$A \vee B$	$\sim B$	$\sim A \supset B$
T	T	T T T	FT	FT T T
F	T	F T T	FT	TF T T
T	F	T T F	TF	FT T F
F	F	F F F	TF	TF F F

Which is it: valid or invalid? VALID  
(If it is invalid, circle the row that proves that it is invalid.)

$$\begin{array}{l} 22. \quad \sim E \supset \sim D \\ \quad D \& \sim E \\ \hline \quad E \end{array}$$

E	D	$\sim E \supset \sim D$	$D \& \sim E$	E
T	T	FT T FT	T F FT	T
F	T	TF F FT	T T TF	F
T	F	FT T TF	F F FT	T
F	F	TF T TF	F F TF	F

Which is it: valid or invalid? VALID  
(If it is invalid, circle the row that proves that it is invalid.)

$$\begin{array}{l} 23. \quad \sim R \vee (M \& \sim N) \\ \quad \sim(M \& R) \\ \hline \quad N \supset (\sim M \vee (N \& \sim R)) \end{array}$$

R	M	N	$\sim R \vee (M \& \sim N)$	$\sim(M \& R)$	$N \supset (\sim M \vee (N \& \sim R))$
T	T	T	FT F T F FT	F T T T	T F FT F T F FT
F	T	T	TF T T F FT	T T F F	T T FT T T TF
T	F	T	FT F F F FT	T F FT	T T T F T F FT
F	F	T	TF T F F FT	T F F F	T T T F T T TF
T	T	F	FT T T T TF	F T T T	F T FT F F FT
F	T	F	TF T T T TF	T T F F	F T FT F F TF
T	F	F	FT F F F TF	T F FT	F T T F T F FT
F	F	F	TF T F F TF	T F F F	F T T F T F TF

Which is it: valid or invalid? VALID  
(If it is invalid, circle the row that proves that it is invalid.)

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Test whether each of these sentences is a tautology, a contradiction, or a contingent sentence by constructing their truth tables. (These problems are worth 3 points each.)

24.  $(A \vee \sim A) \& (\sim A \vee A)$

A	$(A \vee \sim A) \& (\sim A \vee A)$
T	T T FT T FT T T
F	F T TF T TF T F

Which is it: a tautology, a contradiction, or a contingent? TAUTOLOGY

25.  $(A \supset C) \supset (A \supset \sim C)$

A	C	$(A \supset C) \supset (A \supset \sim C)$
T	T	T T T F T F FT
F	T	F T T T F T FT
T	F	T F F T T T TF
F	F	F T F T F T TF

Which is it: a tautology, a contradiction, or a contingent? CONTINGENT

Test each of the following pairs of sentences for logical equivalence by constructing their truth tables. Label for each whether they are equivalent or inequivalent. (These problems are worth 3 points each.)

26.  $(G \& \sim H) \supset \sim G$                        $(G \supset H) \vee \sim G$

G	H	$(G \& \sim H) \supset \sim G$	$(G \supset H) \vee \sim G$
T	T	T F FT T FT	T T T T FT
F	T	F F FT T TF	F T T T TF
T	F	T T TF F FT	T F F F FT
F	F	F F TF T TF	F T F T TF

Which is it: equivalent or inequivalent? EQUIVALENT  
(If they are inequivalent, circle any row that proves that they are inequivalent.)

27.  $\sim(\sim A \vee \sim B)$                        $A \vee B$

A	B	$\sim(\sim A \vee \sim B)$	$A \vee B$
T	T	T FT F FT	T T T
F	T	F TF T FT	F T T
T	F	F FT T TF	T T F
F	F	F TF T TF	F F F

Which is it: equivalent or inequivalent? INEQUIVALENT  
(If they are inequivalent, circle any row that proves that they are inequivalent.)

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28.  $(A \vee B) \& (B \vee A)$                        $(A \& B) \vee (A \& B)$

A	B	$(A \vee B) \& (B \vee A)$	$(A \& B) \vee (A \& B)$
T	T	TTT <b>T</b> TTT	TTT <b>T</b> TTT
F	T	FTT <b>T</b> TTF	FFT <b>F</b> FFT
T	F	TTF <b>T</b> FTT	TFF <b>F</b> TFF
F	F	FTF <b>F</b> FFF	FFF <b>F</b> FFF

Which is it: equivalent or inequivalent? **INEQUIVALENT**  
 (If they are inequivalent, circle any row that proves that they are inequivalent.)

29. Translate the following argument and check to see whether it is valid. Use some or all of the sentence letters provided. (This problem is worth 7 points.)

If Keith is in the band, then Keith plays an instrument.  
 Either Keith and Phil are both in the band, or neither of them is in the band.  
 Unless Phil is in the band, Keith does not play an instrument.  
 Thus, Keith is in the band.

K = "Keith is in the band."  
 P = "Phil is in the band."  
 I = "Keith plays an instrument."

Translate the argument into symbols here:

$K \supset I$ $(K \& P) \vee (\sim K \& \sim P)$ $\sim P \supset \sim I$ Thus, K
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Fill out the truth table here.

K	I	P	$K \supset I$	$(K \& P) \vee (\sim K \& \sim P)$	$\sim P \supset \sim I$	K
T	T	T	T <b>T</b> T	T T T <b>T</b> FT F FT	FT <b>T</b> FT	<b>T</b>
F	T	T	F <b>T</b> T	F F T <b>F</b> TF F FT	FT <b>T</b> FT	<b>F</b>
T	F	T	T <b>F</b> F	T T T <b>T</b> FT F FT	FT <b>T</b> TF	<b>T</b>
F	F	T	F <b>T</b> F	F F T <b>F</b> TF F FT	FT <b>T</b> TF	<b>F</b>
T	T	F	T <b>T</b> T	T F F <b>F</b> FT F TF	TF <b>F</b> FT	<b>T</b>
F	T	F	F <b>T</b> T	F F F <b>T</b> TF T TF	TF <b>F</b> FT	<b>F</b>
T	F	F	T <b>F</b> F	T F F <b>F</b> FT F TF	TF <b>T</b> TF	<b>T</b>
F	F	F	F <b>T</b> F	F F F <b>T</b> TF T TF	TF <b>T</b> TF	<b>F</b>

Which is it: valid or invalid? **INVALID**  
 (If it is invalid, circle the row that proves that it is invalid.)