

Logic—Sample Test A3 with Answers

NAME \_\_\_\_\_

1. Define 'counterexample'. (10 points)

A counterexample is a possible situation where the premises of an argument are all true and the conclusion is false.

2. Define 'valid'. (20 points)

An argument is valid if and only if it has no counterexamples.

Identify the following arguments properly. If the argument is valid, mark it with a V. If it is invalid mark it with an I. (These problems are worth 2 points each)

3. It is cloudy outside.

The ground is wet.

Thus, it has been raining.

Which is it: valid or invalid? INVALID

(counterexample: A dry overcast day with lawn sprinklers running.)

4. He took the test.

She took the test.

Thus, both he and she took the test.

Which is it: valid or invalid? VALID

5. All Tech students are female.

Only students that attend Tech can take this test.

Thus, every student taking this test is female.

Which is it: valid or invalid? VALID

6. It is not the case that both Luke and Henry fell.

Henry did not fell.

Thus, Luke fell.

Which is it: valid or invalid? INVALID

(counterexample: Neither one of them fell.)

7. All teachers are geniuses.

Scott is a genius.

Thus, Scott is a teacher.

Which is it: valid or invalid? INVALID

(counterexample: Scott is a genius sculptor who doesn't teach, but all teachers are geniuses too.)

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Translate the following sentences into the language of sentential logic using the abbreviations given to you. (These problems are worth 2 points each.)

P= “We played the game”

F= “We played fairly”

G= “We won the game”

Q= “We played quickly”

W= “We played well”

8. If we did not win the game, then we did not play well.

$\sim G \supset \sim W$

9. We played quickly and well, but we did not win.

$(Q \ \& \ W) \ \& \ \sim G$

10. Either we played quickly or we played well, but we did not win.

$(Q \ \vee \ W) \ \& \ \sim G$

11. Either we both played well and quickly or we did not win.

$(W \ \& \ Q) \ \vee \ \sim G$

12. We played the game and played it well, but we did not play quickly nor did we win.

$(P \ \& \ W) \ \& \ (\sim Q \ \& \ \sim G)$

13. If we did not play the game, then we did not play well, fairly, or quickly.

$\sim P \supset \sim(W \ \vee \ (F \ \vee \ Q))$

Translate the following sentences into the language of sentential logic using the abbreviations given to you. (These problems are worth 2 points each.)

W = “Water is a solvent.”

E = “Ethelene is a solvent.”

A = “Acetone is a solvent”

Q= “We played quickly”

T = “The test came out positive.”

14. If the test came out positive, then acetone and ethelene are solvents.

$T \supset (A \ \& \ E)$

15. Acetone is a solvent only if water itself is a solvent.

$A \supset W$

16. Unless the test came out positive, acetone is definitely not a solvent.

$\sim T \supset \sim A$

17. Ethelene and water are not solvents, and neither is acetone.

$(\sim E \ \& \ \sim W) \ \& \ \sim A$

18. Ethelene is a solvent if and only if water is not a solvent.

$(E \supset \sim W) \ \& \ (\sim W \supset E)$       or       $E \equiv \sim W$

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19. Water is not a solvent, unless the test did not come out positive.

$$\sim\sim T \supset \sim E$$

20. Either water is a solvent if ethelene is, or the test did not come out positive.

$$(E \supset W) \vee \sim T$$

21. If the test came out positive, then if water is not a solvent, neither is acetone.

$$T \supset (\sim W \supset \sim A)$$

Label each of the following sequences of symbols with a ‘Wff’ if it is a well-formed formula or with an ‘X’ if it is not a well-formed formula. (1 point each)

22.  $(T \ \& \ \sim\sim E) \vee J$     **Wff**

23.  $\sim\sim\sim(K \ \& \ \sim K) \supset J$     **Wff**

24.  $(T \ \sim \ (E \vee L))$

25.  $(Y \vee (\sim\sim E \ \& \ \sim R) \ \& \ \sim J)$

Construct truth tables to test whether these arguments are valid or invalid. *In the case of an invalid argument, indicate the row or rows that show that the argument is invalid by one of them.* (These problems are worth 3 points each.)

26. 
$$\begin{array}{l} \sim B \ \& \ A \\ \sim B \supset C \\ \hline C \ \& \ A \end{array}$$

A	B	C	$\sim B \ \& \ A$	$\sim B \supset C$	$C \ \& \ A$
T	T	T	FT <b>F</b> T	FT <b>T</b> T	T <b>T</b> T
F	T	T	FT <b>F</b> F	FT <b>T</b> T	T <b>F</b> F
T	F	T	TF <b>T</b> T	TF <b>T</b> T	T <b>T</b> T
F	F	T	TF <b>F</b> F	TF <b>T</b> T	T <b>F</b> F
T	T	F	FT <b>F</b> T	FT <b>T</b> F	F <b>F</b> T
F	T	F	FT <b>F</b> F	FT <b>T</b> F	F <b>F</b> F
T	F	F	TF <b>T</b> T	TF <b>F</b> F	F <b>F</b> T
F	F	F	TF <b>F</b> F	TF <b>F</b> F	F <b>F</b> F

Which is it: valid or invalid? **VALID**

(If it is invalid, circle any row that proves that it is invalid.)

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$$\frac{\sim(E \vee \sim D) \quad D \vee \sim E}{E}$$

E	D	$\sim(E \vee \sim D)$	$D \vee \sim E$	E
T	T	FT T FT	T T FT	T
F	T	TF F FT	T T TF	F
T	F	FT T TF	F F FT	T
F	F	FF T TF	F T TF	F

Which is it: valid or invalid? INVALID  
(If it is invalid, circle any row that proves that it is invalid.)

$$\frac{\sim(R \supset M) \supset N \quad \sim(M \& R)}{N}$$

R	M	N	$\sim(R \supset M) \supset N$	$\sim(M \& R)$	N
T	T	T	FT T T T T	F T T T	T
F	T	T	FFT T T T T	T T FF	T
T	F	T	TTF F T T	T F FT	T
F	F	T	FFT F T T	T F FF	T
T	T	F	FTT T T F	F T T T	F
F	T	F	FFT T T F	T T FF	F
T	F	F	TTF F F F	T F FT	F
F	F	F	FFT F T F	T F FF	F

Which is it: valid or invalid? INVALID  
(If it is invalid, circle any row that proves that it is invalid.)

Test whether each of these sentences is a tautology, a contradiction, or a contingent sentence by constructing their truth tables. (These problems are worth 3 points each.)

$$29. (A \& B) \vee (\sim A \& \sim B)$$

A	B	$(A \& B) \vee (\sim A \& \sim B)$
T	T	T T T T FT F FT
F	T	F F T F TF F FT
T	F	T F F F FT F TF
F	F	F F F T TF T TF

Which is it: a tautology, a contradiction, or a contingent? CONTINGENT

$$30. (R \supset C) \vee (R \& \sim C)$$

R	C	$(R \supset C) \vee (R \& \sim C)$
T	T	T T T T TF FT
F	T	F T T T FF FT
T	F	T F F T T T TF
F	F	F T F T FF TF

Which is it: a tautology, a contradiction, or a contingent? TAUTOLOGY

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Test the following pairs of sentences for logical equivalence by constructing their truth tables. Label for each whether they are equivalent or inequivalent. (3 points)

31.  $(\sim G \ \& \ \sim H) \supset G$                        $(G \supset H) \vee G$

G	H	$(\sim G \ \& \ \sim H) \supset G$	$(G \supset H) \vee G$
T	T	FT F FT <b>T</b> T	T T T <b>T</b> T
F	T	TF F FT <b>T</b> F	F T T <b>T</b> F
T	F	FT F TF <b>T</b> T	T F F <b>T</b> T
F	F	TF T TF <b>F</b> F	F T F <b>T</b> F

Which is it: equivalent or inequivalent? **INEQUIVALENT**  
 (If they are inequivalent, circle any row that proves that they are inequivalent.)

Test each of the following pairs of sentences for consistency by constructing their truth tables. Label for each whether they are consistent or inconsistent. (3 points)

32.  $A \vee \sim B$                                        $\sim(\sim A \vee B)$

A	B	$A \vee \sim B$	$\sim(\sim A \vee B)$
T	T	T <b>T</b> FT	<b>F</b> FT T T
F	T	F F FT	<b>F</b> TF T T
T	F	T T TF	<b>T</b> FT F F
F	F	F <b>T</b> TF	<b>F</b> TF T F

Which is it: consistent or inconsistent? **CONSISTENT**  
 (If they are consistent, circle any row that proves that they are consistent.)

Logic—Sample Test A3 with Answers

33. Translate the following argument and check to see whether it is valid. Use only the sentence letters provided. (This problem is worth 7 points.)

Either Alice or Bob ate some of the pizza, and either Cindy or Alice did.  
 If Alice ate some of the pizza then neither Cindy nor Bob ate some of the pizza.  
 Unless both Cindy and Alice failed to eat some of the pizza, Bob ate some.  
 Therefore, Cindy ate no pizza.

A = "Alice ate some of the pizza."  
 B = "Bob ate some of the pizza."  
 C = "Cindy ate some of the pizza."

Translate the argument into symbols here:

$(A \vee B) \& (C \vee A)$   
 $A \supset \sim(C \vee B)$   
 $\sim(\sim C \& \sim A) \supset B$   
 $\sim C$

Fill out the truth table here.

A	B	C	$(A \vee B) \& (C \vee A)$	$A \supset \sim(C \vee B)$	$\sim(\sim C \& \sim A) \supset B$	$\sim C$
T	T	T	TTT <b>T</b> TTT	T <b>F</b> FTTT	TFT F FT <b>T</b> T	<b>FT</b>
F	T	T	FTT <b>T</b> TTF	F <b>T</b> FTTT	TFT F TF <b>T</b> T	<b>FT</b>
T	F	T	TTF <b>T</b> TTT	T <b>F</b> FTTF	TFT F FT <b>F</b> F	<b>FT</b>
F	F	T	FFF <b>F</b> TTF	F <b>T</b> FTTF	TFT F TF <b>F</b> F	<b>FT</b>
T	T	F	TTT <b>T</b> FTT	T <b>F</b> FFTT	TTF F FT <b>T</b> T	<b>TF</b>
F	T	F	FTT <b>F</b> FFF	F <b>T</b> FFTT	FTF T TF <b>T</b> T	<b>TF</b>
T	F	F	TTF <b>T</b> FTT	T <b>T</b> TFFF	TTF F FT <b>F</b> F	<b>TF</b>
F	F	F	FFF <b>F</b> FFF	F <b>T</b> TFFF	FTF T TF <b>T</b> F	<b>TF</b>

Which is it: valid or invalid? INVALID  
 (If it is invalid, circle any row that proves that it is invalid.)