Logic—Sample Test C1 with Answers

NAME _____

1. Define ‘counterexample’. (10 points)
A counterexample is a possible situation where the premises of an argument are all true and the conclusion is false.

2. Define ‘valid’. (20 points)
An argument is valid if and only if it has no counterexamples.

Translate the following sentences into the language of sentential logic using the abbreviations given to you. (These problems are worth 2 points each.)

P*x* = *x* is a person.
F*x* = *x* flees.
T*y* = *y* is threatened.
S*x* = *x* is a serpent.
D*x* = *x* is a demon.
A*x* = *x* is an angel.
a = Angela
j = Juan
m = Melissa

3. “Angela isn’t being threatened.”
~Ta

4. “Melissa, the angel, is fleeing.”
Am & Fm

5. “Juan, Melissa and Angela are not all demons.”
~((Dj & Dm) & Da)

6. “Someone is being threatened.”
$\exists x$(P*x* & T*x*)

7. Not everyone who is threatened, flees.”
~$\forall x$((P*x* & T*x*) $\supset$ F*x*)

8. “All serpents are demons.”
$\forall x$(S*x* $\supset$ D*x*)

9. “Demons never flee when threatened.”
$\forall x$((D*x* & T*x*) $\supset$ ~F*x*)

10. “If Juan flees, there is a serpent demon in existence.”
Fj $\supset$ $\exists x$(S*x* & D*x*)

Px = x is a person.
Fx = x flees.
Ty = y is threatened.
Sx = x is a serpent.
Dx = x is a demon.
Ax = x is an angel.
a = Angela
j = Juan
m = Melissa

11. “No fleeing demons are serpents.”
$\sim\exists x(Dx \,\&\, (Fx \,\&\, Sx))$

12. “Unless threatened, angels never flee.”
$\forall x(Ax \supset (\sim Tx \supset \sim Fx))$

13. “The serpent-demon Juan is threatened.”
$(Sj \,\&\, Dj) \,\&\, Tj$

14. “An angel flees only if it is threatened.”
$\forall x(Ax \supset (Fx \supset Tx))$

15. “Not everybody is an angel or demon.”
$\sim\forall y(Py \supset (Ay \vee Dy))$

16. “Angels flee if threatened, unlike demons, who never do so.”
$\forall x(Ax \supset (Tx \supset Fx)) \,\&\, \forall x(Dx \supset (Tx \supset \sim Fx))$

17. “Both angels and demons flee when threatened.”
$\forall x(((Ax \vee Dx) \,\&\, Tx) \supset Fx)$ or
$\forall x((Ax \,\&\, Tx) \supset Fx) \,\&\, \forall x((Dx \,\&\, Tx) \supset Fx)$
18. “Unless an angel is threatened, Juan the serpent flees.”
$\sim\exists x(Ax \,\&\, Tx) \supset (Sj \,\&\, Fj)$

19. “Neither Juan nor Angela are demons, but someone is.”
$\sim(Dj \vee Da) \,\&\, \exists x(Px \,\&\, Dx)$

20. “Unless no angels are serpents, there will be serpents that are demons as well.”
$\sim\sim\exists x(Ax \,\&\, Sx) \supset \exists x(Sx \,\&\, Dx)$

21. “Juan is not a demon, nor is Angela; they are angels instead.”
$(\sim Dj \,\&\, \sim Da) \,\&\, (Aj \,\&\, Aa)$

22. “All angels are threatened if there is some demon that is not a serpent.”
$\exists x(Dx \,\&\, \sim Sx) \supset \forall x(Ax \supset Tx)$

Use the truth tree method to determine whether the following argument is valid. List a separate translation before doing the tree if the argument is in English. Number all lines. Label all derived lines with the rule and the line from which they were derived. Use the notation provided for your translations. (1 point for each translated sentence, and 6 points per truth tree)

23. Every woman has a right to fair treatment.

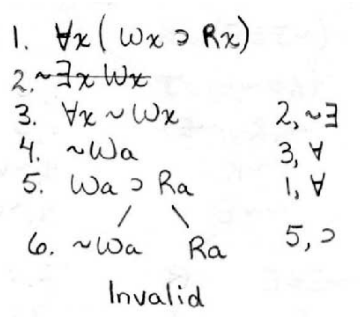
Thus, women exist.

W*x* = *x* is a woman

R*x* = *x* has a right to fair treatment

$\forall x(Wx \supset Rx)$

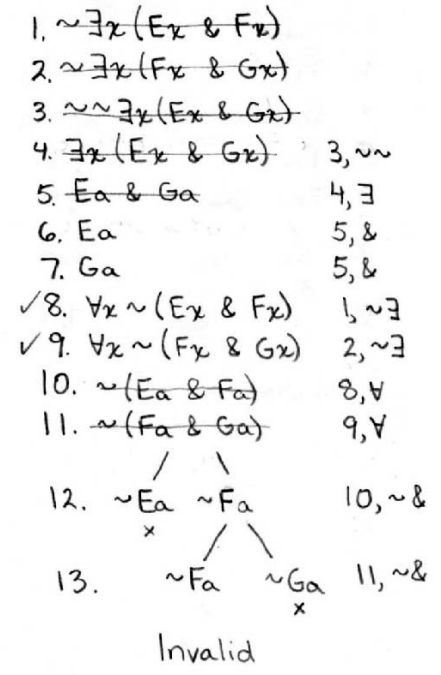
$\exists x Wx$



24. ~∃x(Ex & Fx)

~∃x(Fx & Gx)

~∃x(Ex & Gx)



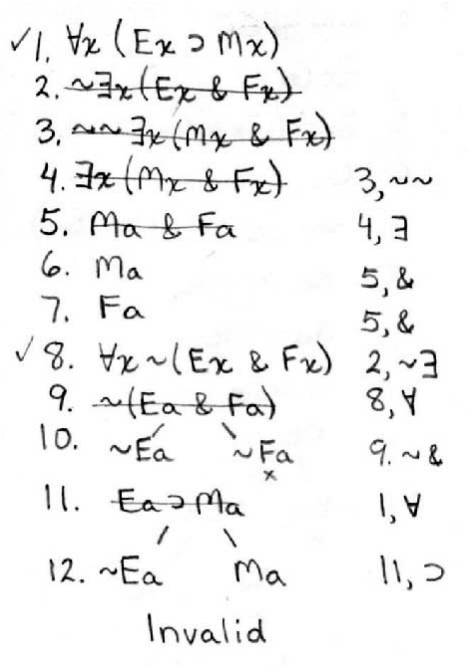
25. All elephants are mammals.
No elephants can fly.

Thus, no mammals can fly.

E*x* = *x* is an elephant
M*x* = *x* is a mammal
F*x* = *x* can fly

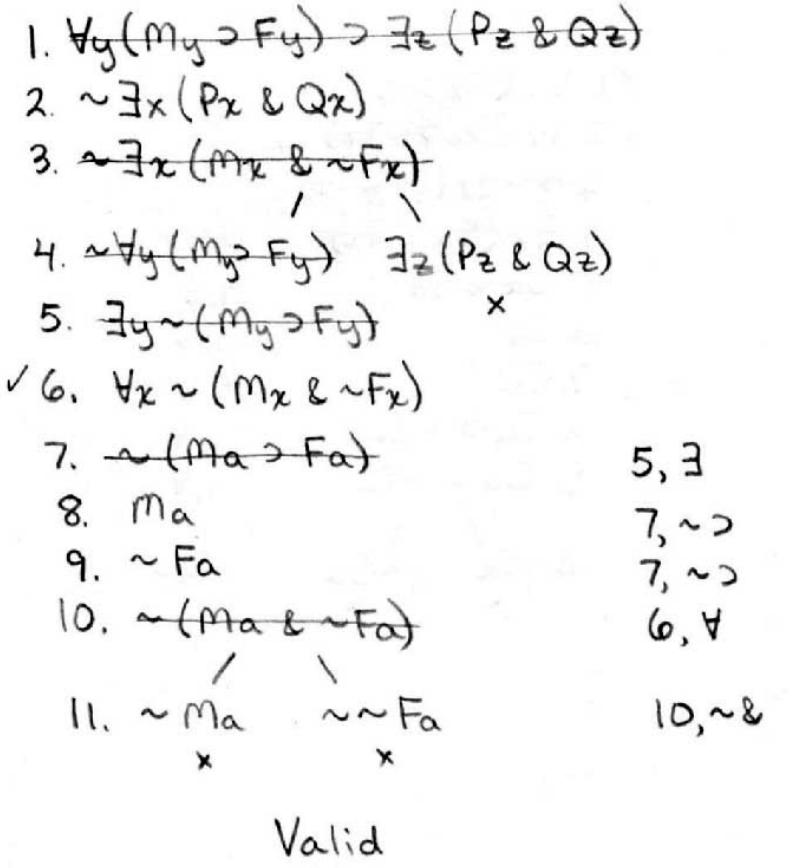
$\forall x(Ex \supset Mx)$
$\sim\exists x(Ex \ \& \ Fx)$

$\sim\exists x(Mx \ \& \ Fx)$



26. $\forall y(My \supset Fy) \supset \exists z(Pz \ \& \ Qz)$
$\sim\exists x(Px \ \& \ Qx)$

$\exists x(Mx \ \& \ \sim Fx)$



Valid