

Logic—Sample Test C2 with Answers

NAME _____

1. Define ‘counterexample’. (10 points)

A counterexample is a possible situation where all the premises are true and the conclusion is false.

2. Define ‘valid’. (20 points)

An argument is valid if and only if it has no counterexamples.

Translate the following sentences into the language of quantifier logic using the given abbreviations. Remember that you do not need to worry about tense. (2 points each.)

$Px = x$ is a person.

$Hx = x$ is happy.

$Bx = x$ barks.

$Sx = x$ is shaggy.

$Dx = x$ is a dog.

$c =$ Chris

$d =$ Duke

3. “Chris and Duke are happy.”

$Hc \ \& \ Hd$

4. “Someone isn’t happy.”

$\exists x(Px \ \& \ \sim Hx)$

5. “Duke is a non-barking dog.”

$Dd \ \& \ \sim Bd$

6. “All shaggy dogs bark.”

$\forall x((Dx \ \& \ Sx) \supset Bx)$

7. “No one who is shaggy is happy.”

$\sim \exists x(Px \ \& \ (Sx \ \& \ Hx))$

8. “There is a dog who is happy, but Chris certainly isn’t.”

$\exists x(Dx \ \& \ Hx) \ \& \ \sim Hc$

9. “Duke is barking, and someone is not happy.”

$Bd \ \& \ \exists x(Px \ \& \ \sim Hx)$

10. “Unless Chris is a dog, he isn’t shaggy.”

$\sim Dc \supset \sim Sc$

11. “If a dog is barking, it isn’t happy.”

$\forall x((Dx \ \& \ Bx) \supset \sim Hx)$

12. “Not all shaggy dogs are happy.”

$\sim \forall x((Dx \ \& \ Sx) \supset Hx)$

13. “Nobody is shaggy, but Duke, the dog, is.”

$\sim \exists x(Px \ \& \ Sx) \ \& \ (Dd \ \& \ Sd)$

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14. "Some dog is shaggy, but nevertheless happy."

$\exists x(Dx \ \& \ (Sx \ \& \ Hx))$

15. "Chris is not a happy dog."

$\sim(Dc \ \& \ Hc)$

16. "Some barking dog is not happy."

$\exists x(Dx \ \& \ (Bx \ \& \ \sim Hx))$

17. "A dog is happy only if it is not barking."

$\forall x((Dx \ \& \ Hx) \supset \ \sim Bx)$

18. "Not all dogs are shaggy or happy."

$\sim \forall x((Dx \supset (Sx \ \vee \ Hx))$

19. "If Chris is not happy, nobody is happy."

$\sim Hc \supset \ \sim \exists x(Px \ \& \ Hx)$

20. "If any dog is shaggy, everybody is happy."

$\exists x(Dx \ \& \ Sx) \supset \ \forall x(Px \supset \ Hx)$

Use the truth tree method to determine whether the set of sentences is consistent. Number all lines. Label all derived lines with the rule and the line from which they were derived. Cross out discharged sentences. (8 points each)

21. $\{ \exists xPx \ \& \ \exists z\sim Pz, \ \forall xz((Px \ \& \ \sim Pz) \supset \ Lxz), \ \forall y\sim Lyy \}$

1. $\exists xPx$
 2. $\exists z\sim Pz$
 ✓ 3. $\forall xz((Px \ \& \ \sim Pz) \supset \ Lxz)$
 ✓ 4. $\forall y\sim Lyy$
 5. Pa 1, \exists
 6. $\sim Pb$ 2, \exists
 7. ~~$(Pa \ \& \ \sim Pb) \supset \ Lab$~~ 3, \forall
 8. ~~$\sim(Pa \ \& \ \sim Pb)$~~ Lab 7, \supset
 9. ~~$\sim Pa$~~ ~~$\sim \sim Pb$~~
 10. $\sim Lab$ 4, \forall
 11. $\sim Lab$ 4, \forall

Consistent

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22. $\{ \forall x(Hx \supset (Bx \ \& \ Sx)), \exists xHx, \exists xSx \}$

- ✓ 1. $\forall x (Hx \supset (Bx \ \& \ Sx))$
 2. ~~$\exists x Hx$~~
 3. ~~$\exists x Sx$~~
 4. H_a 2, \exists
 5. S_b 3, \exists
 6. ~~$H_a \supset (B_a \ \& \ S_a)$~~ 1, \forall
 7. $\sim H_a$ ~~$B_a \ \& \ S_a$~~ 6, \supset
 8. \times ~~$H_b \supset (B_b \ \& \ S_b)$~~ 1, \forall
 9. $\sim H_b$ ~~$B_b \ \& \ S_b$~~ 8, \supset
 10. B_a B_a 7, $\&$
 11. S_a S_a 7, $\&$
 12. B_b 9, $\&$
 13. S_b 9, $\&$
- Consistent

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23. { $\exists x(Ax \& Bx) \supset \forall y(Cy \vee Dy)$, $\sim \forall x(\sim Ax \vee \sim Bx)$, $\sim \exists x Cx$, $\exists x Dx$ }

1. ~~$\exists x(Ax \& Bx) \supset \forall y(Cy \vee Dy)$~~
2. ~~$\sim \forall x(\sim Ax \vee \sim Bx)$~~
3. ~~$\sim \exists x Cx$~~
4. ~~$\exists x Dx$~~
5. Da 4, \exists
- ✓ 6. $\forall x \sim Cx$ 3, $\sim \exists$
7. $\sim Ca$ 6, \forall
8. ~~$\exists x \sim (\sim Ax \vee \sim Bx)$~~ 2, $\sim \forall$
9. ~~$\sim (\sim Ab \vee \sim Bb)$~~ 8, \exists
10. ~~$\sim \sim Ab$~~ 9, $\sim \vee$
11. ~~$\sim \sim Bb$~~ 9, $\sim \vee$
12. Ab 10, $\sim \sim$
13. Bb 11, $\sim \sim$
14. ~~$\sim \exists x(Ax \& Bx)$~~ $\forall y(Cy \vee Dy)$ ✓ 1, \supset
- ✓ 15. $\forall x \sim (Ax \& Bx)$
16. ~~$\sim (Ab \& Bb)$~~
17. $\sim Ab$ $\sim Bb$
18. x x ~~$Ca \vee Da$~~ 14, \forall
19. Ca Da 18, \vee
 x

Consistent

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Use the truth tree method to determine whether the following argument is valid. List a separate translation before doing the tree. Number all lines. Label all derived lines with the rule and the line from which they were derived. Use the notation provided for your translations. (10 points)

$Dx = x$ is a dog.

$Bx = x$ is big

$Mx = x$ is mean

24. All dogs are big and mean.
Some dogs are mean.

$\forall x(Dx \supset (Bx \& Mx))$
 $\exists x(Dx \& Mx)$

\checkmark 1. $\forall x(Dx \supset (Bx \& Mx))$
 2. ~~$\sim \exists x(Dx \& Mx)$~~
 \checkmark 3. $\forall x \sim (Dx \& Mx)$ 2, $\sim \exists$
 4. ~~$Da \supset (Ba \& Ma)$~~ 1, \forall
 5. ~~$\sim (Da \& Ma)$~~ 3, \forall
 6. $\sim Da$ $\sim Ma$ 5, $\sim \&$
 / \ / \
 7. $\sim Da$ ~~$Ba \& Ma$~~ $\sim Da$ ~~$Ba \& Ma$~~ 4, \supset
 8. Ba Ba 7, $\&$
 9. Ma Ma 7, $\&$
 Invalid