

Logic—Sample Test C4 with Answers

NAME _____

1. Define 'counterexample'. (10 points)

A counterexample is a possible situation where all the premises are true and the conclusion is false.

2. Define 'valid'. (10 points)

An argument is valid if and only if it has no counterexamples.

Translate the following sentences into the language of quantifier logic using the given abbreviations. Remember that you do not need to worry about tense. (2 points each.)

$Yx = x$ is yellow.

$Gx = x$ is green.

$Rx = x$ is red.

$Dx = x$ is a dress.

$Bx = x$ is a blouse.

$Cx = x$ is in the closet.

$Sx = x$ is made of silk.

$Lx = x$ is made of leather.

$Jx = x$ is Julie's, (belongs to Julie, is owned by Julie, is possessed by Julie, etc.)

$s =$ Katy's scarf

3. "Katy's scarf is green and yellow."

$Gs \ \& \ Ys$

4. "Julie has a yellow blouse."

$\exists x(Jx \ \& \ (Yx \ \& \ Bx))$

5. "If Katy's scarf is green, it's in the closet."

$Gs \supset C_s$

6. "All of Julie's dresses are in the closet."

$\forall x((Jx \ \& \ Dx) \supset Cx)$

7. "At least some of Julie's red blouses are not in the closet."

$\exists x((Jx \ \& \ Rx) \ \& \ (Bx \ \& \ \sim Cx))$

8. "Julie has a silk dress, not a leather one."

$\exists x((Jx \ \& \ Dx) \ \& \ Sx) \ \& \ \sim \exists x((Jx \ \& \ Dx) \ \& \ Lx)$

9. "Either there is a dress or a blouse in the closet."

$\exists x((Dx \vee Bx) \ \& \ Cx)$ or $\exists x(Dx \ \& \ Cx) \vee \exists x(Bx \ \& \ Cx)$

10. "Any dress in the closet that is red is made of silk."

$\forall x((Dx \ \& \ (Cx \ \& \ Rx)) \supset Sx)$

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11. "Katy's scarf isn't made of silk, but of leather."

$\sim Ss \ \& \ Ls$

12. "None of Julie's blouses are made of leather."

$\sim \exists x((Jx \ \& \ Bx) \ \& \ Lx)$

13. "Not every blouse in the closet is silk."

$\sim \forall x((Bx \ \& \ Cx) \supset Sx)$

14. "Only dresses are in the closet."

$\forall x(Cx \supset Dx)$

15. "Some of Julie's things are yellow, while others are red."

$\exists x(Jx \ \& \ Yx) \ \& \ \exists x((Jx \ \& \ Rx)$

16. "A yellow blouse is in the closet."

$\exists x((Bx \ \& \ Yx) \ \& \ Cx)$

17. "Everything not in the closet doesn't belong to Julie."

$\forall x(\sim Cx \supset \sim Jx)$

18. "If there is a yellow or green dress, it is in the closet."

$\forall x((Bx \ \& \ (Yx \ \vee \ Cx)) \supset Sx)$

19. "Nothing in the closet is made of silk and leather."

$\sim \exists x((Cx \ \& \ Sx) \ \& \ Lx)$

20. "All of Julie's blouses and dresses are green."

$\forall x((Jx \ \& \ (Bx \ \vee \ Dx)) \supset Gx) \ \text{or} \ \forall x((Jx \ \& \ Bx) \supset Gx) \ \& \ \forall x((Jx \ \& \ Dx) \supset Gx)$

21. "If any of Julie's dresses are red, she also has a yellow blouse."

$\exists x((Jx \ \& \ Dx) \ \& \ Rx) \supset \exists x((Jx \ \& \ Bx) \ \& \ Yx)$

22. "All of Julie's dresses are yellow, except possibly those made of leather."

$\forall x((Bx \ \& \ (Dx \ \& \ \sim Lx)) \supset Yx)$

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Use the truth tree method to determine whether the set of sentences is consistent. Number all lines. Label all derived lines with the rule and the line from which they were derived.

26. $\{ \exists x(Ax \ \& \ Bx), \ \forall x(Bx \supset Cjx), \ \forall x(Ax \supset Cxj), \ \forall y \sim \exists x(Cyx \ \& \ Cxy) \}$ (7 points.)

~~1. $\exists x(Ax \ \& \ Bx)$~~
 \checkmark 2. $\forall x(Bx \supset Cjx)$
 \checkmark 3. $\forall x(Ax \supset Cxj)$
 \checkmark 4. $\forall y \sim \exists x(Cyx \ \& \ Cxy)$
~~5. $Aq \ \& \ Bq$~~
 6. Aq 5, &
 7. Bq 5, &
~~8. $\sim \exists x \sim (Cjx \ \& \ Cxj)$~~ 4, $\sim \exists$
 9. $Bq \supset Cjq$ 2, \forall
 10. $Aq \supset Cqj$ 3, \forall
 11. $\sim Bq$ Cjq 9, \supset
 \times
 12. $\sim Aq$ Cqj 10, \supset
 \times
 13. $\checkmark \forall x \sim (Cjx \ \& \ Cxj)$ 8, $\sim \exists$
 14. $\sim (Cjq \ \& \ Cqj)$ 13, \forall
 $\sim Cjq$ $\sim Cqj$ 14, $\sim \&$
 \times \times
 Inconsistent

Logic—Sample Test C4 with Answers

27. { “Every rock is marked.”, “Anything that isn’t marked is indecipherable.”, “Some rocks are decipherable.” } (3 points for translation, 7 for proof)

$Rx = x$ is a rock.

$Mx = x$ is marked.

$Dx = x$ is decipherable.

{ $\forall x(Rx \supset Mx)$, $\forall x(\sim Mx \supset \sim Dx)$, $\exists x(Rx \ \& \ Dx)$ }

\checkmark 1. $\forall x (Rx \supset Mx)$
 \checkmark 2. $\forall x (\sim Mx \supset \sim Dx)$
 3. ~~$\exists x (Rx \ \& \ Dx)$~~
 4. ~~$Ra \ \& \ Da$~~ 3, \exists
 5. Ra 4, $\&$
 6. Da 4, $\&$
 7. ~~$Ra \supset Ma$~~ 1, \forall
 8. $\begin{array}{c} \swarrow \quad \searrow \\ \sim Ra \quad Ma \\ * \end{array}$ 7, \supset
 9. ~~$\begin{array}{c} \swarrow \quad \searrow \\ \sim Ma \supset \sim Da \\ \end{array}$~~ 2, \forall
 10. ~~$\begin{array}{c} \swarrow \quad \searrow \\ \sim Ma \quad \sim Da \\ \end{array}$~~ 9, \supset
 11. $\begin{array}{c} Ma \quad * \\ \end{array}$ 10, $\sim\sim$
 Consistent