Logic—Sample Test D1 with Answers

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1. Define ‘counterexample’. (10 points)
A counterexample is a possible situation where all the premises are true and the conclusion is false.

2. Define ‘valid’. (20 points)
An argument is valid if and only if it has no counterexamples.

Translate the following sentences into the language of quantifier logic using the given abbreviations. Remember that you do not need to worry about tense. (2 points each.)

D*x* = *x* is a dog
P*x* = *x* is a person
C*x* = *x* is a cat.
B*xy* = *x* belongs to *y*
L*xy* = *x* likes *y*
s = Stephanie
p = Penelope

3. “Stephanie has a dog named Penelope.”
$\mathrm{Dp} \;\&\; \mathrm{Bps}$

4. “Everyone has a cat.”
$\forall x(\mathrm{P}x \supset \exists z(\mathrm{C}z \;\&\; \mathrm{B}zx))$

5. “Stephanie has a dog and a cat.”
$\exists z(\mathrm{D}z \;\&\; \mathrm{B}z\mathrm{s}) \;\&\; \exists z(\mathrm{C}z \;\&\; \mathrm{B}z\mathrm{s})$

6. “No one likes all cats.”
$\sim\exists z(\mathrm{P}z \;\&\; \forall x(\mathrm{C}x \supset \mathrm{L}zx))$

7. “There isn’t a single dog that likes a cat.”
$\sim\exists z(\mathrm{D}z \;\&\; \exists x(\mathrm{C}x \;\&\; \mathrm{L}zx)$

8. “Penelope likes Stephanie if Stephanie has a dog.”
$\exists z(\mathrm{D}z \;\&\; \mathrm{B}z\mathrm{s}) \supset \mathrm{Lps}$

9. “No cat belongs to everyone.”
$\sim\exists z(\mathrm{C}z \;\&\; \forall x(\mathrm{P}x \supset \mathrm{B}zx))$

10. “Any cat Stephanie likes likes Penelope.”
$\forall x((\mathrm{C}x \;\&\; \mathrm{Ls}x) \supset \mathrm{L}x\mathrm{p})$

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11. “One of Penelope’s dogs doesn’t like itself.”
$\exists z(\mathrm{D}z \mathbin{\&} (\mathrm{B}z\mathrm{p} \mathbin{\&} {\sim}\mathrm{L}zz))$

12. “Everyone has a dog except maybe Stephanie.”
$\forall x((\mathrm{P}x \mathbin{\&} {\sim}(x = \mathrm{s})) \supset \exists z(\mathrm{D}z \mathbin{\&} \mathrm{B}zx))$

13. “If Stephanie doesn’t like Penelope, she won’t like any of Penelope’s dogs.”
${\sim}\mathrm{Lsp} \supset {\sim}\exists z((\mathrm{D}z \mathbin{\&} \mathrm{B}z\mathrm{p}) \mathbin{\&} \mathrm{Ls}z)$

14. “Anyone who has a dog, likes it.”
$\forall x((\mathrm{P}x \supset \forall z((\mathrm{D}z \mathbin{\&} \mathrm{B}zx) \supset \mathrm{L}xz))$

15. “One of Stephanie’s dogs likes everyone except Penelope.”
$\exists z((\mathrm{D}z \mathbin{\&} \mathrm{B}z\mathrm{s}) \mathbin{\&} \forall x((\mathrm{P}x \mathbin{\&} {\sim}(x = \mathrm{p})) \supset \mathrm{L}zx))$

16. “Penelope’s cats only like her, not anyone else.”
$\forall x((\mathrm{C}x \mathbin{\&} \mathrm{B}x\mathrm{p}) \supset (\mathrm{L}x\mathrm{s} \mathbin{\&} \forall z(\mathrm{P}z \supset {\sim}\mathrm{L}xz)))$

17. “None of Stephanie’s dogs like her.”
${\sim}\exists x(\mathrm{D}x \mathbin{\&} \mathrm{B}x\mathrm{s}) \mathbin{\&} \mathrm{L}x\mathrm{s})$

18. “Stephanie likes every cat except her own.”
$\forall x((\mathrm{C}x \mathbin{\&} {\sim}\mathrm{B}x\mathrm{s}) \supset \mathrm{Ls}x) \mathbin{\&} \forall x((\mathrm{C}x \mathbin{\&} \mathrm{B}x\mathrm{s}) \supset {\sim}\mathrm{Ls}x)$ or just $\forall x((\mathrm{C}x \mathbin{\&} {\sim}\mathrm{B}x\mathrm{s}) \supset \mathrm{Ls}x)$

19. “At least two people like Penelope.”
$\exists x\exists y(((\mathrm{P}x \mathbin{\&} \mathrm{P}y) \mathbin{\&} {\sim}(x = y)) \mathbin{\&} (\mathrm{L}x\mathrm{p} \mathbin{\&} \mathrm{L}y\mathrm{p}))$

20. “Penelope has exactly one cat.”
$\exists x((\mathrm{C}x \mathbin{\&} \mathrm{B}x\mathrm{p}) \mathbin{\&} \forall x\forall y(((\mathrm{C}x \mathbin{\&} \mathrm{B}x\mathrm{p}) \mathbin{\&} (\mathrm{C}y \mathbin{\&} \mathrm{B}y\mathrm{p})) \supset x = y))$

21. “Stephanie has no more than one dog.”
$\forall x\forall y\forall z(((\mathrm{D}x \mathbin{\&} \mathrm{B}x\mathrm{s}) \mathbin{\&} (\mathrm{D}y \mathbin{\&} \mathrm{B}y\mathrm{s})) \supset x = y)$

22. “Stephanie doesn’t like herself and the same goes for Penelope.”
${\sim}\mathrm{Lss} \mathbin{\&} {\sim}\,\mathrm{Lpp}$

Use the truth tree method to determine whether the set of sentences is consistent. Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the book (except that you should cross out each complex sentence after you use it.) (10 points)

23. { Rfg, Rgf, ∀x∀y(~(x = y) ⊃ ~(Rxy & Ryx)) }

1.	Rfg		
2.	Rgf		
✓ 3.	∀xy(x ≠ y ⊃ ~(Rxy & Ryx))		
4.	~~f ≠ g ⊃ ~(Rfg & Rgf)~~		3, ∀
5.	~~~(f ≠ g)~~	~~~(Rfg & Rgf)~~	4, ⊃
6.	f = g		5, ~~
7.		~Rfg / ~Rgf	5, ~&
		× / ×	

Consistent

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1. Rfg
2. Rgf
3. ∀x∀y(~
4. ∀y(~(g=
5. ~(g=f
6. ~~(g=f)
7. g=f
8.

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Use the truth tree method to determine whether the argument is valid. Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the book (except that you should cross out each complex sentence after you use it.) (10 points each)

24. $\forall x(Px \supset \forall y((Py \ \&\ y = b) \supset Lxy))$
$\quad$ Pa & Pb
$\quad$ ———————
$\quad$ Lab

✓1.	$\forall x(Px \supset \forall y((Py \ \&\ y=b) \supset Lxy))$		
2.	~~Pa & Pb~~		
3.	$\sim$Lab		
4.	Pa		2, &
5.	Pb		2, &
6.	$\cancel{Pa \supset \forall y((Py \ \&\ y=b) \supset Lay)}$		1, $\forall$
7.	$\sim$Pa	✓ $\forall y((Py \ \&\ y=b) \supset Lay)$	6, $\supset$
8.		$\cancel{(Pb \ \&\ b=b) \supset Lab}$	7, $\forall$
9.	$\cancel{\sim(Pb \ \&\ b=b)}$	Lab ×	8, $\supset$
10.	$\sim$Pb ×	$b \neq b$ ×	9, $\sim$&

Valid

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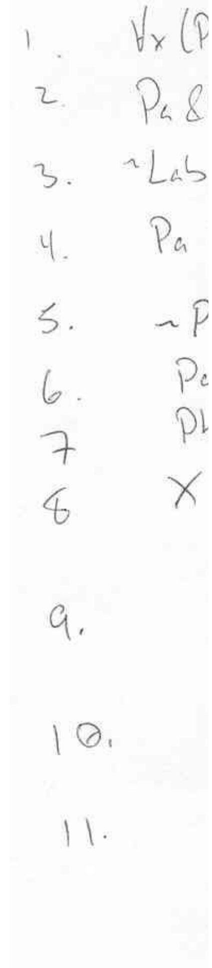

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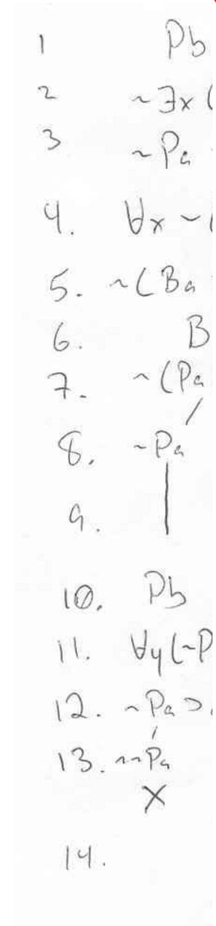
25. Pb & ∀*y*(~P*y* ⊃ *y* = b)
~∃*x*(B*x* ⊃ (P*x* & a = *x*))
Pa

1.	~~Pb & ∀y(~Py ⊃ y=b)~~		
2.	~~~∃x(Bx ⊃ (Px & a=x))~~		
3.	~Pa		
4.	Pb		1,&
✓5.	∀y (~Py ⊃ y=b)		1,&
✓6.	∀x ~ (Bx ⊃ (Px & a=x))		2,~∃
7.	~~~Pa ⊃ a=b~~		5,∀
8.	~~Pa	a=b	7,⊃
	×		
9.		~~~(Bb ⊃ (Pb & a=b))~~	6,∀
10.		Bb	9,~⊃
11.		~~~(Pb & a=b)~~	9,~⊃
12.		~Pb / ~(a=b)	11,~&
		× / ×	

Valid



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