Logic—Sample Test D2 with Answers

NAME ____

1. Define 'counterexample'. (10 points)
A counterexample is a possible situation where the premises of an argument are all true and the conclusion is false.

2. Define 'valid'. (10 points)
An argument is valid if and only if it has no counterexamples.

Translate the following sentences into the language of predicate and relational logic using the abbreviations given to you. (These problems are worth 2 points each.)

P*x* = *x* is a person
H*x* = *x* is happy
K*xy* = *x* knows *y*
G*xyz* = *x* gives *y* to *z*
i = Irene
s = Irene's sandwich
k = Kirk

3. "Kirk knows Irene."
Kki

4. "Irene is giving Kirk her sandwich."
Gisk

5. "Irene doesn't give her sandwich to anyone."
~∃xGisx

6. "Not everyone is happy."
~∀x(Px ⊃ Hx)

7. "Someone gave Kirk something."
∃x(Px & ∃yGxyk)

8. "Everyone knows Irene and Kirk."
∀x(Px ⊃ (Kxi & Kxk))

9. "Kirk and Irene give each other things."
∃xGixk & ∃xGkxi

10. "Nothing is given by Irene to Kirk."
~∃xGixk

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Px = x is a person
Hx = x is happy
Kxy = x knows y
Gxyz = x gives y to z
i = Irene
s = Irene's sandwich
k = Kirk

11. "There is something Kirk doesn't give to anyone."
$\exists x \sim\exists z(Pz \,\&\, Gkxz)$ or $\exists x \forall z(Pz \supset \sim Gkxz)$

12. "If anyone is not happy, Irene will give her sandwich to that person."
$\forall x((Px \,\&\, \sim Hx) \supset Gisx)$

13. "No one gives Irene her sandwich, and Irene is not happy."
$\sim\exists x(Px \,\&\, Gxsi) \,\&\, \sim Hi$

14. "Kirk will give Irene's sandwich to someone if he knows him."
$\forall x((Px \,\&\, Kkx) \supset Gksx)$

15. "If Irene isn't happy, nobody will be happy."
$\sim Hi \supset \sim\exists x(Px \,\&\, Hx)$ or $\sim Hi \supset \forall x(Px \supset \sim Hx)$

16. "Either Kirk or Irene knows somebody who is happy."
$\exists x((Px \,\&\, Hx) \,\&\, (Kkx \vee Kix))$ or $\exists x((Px \,\&\, Hx) \,\&\, Kkx) \vee \exists x((Px \,\&\, Hx) \,\&\, Kix)$

17. "Irene doesn't know anyone who is giving something to Kirk."
$\sim\exists x((Px \,\&\, Kix) \,\&\, \exists y(Gxyk))$

18. "Someone is giving Irene's sandwich to someone he knows."
$\exists x(Px \,\&\, \exists z((Pz \,\&\, Kxz) \,\&\, Gxsz))$

19. "Anyone who knows Irene is happy."
$\forall x((Px \,\&\, Kxi) \supset Hx)$

20. "No one gives Kirk anything."
$\sim\exists xz(Px \,\&\, \sim Gxzk)$

21. "Irene's sandwich is given to her by someone."
$\exists x(Px \,\&\, Gxsi)$

Use the truth tree method to determine whether the following argument is valid. List a separate translation before doing the tree if the argument is in English. Number all lines. Label all derived lines with the rule and the line from which they were derived. Use the notation provided for your translations. (1 point for each translated sentence, and 7 points per truth tree)

22. $\exists x \forall y \mathrm{T}xy$
$\exists xz \mathrm{T}xz \supset \forall y \mathrm{Tm}y$
$\overline{\mathrm{Tmm}}$

	1. ~~∃x∀yTxy~~		
	2. ~~∃xzTxz ⊃ ∀yTmy~~		
	3. ~Tmm		
✓	4. ∀yTay		1,∃
	5. ~~~∃xzTxz~~	∀yTmy ✓	2,⊃
✓	6. ∀xz~Txz		5,~∃
	7.	Tmm	5,∀
		×	
	8. Taa		4,∀
	9. ~Taa		6,∀
	×		

Valid

23. $\exists x \forall y \mathrm{Y}xy$
$\overline{\forall x \exists y \mathrm{Y}yx}$

	1. ~~∃x∀yYxy~~	
	2. ~~~∀x∃yYyx~~	
	3. ~~∃x∀y~Yyx~~	2,~∀/∃
✓	4. ∀yYay	1,∃
✓	5. ∀y~Yyb	3,∃
	6. Yab	4,∀
	7. ~Yab	5,∀
	×	

Valid

24. $\forall x(Gxx \lor \exists yFxy)$
$\sim\exists zGzz$
$\exists xyFxy$

✓1.	$\forall x(Gxx \lor \exists yFxy)$		
2.	$\cancel{\sim\exists zGzz}$		
3.	$\cancel{\sim\exists xyFxy}$		
✓4.	$\forall xy\sim Fxy$		3, $\sim\exists$
✓5.	$\forall z\sim Gzz$		2, $\sim\exists$
6.	$\cancel{Gaa \lor \exists yFay}$		1, $\forall$
7.	Gaa	$\cancel{\exists yFay}$	6, $\lor$
8.	|	Fab	7, $\exists$
9.	$\sim Gaa$	$\sim Gaa$	5, $\forall$
10.	×	$\sim Fab$	4, $\forall$
		×	

Valid

25. Every dog loves to chew a bone.
Thus, there is a bone that every dog loves to chew.

D*x* = *x* is a dog
B*x* = *x* is a bone
C*xy* = *x* loves to chew *y*

$\forall x(Dx \supset \exists y(By \,\&\, Cxy))$
$\exists x(Bx \,\&\, \forall y(Dy \supset Cyx))$

✓1.	$\forall x(Dx \supset \exists y(By \,\&\, Cxy))$				
2.	$\cancel{\sim\exists x(Bx \,\&\, \forall y(Dy \supset Cyx))}$				
✓3.	$\forall x\sim(Bx \,\&\, \forall y(Dy \supset Cyx))$				2, $\sim\exists$
4.	$\cancel{Da \supset \exists y(By \,\&\, Cay)}$				1, $\forall$
5.	$\sim Da$		$\cancel{\exists y(By \,\&\, Cay)}$		4, $\supset$
6.	|		$\cancel{Bb \,\&\, Cab}$		5, $\exists$
7.	|		Bb		6, &
8.	|		Cab		6, &
9.	$\cancel{\sim(Bb \,\&\, \forall y(Dy \supset Cyb))}$		$\cancel{\sim(Bb \,\&\, \forall y(Dy \supset Cyb))}$		3, $\forall$
10.	$\sim Bb$	$\sim\forall y(Dy \supset Cyb)$	$\sim Bb$	$\sim\forall y(Dy \supset Cyb)$	9, $\sim$&
	↑ open		×		

Invalid

26. (12 points).

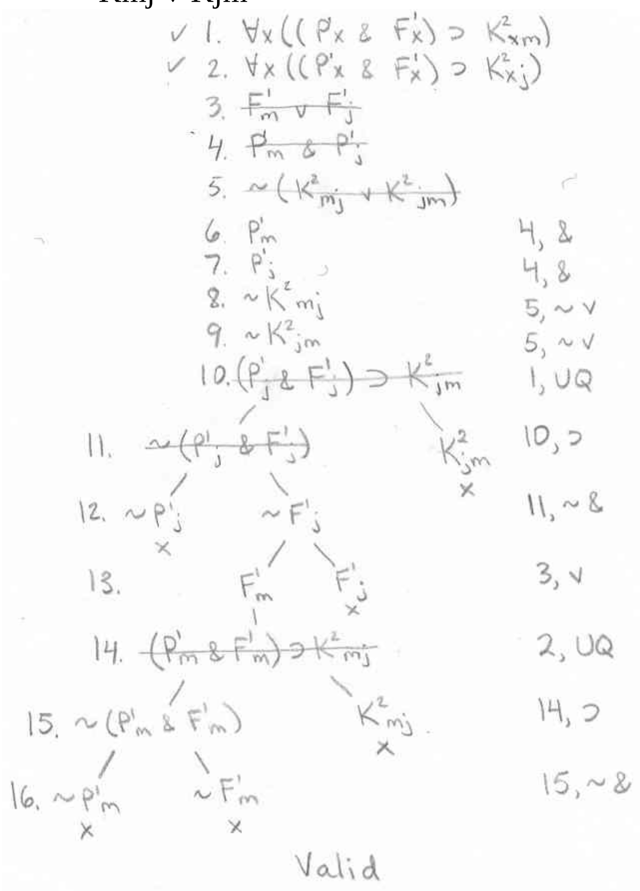
Everyone who is famous knows Monica.
Everyone who is famous knows Jill.
Either Monica or Jill is famous.
Monica and Jill are both people.

Thus, either Monica knows Jill or Jill knows Monica.

P*x* = *x* is a person
F*x* = *x* is famous
K*xy* = *x* knows *y*
m = Monica
j = Jill

$\forall x((Px \,\&\, Fx) \supset Kxm)$
$\forall x((Px \,\&\, Fx) \supset Kxj)$
$Fm \vee Fj$
$Pm \,\&\, Pj$

$Kmj \vee Kjm$



UQ is the universal quantifier rule (∀).
EQ is the existential quantifier rule (∃).
QE is the quantifier exchange rule (~∀/∃).