

Logic—Sample Test D3 with Answers

NAME \_\_\_\_\_

1. Define ‘counterexample’. (10 points)

A counterexample is a possible situation where all the premises are true and the conclusion is false.

2. Define ‘valid’. (20 points)

An argument is valid if and only if it has no counterexamples.

Translate the following sentences into the language of quantifier logic using the given abbreviations. Remember that you do not need to worry about tense. (2 points each.)

$Px = x$  is a person.

$n =$  Nathan

$Lx = x$  is a location.

$t =$  Tim

$Gxy = x$  goes to  $y$ .

$k =$  Kara

$Oxy = x$  is older than  $z$ .

$b =$  the beach

$Ixy = x$  is impressed with  $y$ .

$m =$  the museum

$Fx = x$  is fun.

You can assume (and don’t need to write down) that Nathan, Tim, and Kara are all people, and that the beach and the museum are locations.

3. “Nathan is not going both to the museum and beach.”

$\sim(Gnm \ \& \ Gnb)$

4. “Kara is going somewhere fun.”

$\exists x((Lx \ \& \ Fx) \ \& \ Gkx)$

5. “Kara and Tim are going to the same place.”

$\exists x(Lx \ \& \ (Gkx \ \& \ Gtx))$

6. “Someone is older than Tim, but it isn’t Nathan.”

$\exists x(Px \ \& \ (x \neq n \ \& \ Oxt))$

$\exists x(Px \ \& \ Oxt) \ \& \ \sim Ont$

7. “The beach is fun even if Tim is unimpressed with it.”

$Fb$

8. “Kara is the oldest person going to the beach.”

$Gkb \ \& \ \forall x((Px \ \& \ (Gxb \ \& \ x \neq k)) \supset Okx)$

9. “Tim is older than Kara and someone else.”

$Otk \ \& \ \exists x((Px \ \& \ x \neq k) \ \& \ Otx)$

10. “Anyone younger than Nathan is fun.”

$\forall x((Px \ \& \ Onx) \supset Fx)$

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 $Oxy = x$  is older than  $z$ .                       $b =$  the beach  
 $Ixy = x$  is impressed with  $y$ .                       $m =$  the museum  
 $Fx = x$  is fun.

11. “Tim is unimpressed with someone.”

$\exists x(Px \ \& \ \sim Itx)$

12. “Kara is impressed with anyone who is older than she is.”

$\forall x((Px \ \& \ Oxk) \supset Ikx)$

13. “If Nathan goes anywhere fun, Tim will go there too.”

$\forall x(((Lx \ \& \ Fx) \ \& \ Gnx) \supset Gtx)$

14. “The beach is older than the museum, but it is still fun anyway.”

$Obm \ \& \ Fb$

15. “Tim will be impressed with someone only if he or she is fun.”

$\forall x((Px \ \& \ Itx) \supset Fx)$

16. “Someone is going to the beach but not anywhere else.”

$\exists x((Px \ \& \ Gxb) \ \& \ \sim \exists y((Ly \ \& \ y \neq b) \ \& \ Gxy))$

17. “Someone older than Nathan is going to the museum.”

$\exists x((Px \ \& \ Oxn) \ \& \ Gxm)$

18. “Kara is impressed with anyone who goes to the museum.”

$\forall x((Px \ \& \ Gxm) \supset Ikx)$

19. “Only one person is going to the museum.”

$\exists x((Px \ \& \ Gxm) \ \& \ \forall xy(((Px \ \& \ Py) \ \& \ (Gxm \ \& \ Gym)) \supset x=y))$

20. “Only Tim is impressed with Kara.”

$Itk \ \& \ \forall x((Px \ \& \ x \neq t) \supset \sim Ixk)$

$Itk \ \& \ \sim \exists x((Px \ \& \ x \neq t) \ \& \ Ixk)$

21. “Everyone going to the museum is older than some or other beach-goer.”

$\forall x((Px \ \& \ Gxm) \supset \exists y((Py \ \& \ Gyb) \ \& \ Oxy))$

$\forall x((Px \ \& \ Gxm) \supset \exists y(Gyb \ \& \ Oxy))$

22. “No one who goes to the beach is impressed with all the other people who go to the beach.”  $\sim \exists x((Px \ \& \ Gxb) \ \& \ \forall y((Py \ \& \ (x \neq y \ \& \ Gyb)) \supset Ixy))$

23. “Anyone will be impressed with any location that’s fun.”

$\forall x(Px \supset \forall y((Ly \ \& \ Fy) \supset \sim Ixy))$

24. Use the truth tree method to determine whether the argument is valid. (7 points)



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26. Use the truth tree method to determine whether these two sentences are logically equivalent. (14 points)

$$\sim \exists x(Gx \ \& \ \forall y(Nyx \supset Jy))$$

$$\forall y(Gy \supset \exists z(\sim Jz \ \& \ Nz))$$

1.  ~~$\sim \exists x(Gx \ \& \ \forall y(Nyx \supset Jy))$~~   
 2.  ~~$\sim \forall y(Gy \supset \exists z(\sim Jz \ \& \ Nz))$~~   
 ✓ 3.  ~~$\forall x \sim(Gx \ \& \ \forall y(Nyx \supset Jy))$~~  1,  $\sim \exists$   
 4.  ~~$\exists y \sim(Gy \supset \exists z(\sim Jz \ \& \ Nz))$~~  2,  $\sim \forall$   
 5.  ~~$\sim(Ga \supset \exists z(\sim Jz \ \& \ Nza))$~~  4,  $\exists$   
 6.  ~~$\sim(Ga \ \& \ \forall y(Nya \supset Jy))$~~  3,  $\forall$   
 7.  ~~$Ga$~~  5,  $\sim \supset$   
 8.  ~~$\sim \exists z(\sim Jz \ \& \ Nza)$~~  5,  $\sim \supset$   
 ✓ 9.  ~~$\forall z \sim(\sim Jz \ \& \ Nza)$~~  8,  $\sim \exists$   
 10.  ~~$\sim Ga$~~   ~~$\sim \forall y(Nya \supset Jy)$~~  6,  $\sim \&$   
      ~~$\forall y(Nya \supset Jy)$~~   
 11.  ~~$\exists y \sim(Nya \supset Jy)$~~  8,  $\sim \exists$   
 12.  ~~$\sim(Nba \supset Jb)$~~  11,  $\exists$   
 13.  ~~$Nba$~~  12,  $\sim \supset$   
 14.  ~~$\sim Jb$~~  12,  $\sim \supset$   
 15.  ~~$\sim(\sim Jb \ \& \ Nba)$~~  9,  $\forall$   
 16.  ~~$\sim \sim Jb$~~   ~~$\sim Nba$~~  15,  $\sim \&$   
      ~~$\sim Jb$~~   ~~$\sim Nba$~~   
     x x  
     Valid

✓ 1.  ~~$\forall y(Gy \supset \exists z(\sim Jz \ \& \ Nz))$~~   
 2.  ~~$\sim \exists x(Gx \ \& \ \forall y(Nyx \supset Jy))$~~   
 3.  ~~$\exists x(Gx \ \& \ \forall y(Nyx \supset Jy))$~~  2,  $\sim \sim$   
 4.  ~~$Ga \ \& \ \forall y(Nya \supset Jy)$~~  3,  $\exists$   
 5.  ~~$Ga$~~  4,  $\&$   
 ✓ 6.  ~~$\forall y(Nya \supset Jy)$~~  4,  $\&$   
 7.  ~~$Ga \supset \exists z(\sim Jz \ \& \ Nza)$~~  1,  $\forall$   
      ~~$\sim Ga$~~   ~~$\exists z(\sim Jz \ \& \ Nza)$~~  7,  $\supset$   
     x  
 9.  ~~$\sim Jb \ \& \ Nba$~~  8,  $\exists$   
 10.  ~~$\sim Jb$~~  9,  $\&$   
 11.  ~~$Nba$~~  9,  $\&$   
 12.  ~~$Nba \supset Jb$~~  6,  $\forall$   
      ~~$\sim Nba$~~   ~~$Jb$~~  12,  $\supset$   
     x x  
     Valid