

Logic—Sample Final Examination E1 with Answers

NAME _____

Translate the following sentences into the language of sentential logic using the abbreviations given to you.

A = “The animal is a bovine.”

B = “The animal is a bull.”

C = “The animal is a cow.”

1. “The animal is a bovine, and if it is not a cow, then it is a bull.”

$A \ \& \ (\sim C \supset B)$

2. “Unless the animal is not bovine, it is a cow.”

$\sim\sim A \supset C$

3. “The animal is not both a cow and a bull.”

$\sim(C \ \& \ B)$

4. “Only if the animal is a cow is it bovine.”

$A \supset C$

5. “If the animal is a cow, then it is not a bull, and vice versa.”

$(C \supset \sim B) \ \& \ (B \supset \sim C)$

6. “The animal is bovine if and only if it is a cow or a bull.”

$(A \supset (C \vee B)) \ \& \ ((C \vee B) \supset A)$

T = “Harmon owns a truck.”

S = “Harmon owns a sports car.”

D = “Harmon drives.”

W = “Harmon wants to own a sports car.”

7. “Harmon owns both a truck and a sports car, but he doesn’t drive.”

$(T \ \& \ S) \ \& \ \sim D$

8. “It’s untrue that if Harmon owns a truck, he doesn’t drive.”

$\sim(T \supset \sim D)$

9. “Unless Harmon doesn’t drive, he doesn’t own a sports car.”

$\sim\sim D \supset \sim S$

10. “Harmon neither owns nor wants to own a sports car.”

$\sim(S \vee W)$

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Translate the following sentences into the language of quantifier logic using the abbreviations given to you.

$Ax = x$ is an apple

$Px = x$ is a pear

$Tx = x$ is a tree

$d =$ my desk

$Gx = x$ is green

$Rx = x$ is red

$Oxy = x$ is on y

$S =$ It is September

11. “All apples are either red or green.”

$\forall x(Ax \supset (Rx \vee Gx))$

12. “Not all pears are green.”

$\sim \forall x(Px \supset Gx)$

13. “There are pears on a tree that are neither red nor green.”

$\exists x(\exists y((Px \ \& \ Ty) \ \& \ (Oxy \ \& \ \sim(Rx \ \vee \ Gx))))$

14. “If there is a red apple on my desk, then it is not on a tree.”

$\forall x((Ax \ \& \ Oxd) \supset \sim \exists y(Ty \ \& \ Oxy))$

15. “No apples are red, unless it is September.”

$\sim S \supset \sim \exists x(Ax \ \& \ Rx)$

16. “Even though it is September, none of the apples on the trees are red yet.”

$S \ \& \ \sim \exists x(Ax \ \& \ Rx \ \& \ \exists y(Ty \ \& \ Oxy))$

17. “Every red apple is on some green tree or other.”

$\forall x((Ax \ \& \ Rx) \supset \exists y(Ty \ \& \ Gy \ \& \ Oxy))$

18. “There is no tree that has both pears and apples on it.”

$\sim \exists x(Tx \ \& \ (\exists y(Py \ \& \ Oyx) \ \& \ \exists z(Az \ \& \ Ozx)))$

19. “A green pear or apple is on my red desk.”

$\exists y(Gy \ \& \ ((Py \ \vee \ Ay) \ \& \ Oyd)) \ \& \ Rd$

20. “Each pear on my desk is green, but none of the apples on my desk are.”

$\forall x((Px \ \& \ Oxd) \supset Gx) \ \& \ \sim \exists y(Ay \ \& \ Oyd \ \& \ Gy)$

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Label each of the following sequences of symbols with a check mark if and only if it is a legitimate statement of logic. Mark the expression with an 'X' if and only if it is not a legitimate statement. (1 point each)

21. $\sim(\sim F \vee (\sim\sim E \supset \sim R)) \& \sim J$ X

22. $\sim\sim(T \& \sim\sim E) \vee E$

23. $\forall x \sim \forall y (Q \supset (K_{xy} \vee K_{yx}))$

24. $\exists xy (R_x \& F_{xy})$

25. $\forall x (U \& \sim \forall y S_{yx})$

26. $\forall xy (L_{xc} = y \& P_y)$ X

27. $\exists z \forall y (W_{zy} \supset \sim(z = y)) \& Pa) \vee K$ X

Construct truth tables to test whether these arguments are valid or invalid. *In the case of an invalid argument, indicate the row or rows that show that the argument is invalid by circling at least one of them.* (These problems are worth 3 points each.)

28. $\sim W \& L$
 $\sim W \supset E$
 \hline
 $L \& E$

W	L	E	$\sim W \& L$	$\sim W \supset E$	$L \& E$
T	T	T	F T F T	F T T T	T T T
F	T	T	T F T T	T F T T	T T T
T	F	T	F T F F	F T T T	F F T
F	F	T	T F F F	T F T T	F F T
T	T	F	F T F T	F T T F	T F F
F	T	F	T F T T	T F F F	T F F
T	F	F	F T F F	F T T F	F F F
F	F	F	T F F F	T F F F	F F F

Valid or invalid? VALID

If it is invalid, circle any one row that proves that it is invalid.

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$$\begin{array}{l} 29. \quad E \vee \sim D \\ \quad D \vee J \\ \hline \quad E \vee J \end{array}$$

E	D	J	$E \vee \sim D$	$D \vee J$	$E \vee J$
T	T	T	T I FT	T I T	T I T
F	T	T	F F FT	T I T	F I T
T	F	T	T I TF	F I T	T I T
F	F	T	F I TF	F I T	F I T
T	T	F	T I FT	T I F	T I F
F	T	F	F F FT	T I F	F F F
T	F	F	T I TF	F F F	T I F
F	F	F	F I TF	F F F	F F F

Valid or invalid? VALID

If it is invalid, circle any one row that proves that it is invalid.

Use the truth table method to determine whether the set of sentences is consistent.

30. { “Pedro is not going”, “Laurie is going,” “Pedro is going if and only if Laurie is going,” }

P	L	$\sim P$	L	$(P \supset L) \& (L \supset P)$
T	T	FT	I	T T T I T T T
F	T	IF	I	F T T F T F F
T	F	FT	F	T F F F F T T
F	F	IF	F	F T F I F T F

Consistent or inconsistent? INCONSISTENT

If it is consistent, circle any one row that proves that it is consistent.

31. { “Rachel and Jennifer are camping,” “Rachel is camping unless Jennifer is,” }

R	J	R & J	$\sim J \supset R$
T	T	T I T	FT I T
F	T	F F T	FT I F
T	F	T F F	TF I T
F	F	F F F	TF F F

Consistent or inconsistent? CONSISTENT

If it is consistent, circle any one row that proves that it is consistent.

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For each of the following sentences indicate whether it is a tautology, a contradiction, or a contingent sentence. Show some kind of formal proof.

32. “Unless Audrey lives in a cave, she doesn’t live in a cave.”

$$\sim A \supset \sim A$$

A	$\sim A \supset \sim A$
T	FT T FT
F	TF T TF

TAUTOLOGY

33. “If Indiana is where she lives, she doesn’t live in Indiana.”

$$I \supset \sim I$$

I	$I \supset \sim I$
T	T F FT
F	F T TF

CONTINGENT

34. “Everest is taller than everything in existence.”

1. $\forall xTex$
 2. $\forall x\sim Txx$ Auxiliary premise (Irreflexivity of tallness relation).
 3. Tee 1, \forall
 4. $\sim Tee$ 2, \forall
- x

CONTRADICTION

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Use the truth tree method to determine whether the set of sentences is consistent.

35. $\{ A \vee B, \sim(P \supset A), \sim(P \& B) \}$

1. $A \vee B$
 2. $\sim(P \supset A)$
 3. $\sim(P \& B)$
 4. P 2, $\sim \supset$
 5. $\sim A$ 2, $\sim \supset$
 / \
 6. A B 1, \vee
 \times / \
 7. $\sim P$ $\sim B$ 3, $\sim \&$
 \times \times
 Inconsistent

36. $\{ \sim R \vee T, J \supset R, \sim(T \vee \sim J) \}$

1. $\sim R \vee T$
 2. $J \supset R$
 3. $\sim(T \vee \sim J)$
 4. $\sim T$ 3, $\sim \vee$
 5. $\sim \sim J$ 3, $\sim \vee$
 / \
 6. $\sim R$ T 1, \vee
 / \
 7. $\sim J$ R 2, \supset
 \times \times
 Inconsistent

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37. $\{ \exists xPx \ \& \ \exists z\sim Pz, \ \forall xz((Px \ \& \ Pz) \supset Lxz), \ \forall yLyy \}$

1. ~~$\exists xPx \ \& \ \exists z\sim Pz$~~
- ✓ 2. $\forall xz((Px \ \& \ Pz) \supset Lxz)$
- ✓ 3. $\forall yLyy$
4. $\exists xPx$ 1, \exists
5. ~~$\exists z\sim Pz$~~ 1, \exists
6. Pa 4, \exists
7. $\sim Pb$ 5, \exists
8. ~~$((Pa \ \& \ Pb) \supset Lab)$~~ 2, \forall
9. ~~$\sim(Pa \ \& \ Pb)$~~ Lab 8, \supset
10. ~~$\sim Pa$~~ ~~$\sim Pb$~~ Lab 9, $\sim \&$
11. x Laa Laa 3, \forall

Consistent

38. $\{ \exists x((Px \ \& \ \sim(x = b)) \ \& \ \sim Fbx), \ \forall x\forall z(((Px \ \& \ Pz) \ \& \ \sim(x = z)) \supset Fxz), \ \forall y\sim Fyy \}$

1. ~~$\exists x((Px \ \& \ x \neq b) \ \& \ \sim Fbx)$~~
- ✓ 2. $\forall xz(((Px \ \& \ Pz) \ \& \ x \neq z) \supset Fxz)$
- ✓ 3. $\forall y\sim Fyy$
4. ~~$(Pa \ \& \ a \neq b) \ \& \ \sim Fba$~~ 1, \exists
5. ~~$Pa \ \& \ a \neq b$~~ 4, $\&$
6. $\sim Fba$ 4, $\&$
7. Pa 5, $\&$
8. $a \neq b$ 5, $\&$
9. ~~$((Pb \ \& \ Pa) \ \& \ b \neq a) \supset Fba$~~ 2, \forall
10. ~~$\sim((Pb \ \& \ Pa) \ \& \ b \neq a)$~~ Fba 9, \supset
11. ~~$\sim(Pb \ \& \ Pa)$~~ $b = a$ 10, $\sim \&$
12. ~~$\sim Pb$~~ ~~$\sim Pa$~~ x 11, $\sim \&$
13. $\sim Fbb$ 3, \forall

Consistent

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Use the truth tree method to determine whether the argument is valid. Number all lines. Label all derived lines with the rule and the line from which they were derived. Answers should look just as in the book (except that you should cross out each complex sentence after you use it.)

$$\begin{array}{l}
 39. \quad K \supset L \\
 \quad L \supset M \\
 \hline
 \quad \sim M \supset \sim K
 \end{array}$$

1. $K \supset L$
2. $L \supset M$
3. $\sim(\sim M \supset \sim K)$
4. $\sim M$ 3, $\sim\supset$
5. $\sim\sim K$ 3, $\sim\supset$
6. $\begin{array}{l} \swarrow \quad \searrow \\ \sim K \quad L \end{array}$ 1, \supset
7. $\begin{array}{l} \swarrow \quad \searrow \\ \sim L \quad M \end{array}$ 2, \supset

Valid

$$\begin{array}{l}
 40. \quad \exists x(Px \ \& \ Bx) \\
 \quad \forall y(Py \supset y = b) \\
 \hline
 \quad Ba
 \end{array}$$

1. $\exists x(Px \ \& \ Bx)$
- ✓ 2. $\forall y(Py \supset y = b)$
3. $\sim Ba$
4. $Pc \ \& \ Bc$ 1, \exists
5. $Pc \supset c = b$ 2, \forall
6. Pc 4, $\&$
7. Bc 4, $\&$
8. $\begin{array}{l} \swarrow \quad \searrow \\ \sim Pc \quad c = b \end{array}$ 5, \supset
9. $\begin{array}{l} \swarrow \quad \searrow \\ Pa \supset a = b \end{array}$ 2, \forall
10. $\begin{array}{l} \swarrow \quad \searrow \\ \sim Pa \quad a = b \end{array}$ 9, \supset

↑
open
Invalid

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41. Why do we translate arguments like “All cats meow. Frisky is a cat. Thus, Frisky meows.” with quantifier logic, which is harder, instead of with propositional logic, which is easier?

Because if we translate it with propositional logic, the argument will come out as

A
B _____
Thus, C

Thus, the argument will come out as invalid, but if we translated it with quantifier logic it will come out as valid, which is the correct answer for this particular argument.

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42. Why could we not create a symbol in sentential logic to represent the English connective 'because'?

'Because' is not truth functional. If the two clauses connected by 'because' are both true, one cannot determine the truth value of the whole sentence.