Logic—Sample Final Examination E3 with Answers

NAME _____

Evaluate whether the following arguments are valid. Write a V or an I next to each argument. (1 point each)

1. Kara is qualified if and only if she has a diploma.
Kara doesn't have a diploma.
Thus, Kara is unqualified.

VALID

2. Unless it's round, the gadget won't fit through the hole.
The gadget is spherical.
Thus, the gadget is one object that can fit through the hole.

INVALID

3. All birds have feathers.
The eagle perched on the gutter is a bird.
Thus, some birds do have feathers.

VALID

4. It is not the case that Murray never writes.
Thus, Murray sometime doesn't write.

INVALID

5. The widget is entirely within the container.
No container is entirely within the warehouse.
Thus, the widget is not entirely within the warehouse.

INVALID

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Translate the following sentences into the language of sentential logic using the abbreviations given to you. (These problems are worth 1 point each.)

O = "Oxygen is available."
F = "Fuel is available."
L = "We can light the fire."
D = "The fire will die out soon."

6. "Oxygen and fuel are both unavailable."
~O & ~F

7. "We can light the fire if fuel is available."
F ⊃ L

8. "The fire will die out soon unless oxygen is available."
~O ⊃ D

9. "We can light the fire, but the fire will die out soon if either fuel or oxygen is unavailable."
L & ((~F ∨ ~O) ⊃ D)

10. "The fire will die out soon unless both oxygen and fuel are available."
~(O & F) ⊃ D

11. "Either fuel and oxygen are available, or the fire will die out soon."
(F & O) ∨ D

C = "Gina is capable of flying the plane."
S = "Gina should be flying the plane."
D = "Gina is drunk."
E = "Ed should be monitoring the radar."

12. "If Gina is drunk, she shouldn't be flying the plane."
D ⊃ ~S

13. "Gina shouldn't fly the plane if she isn't capable of it."
~C ⊃ ~S

14. "Gina should be flying the plane, and Ed should be monitoring the radar."
S & E

15. "Unless Gina is drunk and incapable of flying the plane, Ed should be monitoring the radar."
~(D & ~C) ⊃ E

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Translate the following sentences into the language of quantifier logic using the given abbreviations. Remember that you do not need to worry about tense. (These problems are worth 1 point each.)

Wx = x is wood.
Dx = x is damp.
Fx = x can catch on fire.
Cx = x is cedar.
Bx = x is burning.
t = the torch
b = the bonfire

16. "The torch is damp, but it can still catch on fire if it's a cedar torch."
Dt & (Ct ⊃ Ft)

17. "The bonfire is the only thing burning."
$\sim\exists x(x \neq b \ \& \ Bx)$ or equivalently $\forall x(Bx \supset x = b)$

18. "The only kind of wood that can catch on fire is cedar."
$\sim\exists x(Wx \ \& \ Fx \ \& \sim Cx)$ or equivalently $\forall x((Wx \ \& \ Fx) \supset Cx)$

19. "Any wood that isn't damp can catch on fire."
$\forall x((Wx \ \& \sim Dx) \supset Fx)$

20. "Not everything that can catch on fire is burning."
$\sim\forall x(Fx \supset Bx)$

21. "The torch can catch on fire only if it's wooden."
Ft ⊃ Wt

22. "Any wood that isn't cedar can catch on fire."
$\forall x((Wx \ \& \sim Cx) \supset Fx)$

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Px = x is a person.
Bxy = x belongs to y.
Jx = x is a jacket.
Wxy = x is wearing y.
Lx = x is lost.
Dx = x is denim.
s = Steve
c = Chad

23. "One of Chad's jackets is lost."
$\exists x$(Bxc & Jx & Lx)

24. "Chad is lost, but at least he is wearing one of Steve's jacket."
Lc & $\exists x$(Jx & Bxs & Wcx)

25. "All Steve's jackets are denim."
$\forall x$((Bxs & Jx) $\supset$ Dx)

26. "There is a jacket that isn't Chad's."
$\exists x$(Jx & ~Bxc)

27. "Someone is wearing one of Steve's jackets."
$\exists x$(Px & $\exists y$(Bys & Jy & Wxy))

28. "Anyone wearing a denim jacket is lost."
$\forall x$((Px & $\exists y$(Wxy & Jy & Dy)) $\supset$ Lx)

29. "Any jacket that is denim belongs to Steve or Chad."
$\forall x$((Jx & Dx) $\supset$ (Bxc $\vee$ Bxs))

30. "If one of Chad's jackets is lost, it is a denim one."
$\forall x$((Jx & Bxc & Lx) $\supset$ Dx)

31. "Chad doesn't own any demin jacket."
~$\exists x$(Jx & Dx & Bxc)

32. "No one has lost his own jacket."
~$\exists xy$(Px & Jy & Byx & Ly)

Logic—Sample Final Examination E3 with Answers

Label each of the following sequences of symbols with a check mark if and only if it is a legitimate statement of logic. Mark the expression with an 'X' if and only if it is not a legitimate statement. (1 point each)

33. $\forall x \forall \sim y(Q \supset Kxy)$ X

34. $\sim\sim Y \vee \forall x(Gab=x)$ X

35. $\sim(z = y)$ & Pa X

36. $\exists xy(Wx \supset Fxy)$

37. $\sim(\sim F \vee (\sim\sim T \supset \sim(W \ \& \ L)) \ \& \sim E)$ X

38. $\forall k \forall y(k \neq y \supset Pk)$ X

Construct truth tables to test whether these arguments are valid or invalid. *In the case of an invalid argument, indicate the row or rows that show that the argument is invalid by circling at least one of them.* (4 points.)

39. ~K & Q
L ⊃ K

~L ∨ ~Q

K	Q	L	~K & Q	L ⊃ K	~L ∨ ~Q
T	T	T	FT F T	T T T	FT F FT
F	T	T	TF T T	T F F	FT F FT
T	F	T	FT F F	T T T	FT T TF
F	F	T	TF F F	T F F	FT T TF
T	T	F	FT F T	F T T	TF T FT
F	T	F	TF T T	F T F	TF T FT
T	F	F	FT F F	F T T	TF T TF
F	F	F	TF F F	F T F	TF T TF

Valid or invalid? VALID If it is invalid, circle any one row that proves that it is invalid.

40. ~P & (~A ∨ ~P) (3 points)
P ∨ A

A & ~(P ⊃ ~A)

P	A	~P & (~A ∨ ~P)	P ∨ A	A & ~(P ⊃ ~A)
T	T	FT F F T F FT	T T T	T T T T F FT
F	T	TF T F T T TF	F T T	T F F F T FT
T	F	FT F T F T FT	T T F	F F F T T TF
F	F	TF T T F T TF	F F F	F F F F T TF

Valid or invalid? INVALID If it is invalid, circle any one row that proves that it is invalid.

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Use the truth table method to determine whether the set of sentences is equivalent. (4 points)

41. { "Bernie is not going if Wendy is", "Wendy is going if Bernie isn't" }

W	B	W ⊃ ~B	~B ⊃ W
T	T	T F FT	FT T T
F	T	F T FT	FT T F
T	F	T T TF	TF T T
F	F	F T TF	TF F F

Equivalent or inequivalent? INEQUIVALENT
If it is inequivalent, circle any one row that proves that it is inequivalent.

42. Use the truth table method to determine whether the set of sentences is consistent. (4 points)
{ "Unless Cody's fishing, he isn't taking the truck," "Cody is neither fishing nor taking the truck." }

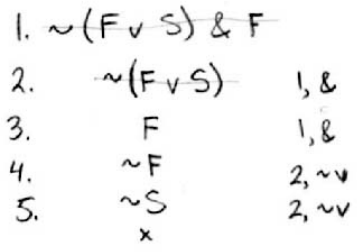
F	T	~F ⊃ ~T	~(F ∨ T)
T	T	FT T FT	F T T T
F	T	TF F FT	F F T T
T	F	FT T TF	F T T F
F	F	TF T TF	T F F F

Consistent or inconsistent? CONSISTENT
If it is consistent, circle any one row that proves that it is consistent.

For each of the following three sentences indicate whether it is a tautology, a contradiction, or a contingent sentence. Show some kind of formal proof. Use auxiliary premises if needed.

43. "Neither fish nor sharks inhabit the cave, but some white fish inhabit the cave." (4 points)

~(F ∨ S) & F



CONTRADICTION

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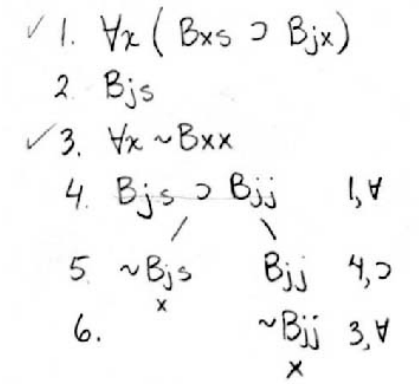
44. “Helen isn’t here, and even if she is here, she isn’t.” (3 points)

1. H & ~H
2. H 1, &
3. ~H 1, &
x

CONTRADICTION

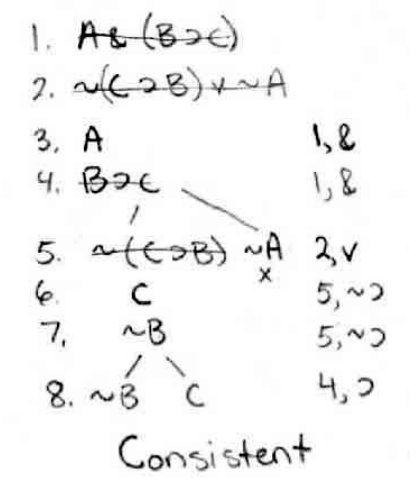
45. “Joe is a brother to all of his sister’s brothers.” (5 points)

∀*x*(B*x*s ⊃ Bj*x*)
Bjs
∀*x*~B*xx*



CONTRADICTION

46. Are the following sentences logically consistent? Show some kind of formal proof. (4 points)
{ A & (B ⊃ C), ~(C ⊃ B) ∨ ~A}



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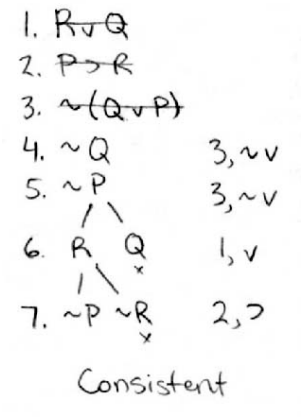
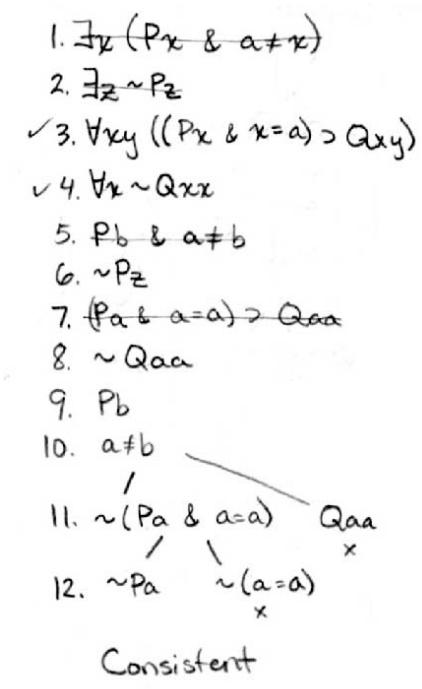
Use the truth tree method to determine whether the set of sentences is consistent.

47. { R ∨ Q, P ⊃ R, ~(Q ∨ P) } (4 points)



48. { ∃*x*(P*x* & a≠*x*), ∃*z*~P*z*, ∀*xy*((P*x* & *x*=a) ⊃ Q*xy*), ∀*x*~Q*xx* } (6 points)

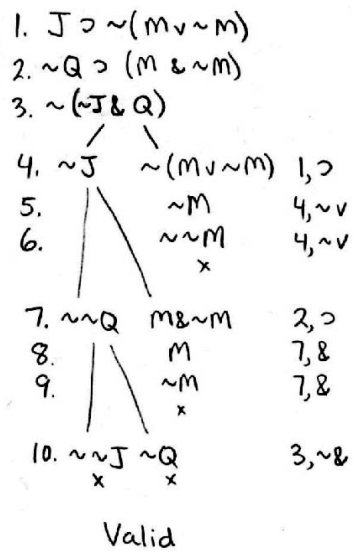


Logic—Sample Final Examination E3 with Answers

Use the truth tree method to determine whether the argument is valid.

49. (4 points)

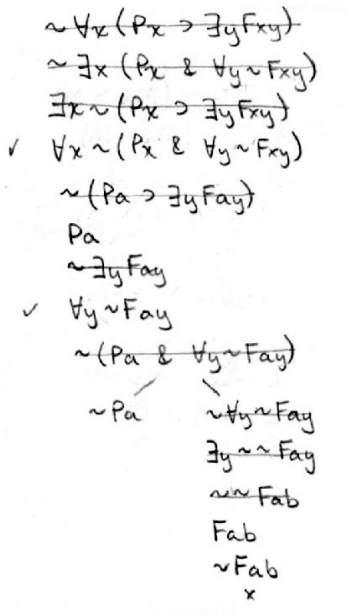
$J \supset \sim(M \vee \sim M)$
$Q \supset (M \,\&\, \sim M)$
$\therefore \sim J \,\&\, Q$



Valid

50. (6 points)

$\sim\forall x(Px \supset \exists y Fxy)$
$\therefore \exists x(Px \,\&\, \forall y \sim Fxy)$



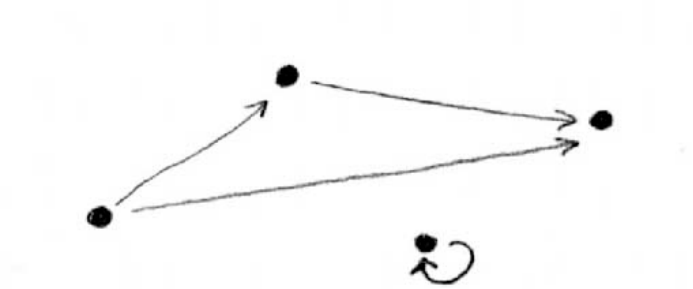
Logic—Sample Final Examination E3 with Answers

51. What is the definition of an equivalence relation? (1 point)
An equivalence relation is a relation that is reflexive, symmetric, and transitive.

The following questions are worth 3 points each.

52. The relation S, where S*xy* means "*x* is similar in appearance to *y*."
Reflexive? (**Reflexive** / Irreflexive / Neither)
Symmetric? (**Symmetric** / Anti-symmetric / Neither)
Transitive? (Transitive / **Not-transitive**)

53. The relation R, where R is defined over the universe pictured below.
Reflexive? (Reflexive / Irreflexive / **Neither**)
Symmetric? (Symmetric / Anti-symmetric / **Neither**)
Transitive? (**Transitive** / Not-transitive)



54. The relation Q, where Q is defined over the universe pictured below.
Reflexive? (**Reflexive** / Irreflexive / Neither)
Symmetric? (**Symmetric** / Anti-symmetric / Neither)
Transitive? (Transitive / **Not-transitive**)