Kidney Exchange

Itai Ashlagi

Market Design: Theory and Applications
Brown

Many slides adopted
by Al Roth

Kidney Exchange--Background

- On 06/Oct/10 there were **86,254** patients on the waiting list for cadaver kidneys in the U.S.
- In 2009 **33,671** patients were added to the waiting list, and **27,066** patients were removed from the list.
- In 2009 there were 10,442 transplants of cadaver kidneys performed in the U.S.
- In the same year, 4,644 patients died while on the waiting list. 1,940 others were removed from the list as “Too Sick to Transplant”.
- Transplant is the preferred treatment.
- No money transfers.

Section 301,National Organ Transplant

“it shall be unlawful for any person

to knowingly acquire, receive or otherwise transfer any human organ for valuable consideration for use in human transplantation”.

``The preceding sentence does not apply with respect to human organ paired donation.‘’

Sources of Donation

- Deceased: In the U.S. and Europe a centralized priority mechanism is used for the allocation of deceased donor kidneys.
- Living Donors: In 2009 there were also **6387** transplants of kidneys from living donors in the US.

Living donors distribution:

	Sibling/parent/offspring	**Relative**	**Unrelated**
1990	84%	11%	5%
1994	79%	13%	8%
2004	61%	18%	21%

Compatibility

Two tests to decide whether a donor is compatible with the patient:

1. *Blood compatibility test.*

   O can give A,B,AB,O

   A can give A,AB

   B can give B,AB

   AB can give AB

2. *Tissue type compatibility test (crossmatch test).*

   ~89% chance between random two people.

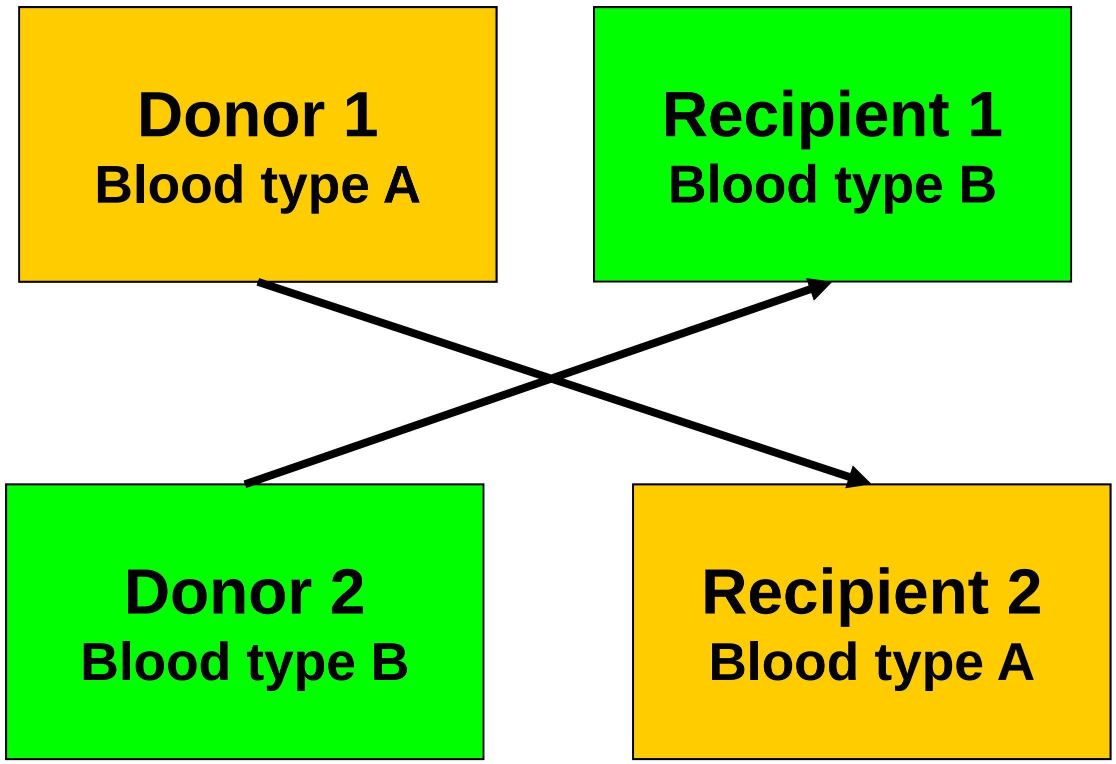
   <20% chance for a highly sensitized patient with a random donor.

If one test fails the patient and donor are incompatible.

How to increase the number of transplants?

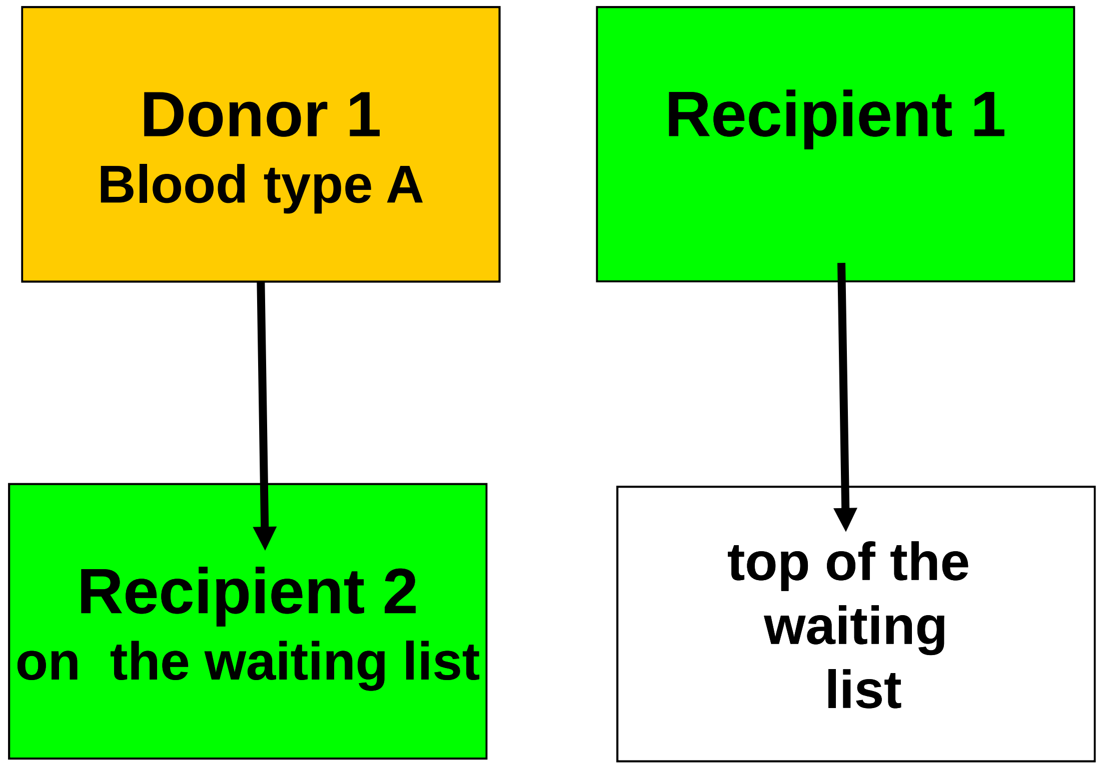
Kidney Exchange

Two pair kidney exchange



3-way exchanges (and larger) have been conducted

List Exchange



Related Literature

Economics literature:

Roth, Sonmez & Ünver *QJE 2004, JET 2005* - mechanisms for kidney exchange.

Roth, Sonmez & Ünver - AER P&P, A Kidney Exchange Clearinghouse in New England

Roth, Sonmez & Ünver, *AER* 2007 – efficient kidney exchange

Ünver, ReStud 2009 - efficient dynamic kidney exchange

Ashlagi & Roth Participation (versus free riding) in large scale, multi-hospital kidney exchange (in preperation)

More Related Literature (many fields)

CS literature*:*

Abraham, Blum & Sandholm, EC 07 – algorithm for large pools

Ashlagi, Fischer, Kash & Procaccia - EC 10 – Randomized strategyproof mechanism

Medical literature:

Roth et. al*, A.J of Transplantation 2006* – list exchanges

Rees et. al, NE J. of Medicine 2009 – long chain

Ashlagi, Gilchrist, Roth & Rees. - importance of ``open chains"

Operations literature:

Zenios et al. OR 2000, allocating to the waiting list

Su and Zenios, OR 2005, waiting list patients' choices

Kidney Exchange Institutions

- New England Program for Kidney Exchange—approved in 2004, started 2005.
    - Organizes kidney exchanges among the 14 transplant centers in New England
- Ohio Paired Kidney Donation Consortium, Alliance for Paired Donation, 2006-07 (Rees)
    - 81 transplant centers and growing…
- National (U.S.) kidney exchange—2010??
    - A national exchange has been proposed, a pilot is tentatively scheduled, but obstacles remain…

Centralized Kidney Exchange

No. of Hospitals	Num Of Pairs	Decentralized k=2	Centralized k=2	Decentralized Exchange k=3	Centralized Exchange k=3
2	21	3.46	5.26	4.36	6.89
4	42	6.6	13.58	8.32	18.67
6	67	11.72	25.62	14.73	35.97
8	85	14.4	36.52	18.04	49.75
10	108	17.52	47.74	22.87	64.34
12	131	22.32	60.6	28.16	81.83
14	154	26.44	74.72	33.85	98.07
16	173	28.76	84.2	36.58	109.41
18	191	31.78	95.62	39.75	122.1
20	227	38.7	116.68	49.79	144.35
22	252	44.52	131.5	55.85	161.07

The initial problem: How might more frequent and larger-scale kidney exchanges be organized?

- First, how can the market be made thicker?
  - Task 1: Assembling appropriate databases
  - Task 2: Coordinating hospital logistics
- Efficient outcomes (Pareto, maximum transplants)
- Incentives –individuals, hospitals

Making it safe to participate…

On 2004 – the establishment of a clearinghouse for kidney exchange in New England was approved

Model (for individuals) –housing markets

Shapley & Scarf [1974] housing market model: n agents each endowed with an indivisible good, a “house”.

Each patient has preferences over all the houses and there is no money, trade is feasible only in houses.

A matching function μ maps for each agent which house it will get.

A mechanism is a procedure that selects a matching for each preference profile.

Housing markets

A matching μ is in the indidivualy rational if every agent obtains a house at least as good as his own.

A matching μ is in the core if the is no coalition of agents B that

can block the matching – trade among themselves while making every agent in B not worse off and at least one of the agents in B strictly better off.

Any matching in the core is individually rational, and Pareto efficient

A mechanism is strategyproof if it makes it for every agent a dominant strategy to report its true preference.

Housing markets

Theorem[Shapley and Scarf 1974]: The core is not empty.

Top Trading Cycles - TTC (Gale):

Step 1: Each agent points to her most preferred house (and each house points to its owner). There is at least one cycle in the resulting directed graph (a cycle may consist of an agent pointing to her own house.) In each such cycle, the corresponding trades are carried out and these agents are removed from the market together with their assignments.

Step t: each agent points to her most preferred house that remains on the market….

….

Proof of Shapley and Scarf's Theorem

Suppose there is a coalition B that can block the outcome of the TTC.

Let a be the agent to be matched first in B under TTC which prefers its new outcome ν(a) over what it gets in TTC.

Then ν(a) is owned by b$\in$B who is removed in a strictly earlier step in the TTC, say in cycle C.

b obtains a house of some b'$\in$B$\cap$C under both matchings, some b' obtains the house of b''$\in$B$\cap$C under both matchings and so on…. Contradiction.

Uniqueness of the Core

Theorem[Roth and Postelwaite] For strict preferences the matching produced by TTC is the unique matching in the core.

Proof:

Suppose there is another matching ν

Let a be the first agent that gets a different house than in TTC.

Each agent that is matched in a cycle before a's will get the same under both matchings.

Given what is left, each agent in a's cycle prefers the house it gets by TTC to the house in ν

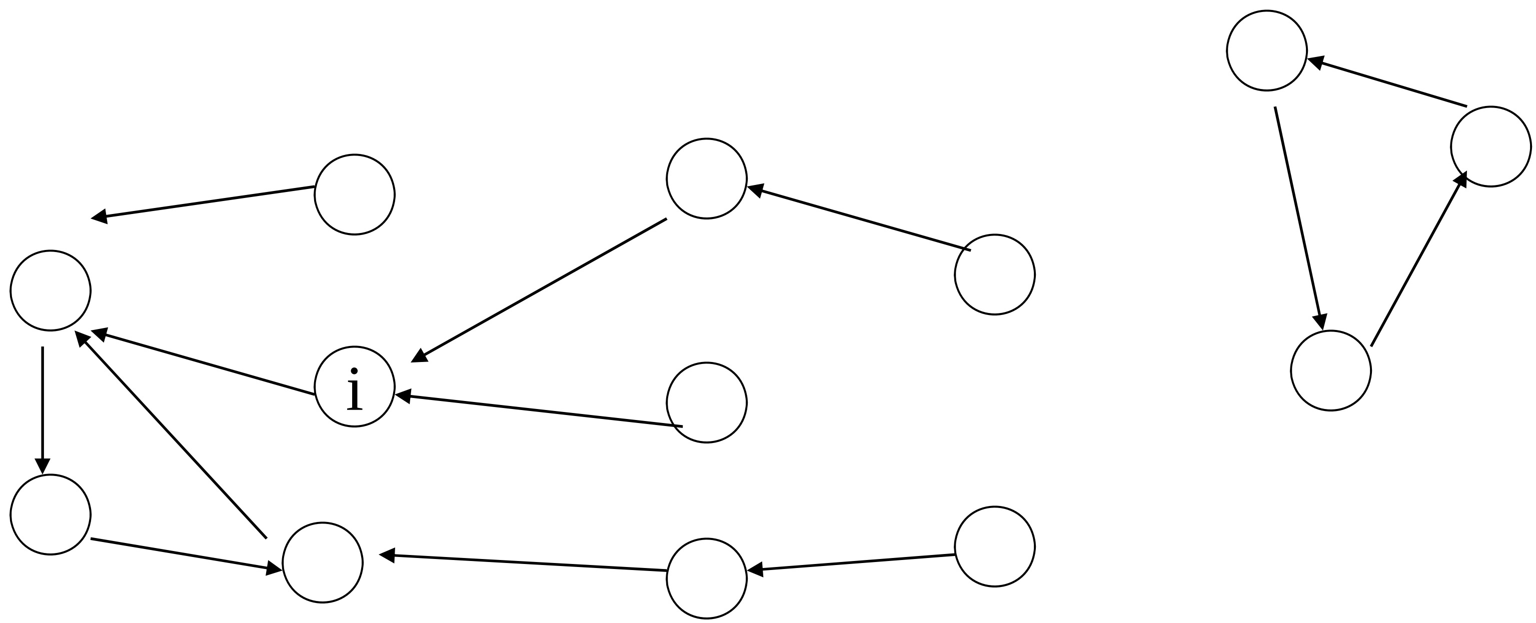
Since also a prefers strictly ν(a) to TTC(a) (why?) the agents in its cycle is a block to TTC (contradiction).

Theorem (Roth '82): if the top trading cycle procedure is used, it is a dominant strategy for every agent to state his true preferences.

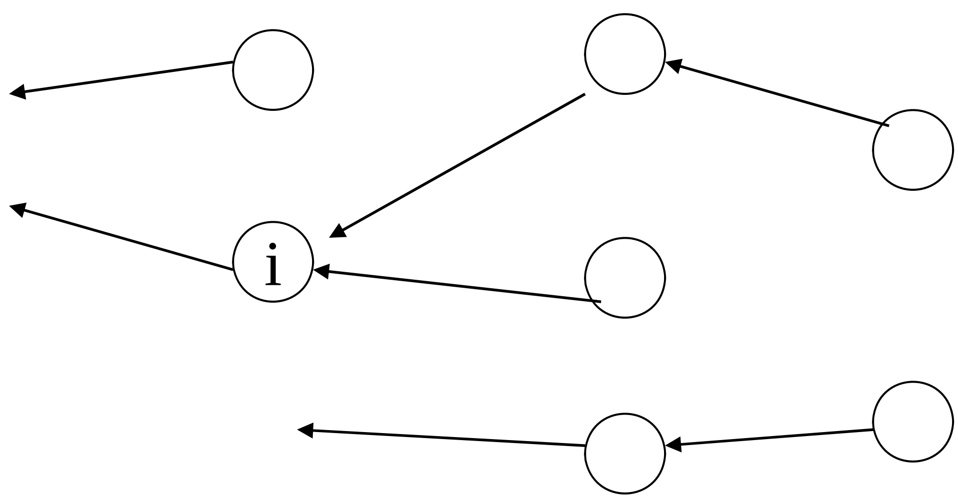
All together: Top Trading Cycles, individually rational strategyrpoof and Pareto-efficient.

Apply the kidney settings:
House – kidney/donors
Agent – patient

Cycles and chains

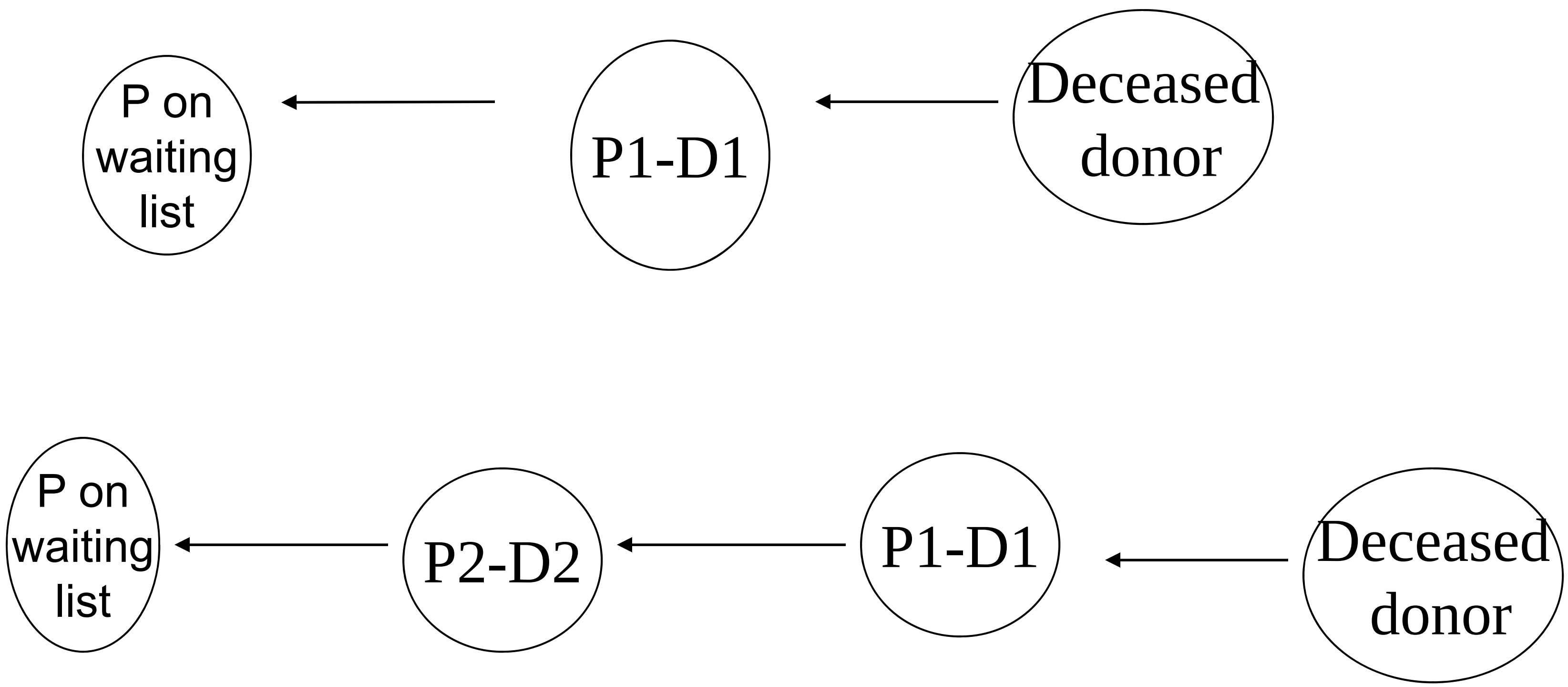


The cycles leave the system (regardless of where i points), but i's choice set (the chains pointing to i) remains, and can only grow



Chains that integrate exchange with the waiting list

- Paired exchange and list exchange



Top trading cycles and chains

- Unlike cycles, chains can intersect, so a kidney or patient can be part of several chains, so an algorithm will have choices to make.
- Theorem: Strategy proof and efficient “TTCC” mechanisms exist for selecting cycles and chains.
- That is, it’s possible to organize kidney exchange to integrate cycles and chains in a way that makes it safe for doctors and patients to reveal information.

After talking to Doctors

- For incentive and other reasons, such exchanges have been done simultaneously – why is this a problem?
- Patients have dichotomous preferences (0-1) - compatible are equally good, incompatible are equally bad.

Suppose exchanges involving more than two pairs are impractical?

- The New England surgical colleagues have (as a first approximation) 0-1 (feasible/infeasible) preferences over kidneys.
- Initially, exchanges were restricted to pairs.
  - This involves a substantial welfare loss compared to the unconstrained case
  - But it allows us to tap into some elegant graph theory for constrained efficient and incentive compatible mechanisms.

Pairwise matchings and matroids

- Let (V,E) be the graph whose vertices are incompatible patient-donor pairs, with mutually compatible pairs connected by edges.
- A *matching* M is a collection of edges such that no vertex is covered more than once.
- Let *S* ={S} be the collection of subsets of V such that, for any S in *S*, there is a matching M that covers the vertices in S
- Then (V, *S*) is a matroid:
  - If T is in *S*, so is any subset of T.
  - If T and T' are in *S*, and |T'|>|T|, then there is a point in T' that can be added to T to get a set in *S*.

Pairwise matching with **0-1** preferences (Roth et. al 2005)

- All maximal matchings match the same number of couples.
- If patients (nodes) have priorities, then a "greedy" priority algorithm produces the efficient (maximal) matching with highest priorities (or edge weights, etc.)
- Any priority matching mechanism makes it a dominant strategy for all couples to
  - accept all feasible kidneys
  - reveal all available donors

Efficient Kidney Matching

- Two genetic characteristics play key roles:

1. ABO blood-type: There are **four** blood types A, B, AB and O.
   - Type O kidneys can be transplanted into any patient;
   - Type A kidneys can be transplanted into type A or type AB patients;
   - Type B kidneys can be transplanted into type B or type AB patients; and
   - Type AB kidneys can only be transplanted into type AB patients.

- So type O *patients* are at a disadvantage in finding compatible kidneys.
- And type O donors will be in *short* supply.

2. Tissue type or HLA type:

- Combination of six proteins, two of type A, two of type B, and two of type DR.
- Prior to transplantation, the potential recipient is tested for the presence of antibodies against HLA in the donor kidney. The presence of antibodies, known as a *positive crossmatch*, significantly increases the likelihood of graft rejection by the recipient and makes the transplant infeasible.

A. Patient ABO Blood Type	**Frequency**
O	48.14%
A	33.73%
B	14.28%
AB	3.85%
B. Patient Gender	**Frequency**
Female	40.90%
Male	59.10%
C. Unrelated Living Donors	**Frequency**
Spouse	48.97%
Other	51.03%
D. PRA Distribution	**Frequency**
Low PRA	70.19%
Medium PRA	20.00%
High PRA	9.81%

Incompatible patient-donor pairs in long and short supply in a sufficiently **large** market

- Long side of the market— (i.e. some pairs of these types will remain unmatched after any feasible exchange.)
  - hard to match: looking for a harder to find kidney than they are offering
  - O-A, O-B, O-AB, A-AB, and B-AB,
  - |A-B| > |B-A|
- Short side:
  - Easy to match: offering a kidney in more demand than the one they need.
  - A-O, B-O, AB-O, AB-A, AB-B
- Not especially hard to match whether long or short
  - A-A, B-B, AB-AB, O-O
- All of these would be different if we weren't confining our attention to *incompatible* pairs.

The structure of efficient exchange

- Assumption 1 (Large market approximation). No patient is tissue-type incompatible with another patient's donor
- Assumption 2. There is either no type A-A pair or there are at least two of them. The same is also true for each of the types B-B, AB-AB, and O-O.
- Theorem[Roth et. al 2007]: every efficient matching of patient-donor pairs in a **large** market can be carried out in exchanges of no more than 4 pairs.
  - The easy part of the proof has to do with the fact that there are only four blood types, so in any exchange of five or more, two patients must have the same blood type.

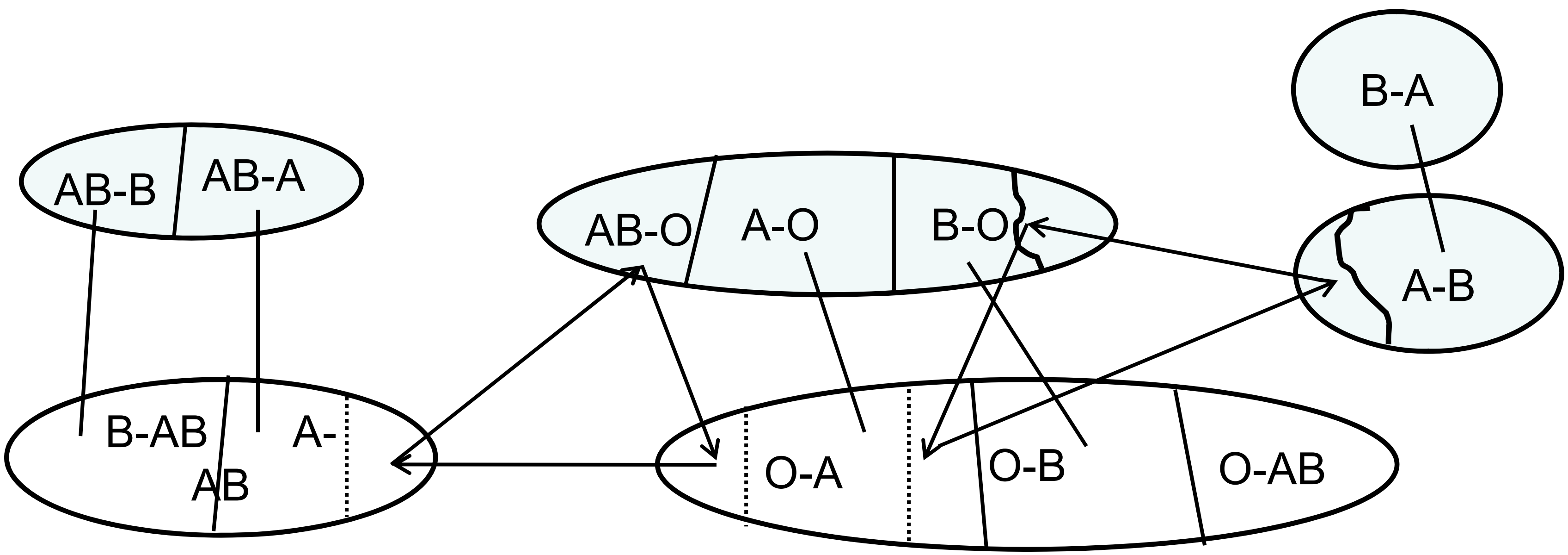
Efficient Exchange Size

Pop. size	Method	Two-way	Two-way, three-way	Two-way, three-way, four-way
	Simulation	8.86 (3.4866)	11.272 (4.0003)	11.824 (3.9886)
$n = 25$	Upperbound 1	12.5 (3.6847)	14.634 (3.9552)	14.702 (3.9896)
	Upperbound 2	9.812 (3.8599)	12.66 (4.3144)	12.892 (4.3417)
	Simulation	21.792 (5.0063)	27.266 (5.5133)	27.986 (5.4296)
$n = 50$	Upperbound 1	27.1 (5.205)	30.47 (5.424)	30.574 (5.4073)
	Upperbound 2	23.932 (5.5093)	29.136 (5.734)	29.458 (5.6724)
	Simulation	49.708 (7.3353)	59.714 (7.432)	60.354 (7.3078)
$n = 100$	Upperbound 1	56.816 (7.2972)	62.048 (7.3508)	62.194 (7.3127)
	Upperbound 2	53.496 (7.6214)	61.418 (7.5523)	61.648 (7.4897)

Efficient Allocations

Without assuming no tissue type incompatibilities, in a large exchange pool:

Theorem[Ashlagi and Roth 2010]: In almost every large enough exchange pool there exist an efficient allocation with exchanges of size at most 3.



Thicker market and more efficient exchange?

- Make kidney exchange available not just to incompatible patient-donor pairs, but also to those who are compatible but might nevertheless benefit from exchange
  - E.g. a compatible middle aged patient-donor pair, and an incompatible patient-donor pair with a 25 year old donor could both benefit from exchange.
  - This would also relieve the present shortage of donors with blood type O in the kidney exchange pool, caused by the fact that O donors are only rarely incompatible with their intended recipient.
    - Adding compatible patient-donor pairs to the exchange pool has a big effect: Roth, Sönmez and Ünver (2004a and 2005
- Establish a national exchange

Computational Issues

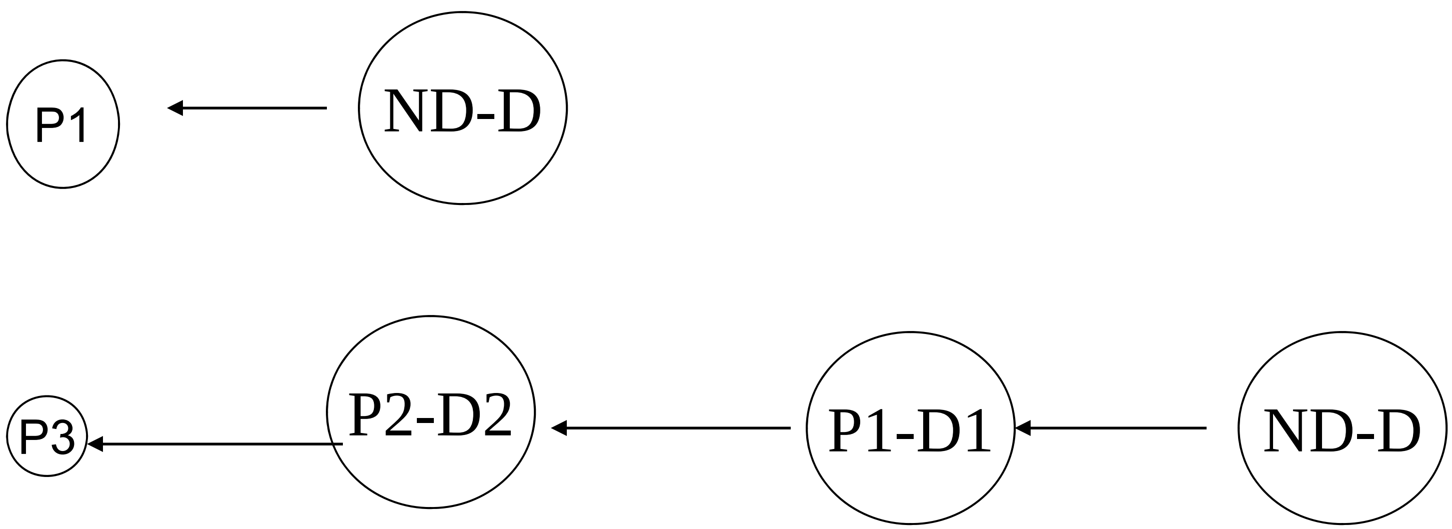
Finding a 2-way matching is easy.

Finding a maximum matching using up to k-way exchanges is computationally difficult (NP hard).

Abraham Blum and Sandholm (2007), present an algorithm that finds such (almost) maximum matchings up to 10000 pairs.

Other sources of efficiency gains

- Non-directed donors



The graph theory representation doesn't capture the whole story

Rare 6-Way Transplant Performed

Donors Meet Recipients

March 22, 2007

BOSTON -- A rare 3-way exchange was a success in Boston.

NewsCenter 5's Heather Unruh reported Wednesday that three people donated their kidneys to three people they did not know. The transplants happened one month ago at Massachusetts General Hospital and Beth Israel Deaconess.

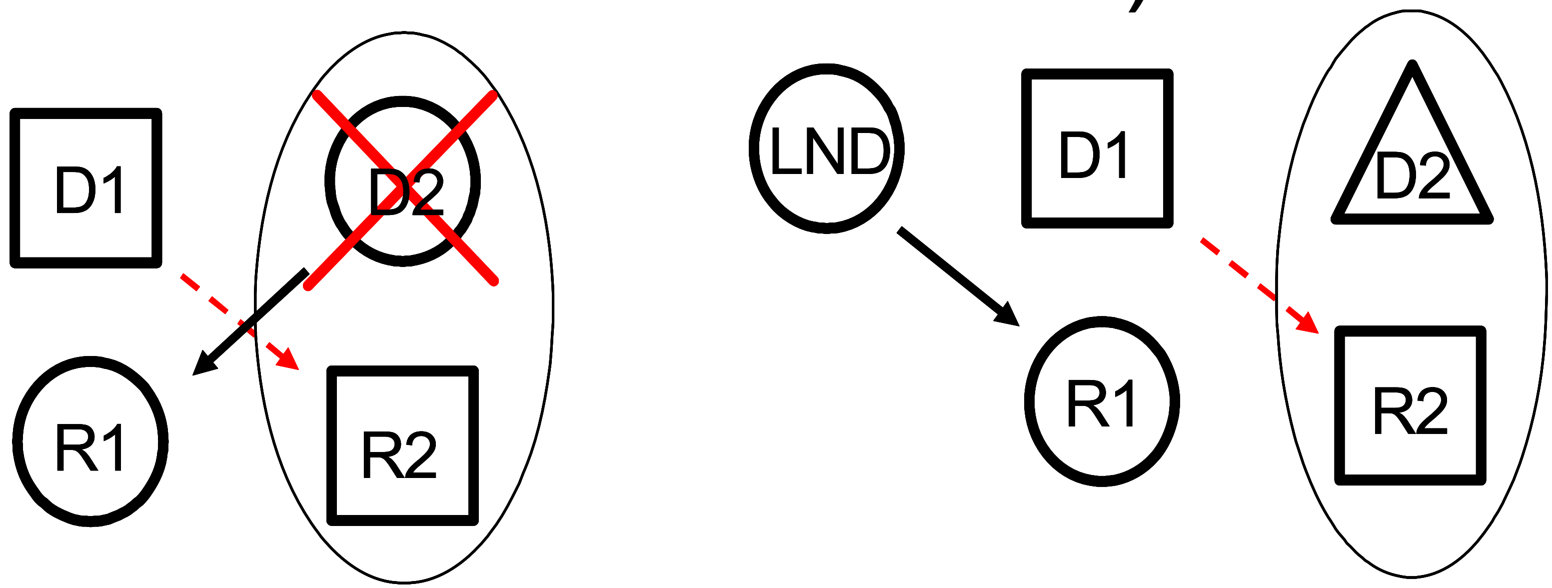
The donors and the recipients met Wednesday for the first time.

Simultaneity congestion: 3 transplants + 3 nephrectomies = 6 operating rooms, 6 surgical teams…

Can simultaneity be relaxed in Non-directed donor chains?

- “*If something goes wrong in subsequent transplants and the whole ND-chain cannot be completed, the worst outcome will be no donated kidney being sent to the waitlist and the ND donation would entirely benefit the KPD [kidney exchange] pool*.” (Roth et al. *AJT* 2006).

Non-simultaneous extended altruistic donor chains (reduced risk from a broken link)

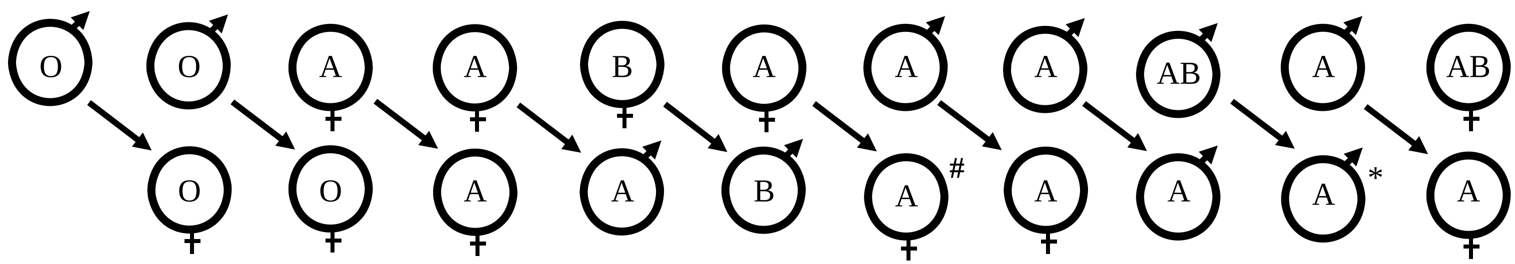


A. Conventional 2-way Matching   B. NEAD Chain Matching

Since **N**EAD chains don't require simultaneity, they can be longer…

The First NEAD Chain (Rees, APD)

		1	2	3	4	5	6	7	8	9	10
		July 2007	*July 2007*	*Sept 2007*	*Sept 2007*	*Feb 2008*	*Feb 2008*	*Feb 2008*	*Feb 2008*	*March 2008*	*March 2008*
	MI	AZ	OH	OH	OH	MD	MD	MD	NC	MD	OH



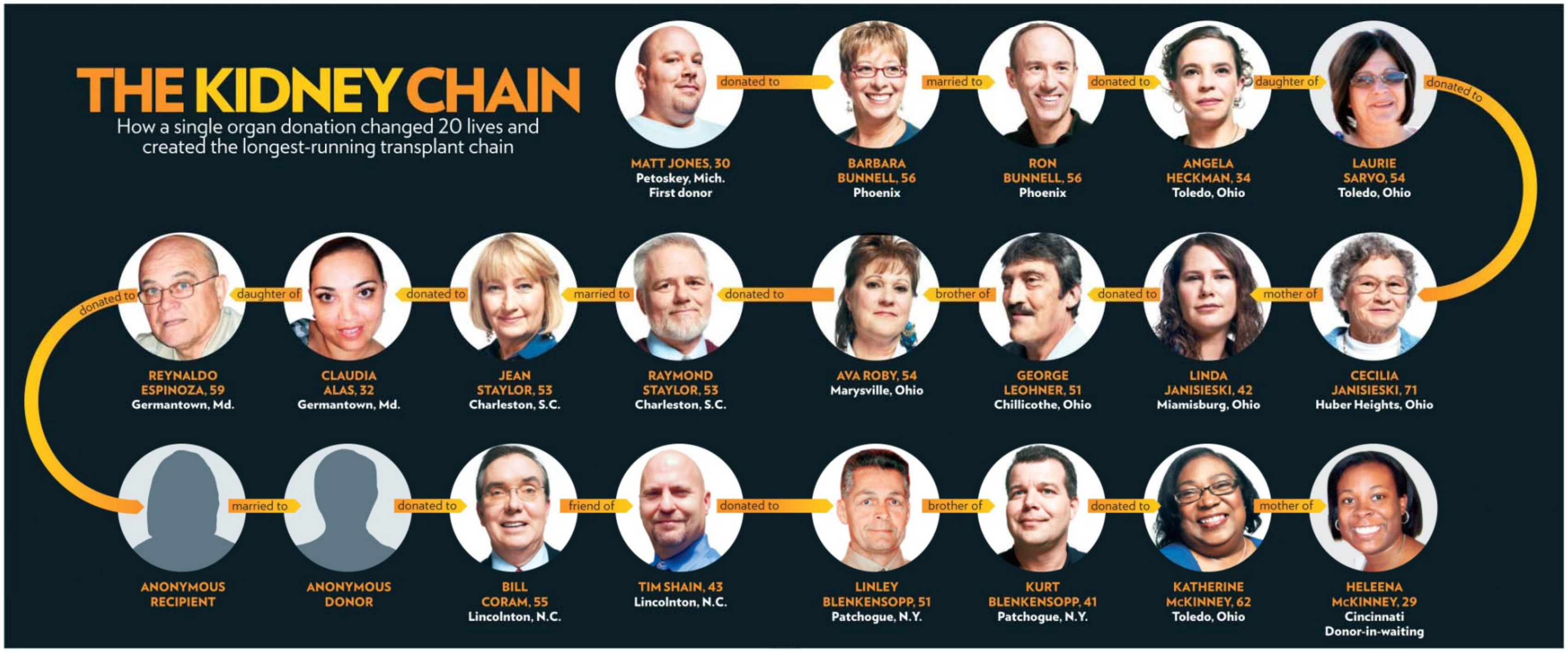
	1	2	3	4	5	6	7	8	9	10
Recipient PRA	62	0	23	0	100	78	64	3	100	46
Recipient Ethnicity	Cauc	Cauc	Cauc	Cauc	Cauc	Hisp	Cauc	Cauc	Cauc	AA
Relationship	Husband Wife	Mother Daughter	Daughter Mother	Sister Brother	Wife Husband	Father Daughter	Husband Wife	Friend Friend	Brother Brother	Daughter Mother

* This recipient required desensitization to Blood Group (AHG Titer of 1/8).

This recipient required desensitization to HLA DSA by T and B cell flow cytometry.

THE KIDNEY CHAIN

How a single organ donation changed 20 lives and created the longest-running transplant chain

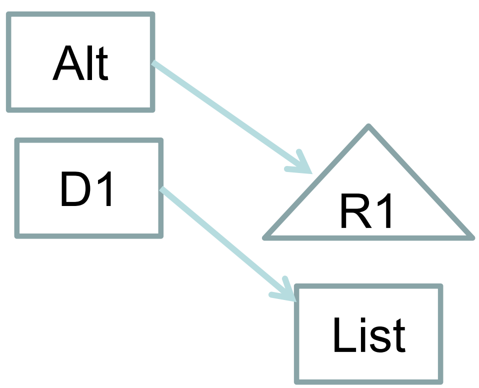


Dr. Mike Rees (center, left) and his team perform a kidney transplant.

Chains

- Bridge Donors Can Renege

Doctors (Gentry et. al) offer Domino chains (2 or 3 way dominos), combined with 2 and 3 way exchanges.

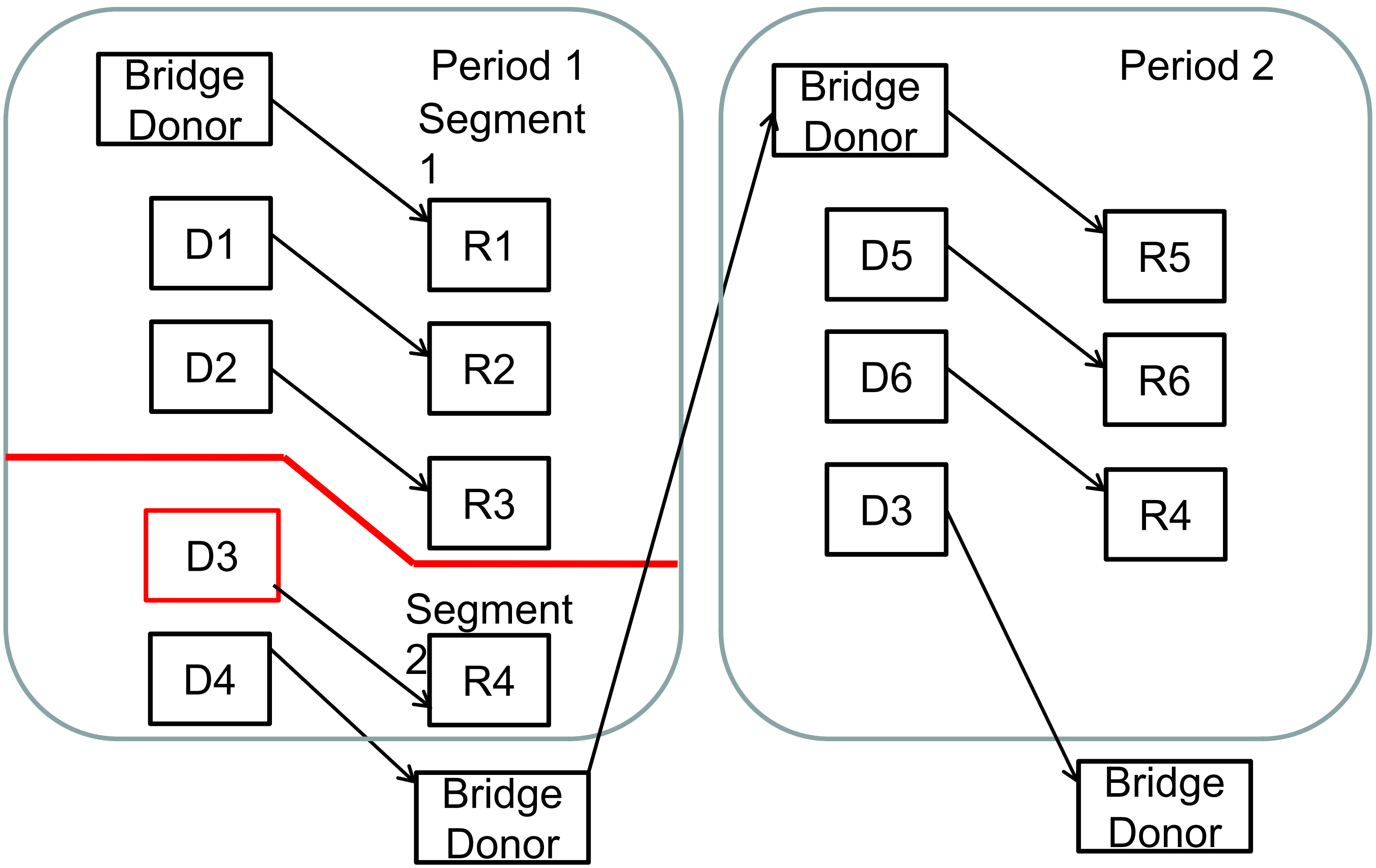


2-way Domino Chain

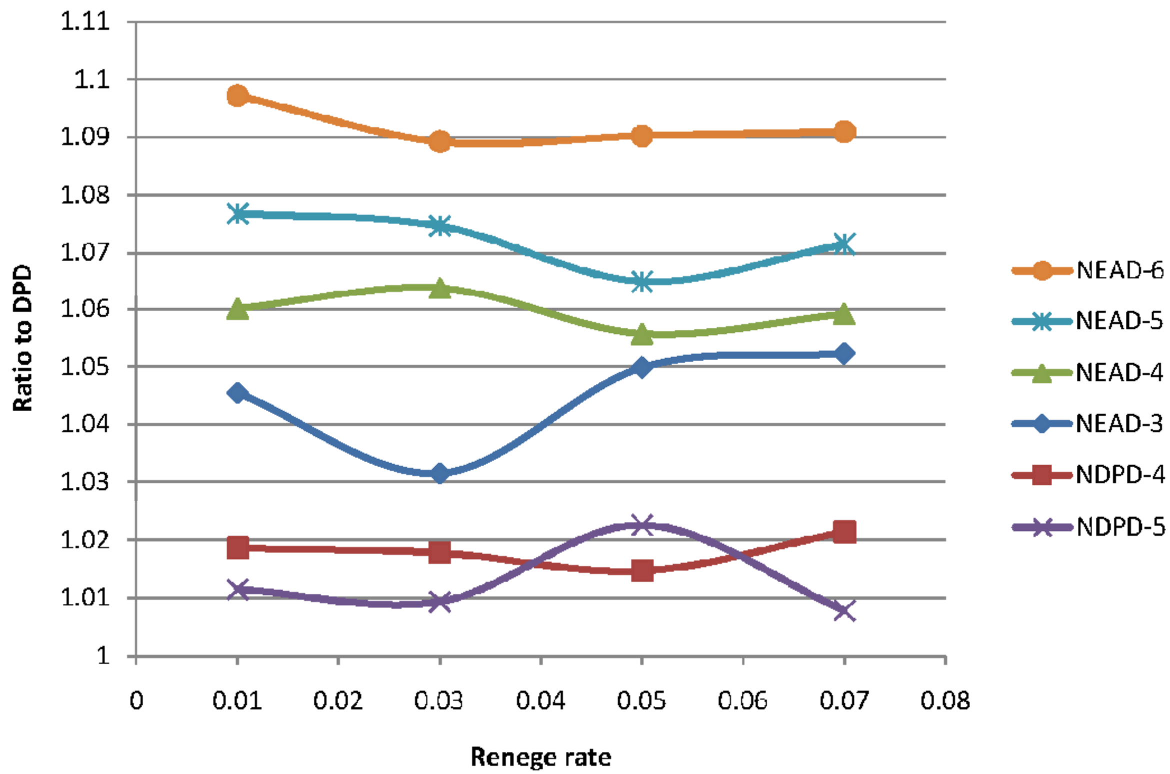
80,000 people

Is there a better policy? We try to answer (Ashlagi et. al 2010)

Chains – Simulated Policy



Ratio of #transplants between policies



Compared a domino with two pairs, to various policies

Quality of Policies

	DPD	NEAD-3	NDPD-4	NEAD-4	NDPD-5	NEAD-5	NEAD-6
Percentage of high PRA patients receiving transplants	6.4/87.1 (7.3%)	8.4/87.1 (9.7%)	7.5/87.1 (8.7%)	9.7/87.1 (11.2%)	7.6/87.1 (8.8%)	10.2/87. (11.7%)	10.6/87.1 (12.2%)

Quality of Policies

	DPD	NEAD-3	NDPD-4	NEAD-4	NDPD-5	NEAD-5	NEAD-6
Percentage of high PRA patients receiving transplants	6.4/87.1 (7.3%)	8.4/87.1 (9.7%)	7.5/87.1 (8.7%)	9.7/87.1 (11.2%)	7.6/87.1 (8.8%)	10.2/87. (11.7%)	10.6/87.1 (12.2%)

Currently under debate in UNOS whether to do open or closed chains…

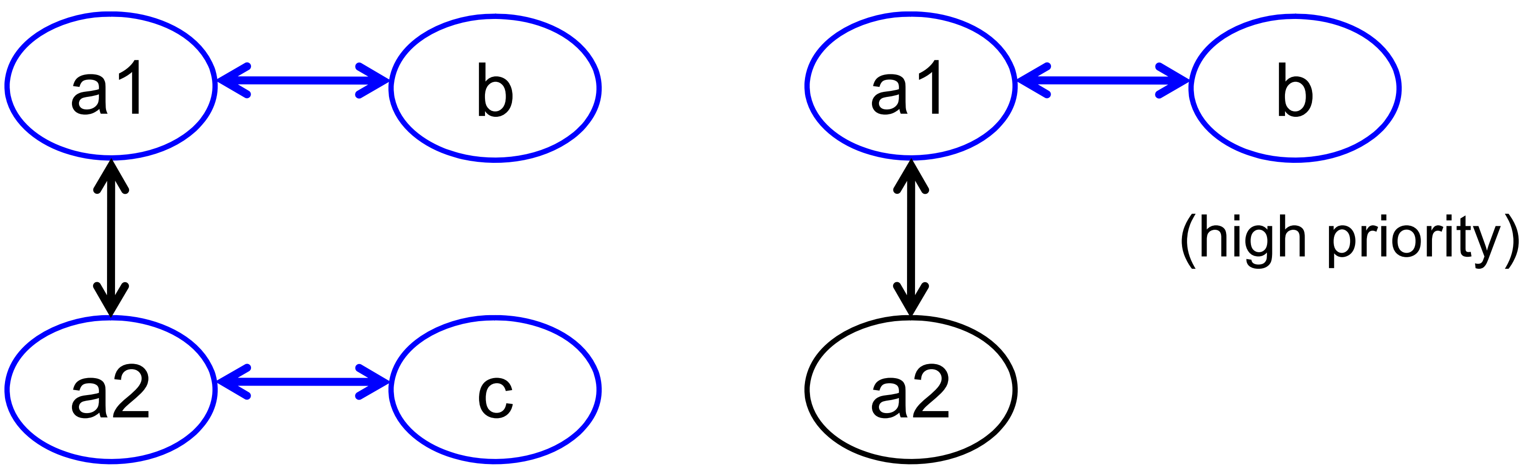
Problems going forward

- What are the problems facing a big, multi-transplant-center kidney exchange program?

Hospitals have Incentives

a1,a2 are pairs from the same hospital

Pairs b and c are from different hospitals



Centralized Kidney Exchange

Mike Rees (APD director) writes us: “As you predicted, competing matches at home centers is becoming a real problem. Unless it is mandated, I'm not sure we will be able to create a national system. I think we need to model this concept to convince people of the value of playing together”.

Centralized Kidney Exchange

Mike Rees (APD director) writes us: “As you predicted, competing matches at home centers is becoming a real problem. Unless it is mandated, I'm not sure we will be able to create a national system. I think we need to model this concept to convince people of the value of playing together”.

Need to deal with hospitals’ incentives.

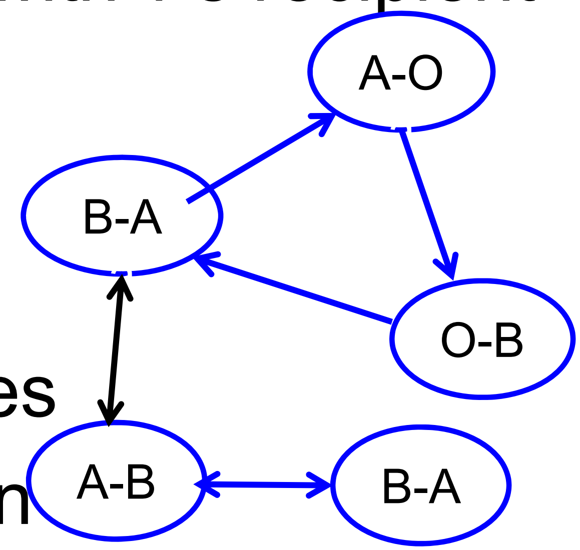
Goal: Design an efficient mechanism for kidney exchange in which hospitals are the players.

Exchange Pools – Compatibility Graphs

Compatibility graph - a directed graph G(V,E):

V - set of incompatible (patient-donor) pairs (nodes)

(u,v)$\in$E if u's donor is compatible with v's recipient (edges)



Exchange – cycle

Allocation – set of disjoint exchanges

nodes are matched by the allocation

Assumption: maximum exchange of size k>0

Definitions (Cont.)

efficient allocation - maximum allocation

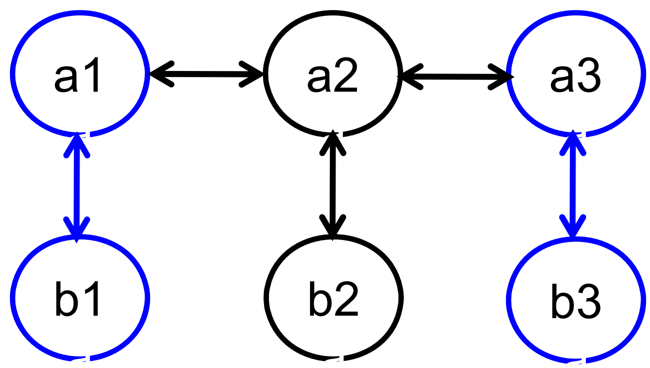
Pareto-efficient allocation – maximal (inclusion) allocation

Set of hospitals $H=\{1,\ldots,n\}$, each $h \in H$ with a set of pairs V_h

$\cup_{h \in H} V_h$ induces the underlying compatibility graph

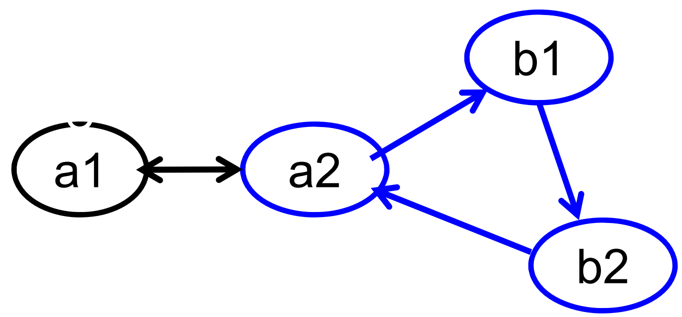
Individual Rationality

An allocation is individually rational (IR) if it matches for every hospital the number of nodes it can match on its own by an efficient allocation



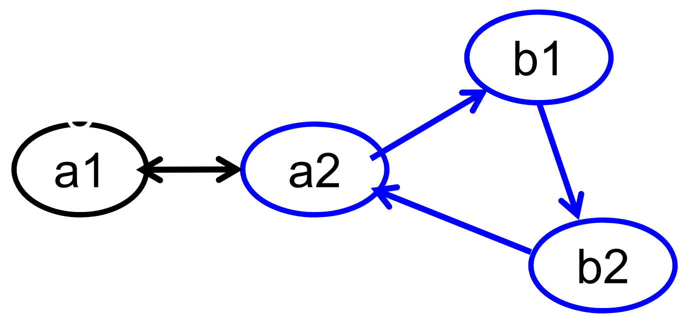
IR & Efficiency

Proposition: for every k$\geq$3 there exists a compatibility graph such that every efficient allocation is not IR.



IR & Efficiency

Proposition: for every $k \geq 3$ there exists a compatibility graph such that every efficient allocation is not IR



worst cast efficiency loss (tight): $1/(k-1)$

Proposition: For every $k>1$, and every compatibility graph there is a Pareto-efficient allocation which is IR.

Proof:

Augmenting algorithm:

1. Choose an IR allocation in each Hospital.
2. Repeat – find an augmenting allocation.

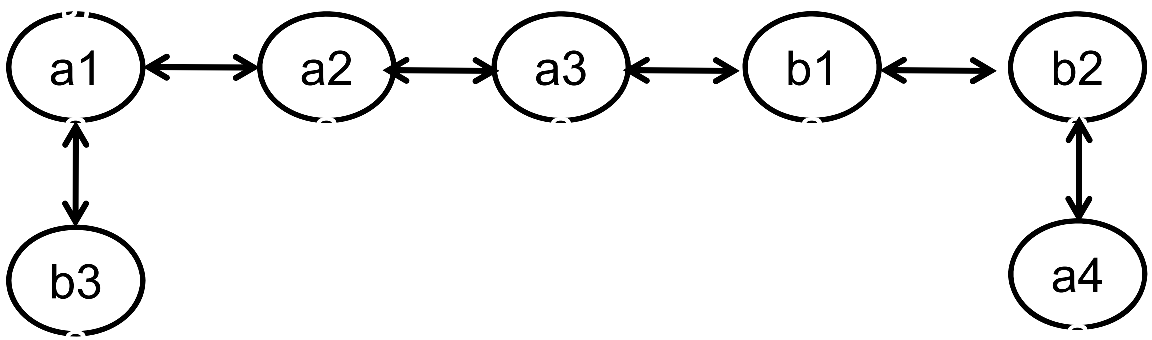
2-way Exchanges

Proposition(Edmonds): for k=2 every Pareto-efficient allocation is efficient.

Therefore for k=2, there is an efficient IR allocation.

Impossibilities

Theorem[Roth, Sönmez, Ünver]: For any k$\geq$2 no IR mechanism which always outputs a Pareto-efficient allocation is strategyproof.



At least one a or one b are not chosen with probability at least ½.

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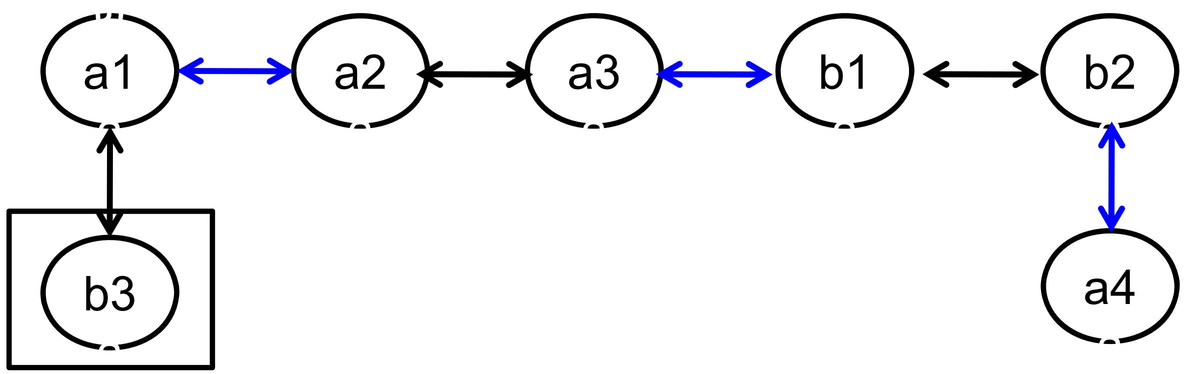

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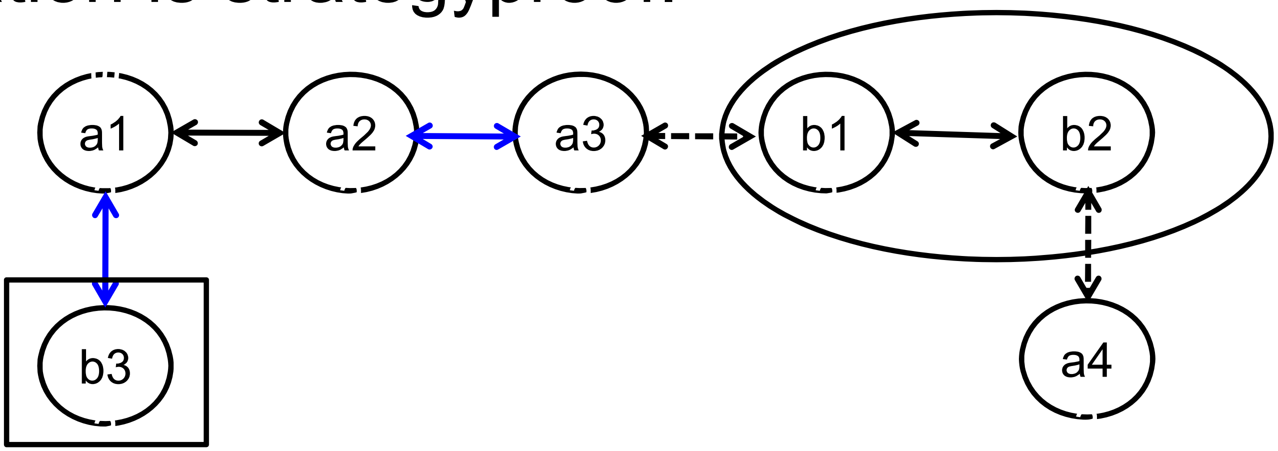
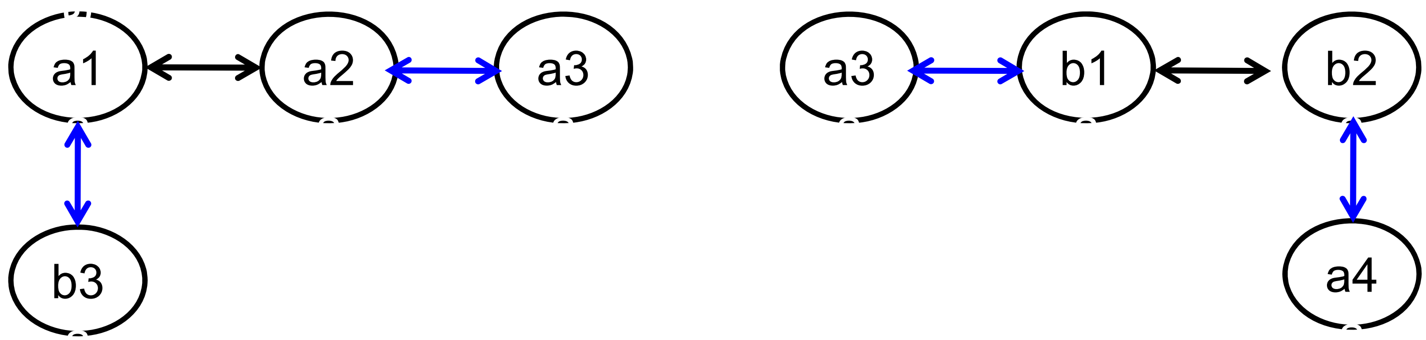


Maybe we can get close to efficiency?

No: (Ashlagi and Roth, 2010)

Theorem: For any k>1 no strategyproof IR mechanism can guarantee more than 1/2 of the efficient allocation.

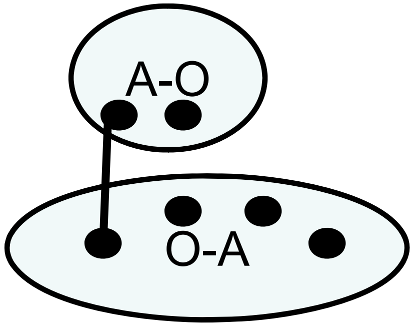


How about randomized mechanisms?
No, cannot guarantee more than 7/8 of the efficient allocation.

Status Quo

- **Current mechanism:** Choose (randomly) a maximum allocation.

Proposition: Withholding internal exchanges can (often) be strictly better off for a hospital regardless of the number of hospitals



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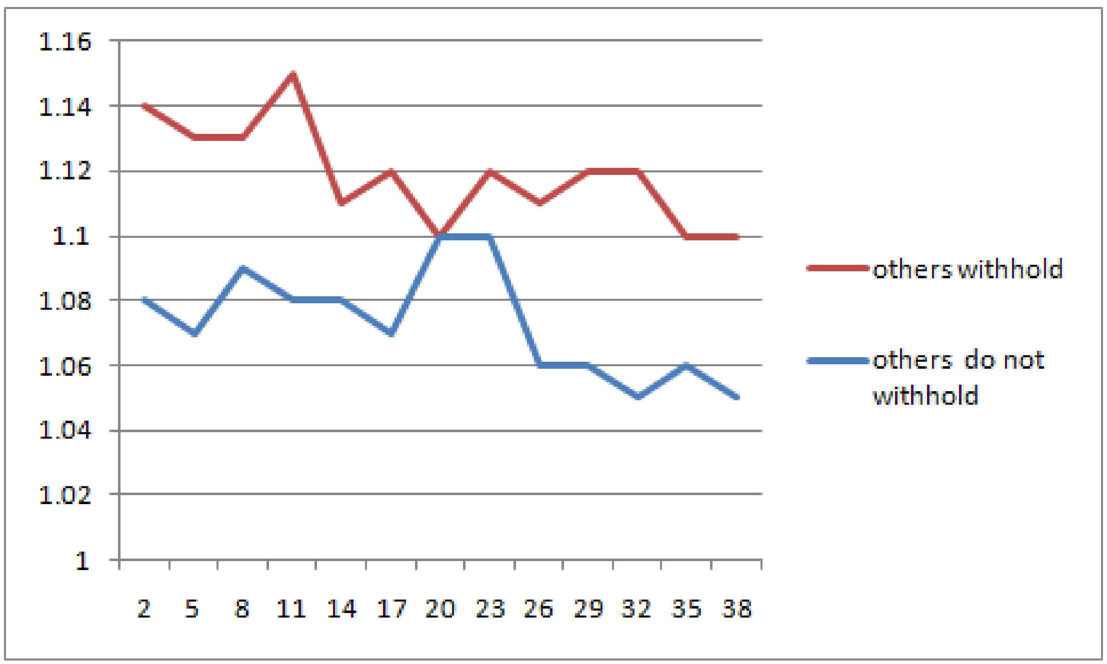

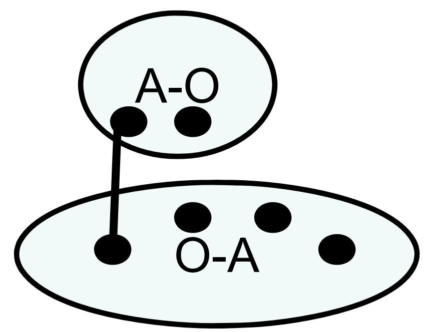

Status Quo (cont.)

No. of Hospitals	Num Of Pairs	Profitable Naïve Strategy, k=2	IR, k=2	Profitable Naïve Strategy, k=3	IR, k=3	Non IR & non strategic hostpials, k=3
2	21	5.08	5.26	5.99	6.8	6.89
4	42	12.66	13.58	15.85	18.37	18.67
6	67	23.78	25.62	30.01	35.42	35.97
8	85	33.54	36.52	41.62	49.3	49.75
10	108	44.04	47.74	54.21	63.68	64.34
12	131	55.84	60.6	70.15	81.43	81.83
14	154	68.64	74.72	85.44	97.82	98.07
16	173	77.44	84.2	96.57	109.01	109.41
18	191	87.84	95.62	109.76	121.81	122.1
20	227	107.74	116.68	132.32	144.09	144.35

Some good news (Ashlagi Roth 2010)

- Theorem: there exist allocations that are close up to the expected number of AB-O pairs from the efficient allocation in large pools.

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Simulation results:

No. of Hospitals	2	4	6	8	10	12	14	16	18	20	22
IR,k=3	6.8	18.37	35.42	49.3	63.68	81.43	97.82	109.01	121.81	144.09	160.74
Non IR, k=3	6.89	18.67	35.97	49.75	64.34	81.83	98.07	109.41	122.1	144.35	161.07

Some more good news (Ashlagi Roth 2010)

- Mechanisms that give priority to internally matchable pairs and under demanded pairs have good incentive and efficiency properties in large markets:
- Theorem: for k =2 (pairwise exchange), full participation is almost an equilibrium.
- Conjecture: for k>2, “close to full participation”.

Loss is Small - Simulations

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Summary of participation incentives

- As kidney exchange institutions grow to include more transplant centers, they will have to fight increasingly hard to get the centers to reveal their most easily match-able patient-donor pairs. This will be an uphill battle as long as the matching algorithm tries to maximize total (or weighted) number of transplants, without regard to internally matchable pairs.
- But the fight will be less hard if the matching algorithms pay attention to internally matchable pairs.

Progress to date

There are several potential sources of increased efficiency from making the market thicker by assembling a database of incompatible pairs (aggregating across time and space), including

1. More 2-way exchanges
2. longer cycles of exchange, instead of just pairs

It appears that we will initially be relying on 2- and 3-way exchange, and that this may cover most needs.

3. Integrating non-directed donors with exchange among incompatible patient-donor pairs.
4. Non-simultaneous non-directed donor chains
5. future: integrating compatible pairs (and thus offering them better matches…)

But progress is still slow☹

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
#Kidney exchange transplants in US*	2	4	6	19	34	27	74	121	240	304
Deceased donor waiting list (active + inactive) in thousands	54	56	59	61	65	68	73	78	83	88

*http://optn.transplant.hrsa.gov/latestData/rptData.asp

Thank you