Ad Auctions

October 8, 2010

Ad Auction Theory: Literature

Old:

Shapley-Shubik (1972)

Leonard (1983)

Demange-Gale (1985)

Demange-Gale-Sotomayor (1986)

New:

Varian (2006)

Edelman-Ostrovsky-Schwarz (2007)

Edelman-Ostrovsky(2007)

The Model

S slots labeled $s = 1 \ldots S$

A agents (bidders) $a = 1 \ldots A$

Agent A values slot s by $u_{as} = v_a \cdot x_s$.

Assume $x_1 > x_2 > \ldots > x_S$

$\Rightarrow$ agents agree on order of slots, but value them differently.

Interpretation:

x_s is click through rate associated w/ position s.

v_a is the value per click for agent a

Example: Assume 1000 searches in 1 day.

1000 searches $\times$ x_s (clicks/searches) $\times$ v_a (dollars/click)

Modification: Ad "Quality"

Consider two different ads in slot 1.

Should we expect CTR to be the same?

One ad "creative" may be more appealing/relevant.

Model: Decompose CTR into underlying slot appeal and ad quality.

$\text{CTR} = q_a \times x_s$

$\Rightarrow$ can combine q_a with v_a!

For now, ignore q_a.

The Auction

Each agent a bids b_a

Slots assigned in order of bids, high to low

Agent a's price (per click!) is bid of agent in next slot down

Edelman et al.: Generalized Second Price auction (GSP)

Payoffs (per search)

(renumber agents so that agent 1 has high bid etc.)

Payoff of agent who wins slot s:

$v_s \cdot x_s - b_{s+1} \cdot x_s$

$= (v_s - b_{s+1})x_s$

Incentives

Compared to first price auction, less need for monitoring/bid adjustment.

However, room for “squeezing’.’

Is this a Vickrey Auction?

Is truth telling a dominant strategy?

Example:

3 bidders with values per click $v_1 = 15, v_2 = 10, v_1 = 5$.

Suppose CTRs are $x_1 = .401, x_2 = .399, x_3 = .2$

Truthful bidding gives agent 1 payoff $(15 - 10) \times .401$.

Bidding 9 gives payoff $(15 - 5) \times .399$

In fact, no dominant strategy exists.

Bid shading

Vickrey Auction

Let z be an allocation.

Let $u_a(v_a, z)$ give a's utility from allocation z.

Vickrey Mechanism:

1. Each agent reports value r_a.
2. Mechanism chooses allocation z to maximize total value; that is $u_a(r_a, z) + \sum_{b \neq a} u_b(r_b, z)$.
3. Payment to a is sum of utilities reported by other agents; hence, a's payoff is $u_a(v_a, z) + \sum_{b \neq a} u_b(r_b, z)$

Truthful reporting is a dominant strategy for a.

VCG Pivot Mechanism

Recall: can include in payment to a any term that only involves announcements of others (without changing incentives.)

Payoff to a under pivot mechanism:

$$u_a(v_a, z) - \left(\max_y \sum_{b \neq a} u_b(r_b, y) - \sum_{b \neq a} u_b(r_b, z) \right).$$

Payment *by* a is harm he imposes on other agents.

VCG Payments in Ad Auctions

In this model, a sends all bidders with reported (value) below him down a slot.

Payment of agent in slot s:

$$\sum_{t>s} v_t(x_{t-1} - x_t)$$

So why not use VCG?

GSP Equilibrium Analysis

- Full information setting: agents know values v.

Equilibrium $\Leftrightarrow$ no gain from changing slots

Consider agent in slot s:

Move up a slot $\Leftrightarrow$ must beat bid of $s - 1$.

Move down a slot $\Leftrightarrow$ must beat bid of $s + 2$

Nash equilibrium

A **Nash equilibrium** is a bid for each agent s such that

$$\begin{aligned}(v_s - b_{s+1})x_s &\geq (v_s - b_{t+1})x_t \ for \ t > s \\ (v_s - b_{s+1})x_s &\geq (v_s - b_t)x_t \ for \ t < s\end{aligned}$$

In general, there is a range of Nash equilibribria.

Some equilibria involve low value agents outbidding high value agents. (Exercise: find a simple example)

$\Rightarrow$ No revenue equivalence.

Refinement: Locally Envy-Free (Edelman et al. 2007)

Definition: An equilibrium induced by GSP is **locally envy-free** if no player can improve his payoff by exchanging bids with the player ranked one position above him.

Motivation: "Squeezing"

No room for "safe squeezing" $\Leftrightarrow$ locally envy-free.

Stable Assignments

Treat positions as players. Coalitional value from a position-bidder pair is given by $v_a x_s$. Payoff to agent is $(v_a - p)x_s$, and payoff to position is px_s.

Then

1. The outcome of any locally envy-free equilibrium is a stable assignment.
2. Provided $|A| > |S|$, then any stable assignment is an outcome of a locally envy-free equilibrium.

Revenue and Payments in GSP Auction

Theorem:

1. $\exists$ a "best" locally envy-free equilibrium for the bidders - that is; any other eqm involves weakly lower payoffs for all bidders. Correspondingly, this is the worst eqm for the search engine.

2. Positions and payments are equal to VCG positions and payments.

Revenue and Payments

Implication: Maybe this is why we don't see VCG...

VCG yields higher bids (no shading), but not higher revenue!

Varian Refinement

“Symmetric Nash Equilibria”

- Nice algebraic properties

- Intuition?

Incomplete Information

VCG: Still a dominant strategy to bid truthfully.

Edelman, Ostrovsky and Schwarz: Generalized English Auction.
Equilibrium payments and positions same as in VCG.

Research Questions

Empirical Analysis: Backing values from bids; do players bid in eqm?

Learning: How should my bid change in a dynamic environment: costly to learn.

Reserve prices/min bids

Bid Transparency

Complementarities (regarding what ads search engine should display)

Affiliate bidding