

# Market Design: Theory and Applications

## Introduction

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Fall 2010

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- ▶ But bridge design also concerns metal fatigue, soil mechanics, and the sideways forces of waves and wind. Many questions concerning these complications can't be answered analytically, but must be explored using physical or computational models.
- ▶ These complications, and how they interact with that part of the physics captured by the simple model, are the concern of the engineering literature. Some of this is less elegant than the simple model, but it allows bridges designed on the same basic model to be built longer and stronger over time, as the complexities and how to deal with them become better understood.

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- ▶ Design involves a responsibility for detail.
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- ▶ Game theory, the part of economics that studies the “rules of the game,” provides a framework with which design issues can be addressed.
- ▶ But dealing with complexity will require new tools, to supplement the analytical toolbox of the traditional theorist.

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- ▶ In recent years there have been some great advances in the theory of each of these, that brings them much closer together.
- ▶ A lot of these theoretical insights have come from the difficulties faced in designing complex labor markets and auctions (e.g. labor markets in which there may be two-career households, and auctions in which bidders may wish to purchase packages of goods).

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- ▶ Some kinds of transactions are repugnant. . . This can be an important constraint on market design

# Examples

- ▶ Kidney exchange (thickness, congestion, incentives)
  - ▶ New England and Ohio (2005)
  - ▶ National US (2010/2011??)
  - ▶ Is buying a kidney repugnant?
- ▶ Medical labor markets
  - ▶ NRMP in 1995 (thickness, congestion, incentives)
  - ▶ Gastroenterology in 2006 (thickness, incentives)
  - ▶ Is renegeing on early acceptances repugnant?
- ▶ School choice systems:
  - ▶ New York City since Sept. 2004 (congestion & incentives)
  - ▶ Boston since Sept. 2006 (incentives)
  - ▶ Is exchanging priorities repugnant? (particularly sibling priorities)
  - ▶ SFUSD—presently underway
- ▶ American market for new economists
  - ▶ Scramble (thickness) March 2006
  - ▶ Signaling (congestion) December 2007

## More Examples

- ▶ Online dating
- ▶ eBay and Amazon
- ▶ Google's internet ad auctions
- ▶ Landing slots in airports
  - ▶ a combinatorial auction?

# Zooming through a matching example: matching doctors to first positions in U.S. and Canada

- ▶ 1995: redesign of the
  1. U.S. National Resident Matching Program (NRMP) (approx. 23,000 positions, 500 couples)
  2. Canadian Resident Matching Service (CaRMS) (1,400 Canadian medical grads, including 41 couples, 1,500 positions in 2005)
- ▶ 2005: redesign of the fellowship market for Gastroenterologists
- ▶ Contemporary issues in labor markets for Orthopedic surgeons, neuropsychologists, and law clerks for appellate judges.

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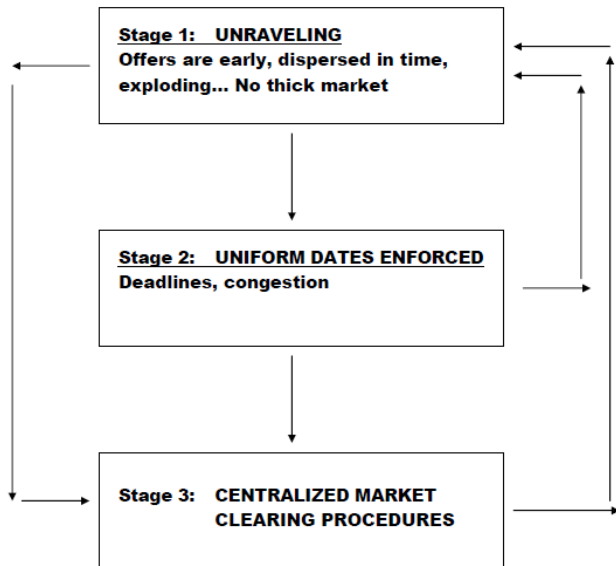
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  - ▶ Roth-Peranson clearinghouse algorithm adopted, and employed

# Stages and transitions observed in various markets



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- ▶ Hypothesis: successful clearinghouses produce stable matchings.
- ▶ How to test this?

# Gale, David and Lloyd Shapley [1962], Two-Sided Matching Model

- ▶ Men =  $\{m_1, m_2, \dots, m_n\}$       Women =  $\{w_1, w_2, \dots, w_p\}$
- ▶ PREFERENCES (complete and transitive):
  - ▶  $P(m_i) = w_3, w_2, \dots, m_i, \dots$        $[w_3 \succ_{m_i} w_2]$
  - ▶  $P(w_j) = m_2, m_4, \dots, w_j, \dots$
- ▶ Outcomes = matchings:  $\mu : M \cup W \rightarrow M \cup W$  such that
  1.  $w = \mu(m)$  iff  $\mu(w) = m$ , and
  2. either  $\mu(w)$  is in  $M$  or  $\mu(w) = w$ , and
  3. either  $\mu(m)$  is in  $W$  or  $\mu(m) = m$



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- ▶ NB: A stable matching is efficient, and in the **core**, and in this simple model the set of (pairwise) stable matchings equals the core.

# Stability as a criterion for a successful clearinghouse

Market	Stable	Still in use (halted unraveling)
• NRMP	yes	yes (new design in '98)
• <i>Edinburgh ('69)</i>	<i>yes</i>	<i>yes</i>
• <i>Cardiff</i>	<i>yes</i>	<i>yes</i>
• <i>Birmingham</i>	<i>no</i>	<i>no</i>
• <i>Edinburgh ('67)</i>	<i>no</i>	<i>no</i>
• <i>Newcastle</i>	<i>no</i>	<i>no</i>
• <i>Sheffield</i>	<i>no</i>	<i>no</i>
• Cambridge	no	yes
• London Hospital	no	yes
• Medical Specialties	yes	yes (~30 markets, <b><u>1 failure</u></b> )
• Canadian Lawyers	yes	yes (Alberta, no BC, Ontario)
• Dental Residencies	yes	yes (5 ) (no 2)
• Osteopaths (< '94)	no	no
• Osteopaths ( $\geq$ '94)	yes	yes
• Pharmacists	yes	yes
• Reform rabbis	yes (first used in '97-98)	yes
• Clinical psych	yes (first used in '99)	yes
• <b>Lab experiments</b>	<b>yes</b>	<b>yes.</b>
<b>(Kagel&amp;Roth QJE 2000)</b>	<b>no</b>	<b>no</b>

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  2. Each woman holds her most preferred acceptable offer to date, and rejects the rest.
- ▶ When no further proposals are made, stop and match each woman to the man (if any) whose proposal she is holding.

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  2. The matching  $\mu_M$  produced by the deferred acceptance algorithm with men proposing is the M-optimal stable matching. The W-optimal stable matching is the matching  $\mu_W$  produced by the algorithm when the women propose.

## Incentives: many-to-one matching

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- ▶ **Capacity manipulation** (Sönmez, 1997) No stable matching mechanism makes it a dominant strategy for a hospital to always reveal its capacity.



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- ▶ **New theory** (Immorlica and Mahdian 2005, Kojima and Pathak 2009): as the market grows (in a very specific way), the proportion of hospitals that might profit from preference or capacity manipulation goes to zero in the student proposing deferred acceptance algorithm.

## Some NRMP "match variations"

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2. In the second category are requests by residency programs to have an even or an odd number of matches, and reversions of unfilled positions from one program to another.

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- ▶ Computational explorations can help to see how close an approximation the simple theory provides for the complex market.
- ▶ Experiments can help to see how real people react to the market and which details are important.

## Zooming through an auction example: ad auctions

- ▶ 98% of Google's and ~50% of Yahoo's revenues
- ▶ The “future of advertising”
- ▶ Unusual auction rules: multiple units, but only one bid.  
Continuous time.
- ▶ Purely electronic market:
  - ▶ Good data, almost like a lab.
  - ▶ Flexibility to change auction rules from time to time

Google Search: computer - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address http://www.google.com/search?hl=en&q=computer


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# Generalized Second-Price (GSP) auction

First implemented by Google (2002), later adopted by Yahoo

- ▶ Each slot has a click-through-rate

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- ▶ Each bidder pay the bid of the next-highest bidder (per-click)

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# Nobel Prize-winning economic theory - Vickery's 2nd price auction

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- ▶ A dominant strategy: bid your value

## 'Bid your value' is not a dominant strategy under GSP

- ▶ **Idea:** sometimes, bidding below your true valuation can be profitable - you will get less traffic, but earn greater profits.



## 'Bid your value' is not a dominant strategy under GSP

- ▶ **Idea:** sometimes, bidding below your true valuation can be profitable - you will get less traffic, but earn greater profits.
- ▶ **Example:** suppose that there are 3 bidders and 2 positions. Positions have click-through rates 100 and 80.

bidder    bid

C's valuation: \$10

A        \$8   ← C bids \$10, pays \$8   → payoff  $(\$10 - \$8) * 100 = \$200$

B        \$5   ← C bids \$6, pays \$5   → payoff  $(\$10 - \$5) * 80 = \$400$

$\$400 > \$200$ . So C should place a bid below its valuation.

# Course website

<https://sites.google.com/a/brown.edu/itay-fainmesser/teaching/econ-1465>

## Course structure (tentative) - many advanced topics

1. Matching Theory
2. NRMP design
3. Congestion in Matching Markets: the Market for Clinical Psychologists, College admission, AEA signaling, online dating
4. Unraveling: college football, the market for Gastroenterologists, the market for judicial clerks
5. Kidney exchange
6. School choice
7. Auction Theory
8. eBay vs. Amazon
9. Ad auctions
10. Combinatorial exchanges
11. Students' presentations

# Guests

1. Itai Ashlagi

[http://mitsloan.mit.edu/faculty/detail.php?in\\_spseqno=sp0027513&](http://mitsloan.mit.edu/faculty/detail.php?in_spseqno=sp0027513&)

2. David Parkes

<http://www.eecs.harvard.edu/~parkes/>

# Prerequisites

1. ECON 1110 or ECON 1130
2. MATH 0090 or equivalent
3. Background in Game Theory is a plus
4. Curiosity, open mindedness, and willingness to work hard

Related course: two-sided matching theory

## Required reading

- ▶ Al Roth and Merilda Sotomayor, "Two-Sided Matching," Cambridge University Press, 1990
- ▶ Paul Klemperer, Auctions: Theory and Practice, Princeton University Press, 2004. (available online at: <http://www.gqq10.dial.pipex.com/>)
- ▶ Many research papers
- ▶ Slides (when posted in advance)

Course website:

<https://sites.google.com/a/brown.edu/itay-fainmesser/teaching/econ-1465>

# Assignments

- ▶ Problem sets
- ▶ Referee report (check out the guidelines on the course website)
- ▶ Final paper proposal (October 20)
- ▶ In class presentation (November 10 - December 1.  
Presentation slots will be assigned by the instructor)
- ▶ Final paper (December 12)
  1. A review of a real-world market, with focus on the markets rules, strengths and weaknesses of the existing design, and relevant academic literature.
  2. A review of a question that is discussed in the academic literature on market design.
  3. A research proposal that is related to Market Design (recommended only for students that plan to follow up with an honors thesis).

## An idea for a term paper

- ▶ Here is the web site of the American Association of Colleges of Podiatric Medicine:  
<http://www.casprcrip.org/html/casprcrip/students.asp>
- ▶ They run a match, and here is the description of their algorithm:  
<http://www.casprcrip.org/html/casprcrip/pdf/MatchExpl.pdf>
- ▶ Is their algorithm equivalent to the hospital proposing deferred acceptance procedure?
- ▶ Does it produce the same matching, when it produces a matching?
- ▶ Does it always (for every preference profile) produce a matching?
- ▶ Is the description of the algorithm complete enough to be sure?