Multiunit Auctions: Package Bidding

Examples of Multiunit Auctions

- Spectrum Licenses
- Bus Routes in London
- IBM procurements
- Treasury Bills

Note: Heterogenous vs Homogenous Goods

Challenges in Multiunit Auctions

- Complexity
  1. How to partition object for sale
  2. How to bid
  3. Determine winning bids
- Demand Reduction
- Exposure Problem
- Efficiency, core outcomes

The Simultaneous Ascending Auction

- Used e.g. to auction spectrum licenses
  - 10 paging licenses in 1994 - $617 mil
  - 99 broadband PCS licenses in 1998 - $7 bil
  - Many additional auctions in Europe
- Auction Format
  - Bidders bid separately for each license
  - Each round of bidding takes place by sealed bid
  - "Standing high bids" announced each round
  - Activity rules, minimum increments ...
  - Bids are binding! Penalty for withdrawal.
- For details, see Milgrom JPE 2000

Exposure Problem in the Netherlands

- 1998 Netherlands Spectrum Auction
  - Simultaneous Ascending Auction
  - Raised \$1.84 billion
  - 2 large lots (A and B), 16 smaller lots
- Outcome: Price per unit bandwidth in millions of NL Guilder
  - Lot A: 8.0
  - Lot B: 7.3
  - Lots 1-16: 2.9-3.6
- Low outcomes? Arbitrage?
- ??

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- Small lots are complements.

Package Bidding

- Idea: Bidders specify bids for each package
- Example: If A and B are complements, may bid high for the package AB, but low for A and low for B.
- Immediate concern: complexity.
  - N items $\rightarrow 2^N - 1$ bids
  - One solution: 'volume discounts' for bus routes

Three Auction Formats

- Menu Auctions (Bernheim-Whinston QJE 1986)
  - 'Pay-as-bid' or first price sealed bid
  - Assumption: Common knowledge of bidder values
- Vickrey Auction
  - Clarke-Groves pivot mechanism
  - Report values, pay externality you impose on others
- Ascending Auctions with Package Bidding (Ausubel-Milgrom FTE 2002)
  - Shares many good qualities with the above auctions, solves some of the problems
  - Fits into the "Matching with Contracts" framework

First Price Sealed Bid Auction: Example

Object X for sale, can be divided into two pieces X_1 and X_2.

Two bidders, A and B

	value to A	value to B
X_1	6	5
X_2	5	6
X	8	7
Nothing	0	0

Loosely speaking, X_1 and X_2 are substitutes.

Menu Auction: Rules

1. Players bid on all packages

2. Seller selects feasible bids that maximize revenue

3. Winning bidders pay bids

MENU AUCTIONS AND RESOURCE ALLOCATION

TABLE I

Equilibrium	(a)	(b)	(c)	(d)	(e)	(f)
A's offer for:						
nothing	0	0	0	0	0	0
X_1	0	0	2	0	5	1
X_2	0	3	3	2	0	0
X	7	6	6	5	7	3
B's offer for:						
nothing	0	0	0	0	0	0
X_1	0	3	3	3	0	1
X_2	0	0	2	0	2	2
X	7	6	6	5	7	3
Equilibrium allocation:						
A	X	X_2	X_2	X_2	X_1	X_1
B	0	X_1	X_1	X_1	X_2	X_2
Net payoffs:						
A	1	2	2	3	1	5
B	0	2	2	2	4	4
Auctioneer	7	6	6	5	7	3

Equilibrium Analysis

Observations:

- Multiple equilibria
- Not all equilibria are efficient; $(a) - (d)$ are not
- Even among efficient equilibria, seller revenue can vary

Formal Model

The Model:

- M bidders
- Seller can choose a single allocation s from menu S
- $g_i(s)$ gives i's value for allocation s (common knowledge)
- Define $S^* \equiv \arg\max_S \sum_i g_i(s)$

The Game

1. Each bidder i names $b_i : S \to \mathbb{R}$.

2. Define $I^*(\{b_i\}_{i=1}^M) \equiv \arg\max_S \sum_i b_i(s)$

3. Auctioneer chooses $s \in I^*(\{b_i\}_{i=1}^M)$ (tiebreaker?)

4. Allocation s, each bidder i pays $b_i(s)$

Profit-Targeting Strategies

Definition: $f_i(\cdot)$ is the π^i-profit-targeting strategy if for all $s \in S$

$$b_i(s) = \max[g_i(s) - \pi^i\,, 0]$$

Appeal:

- Simple bidding strategies
- Theorem: Given strategies of others, $\exists$ a profit-targeting strategy in the set of best responses.
- Robust to demand reduction
- Theorem: The set of "profit-targeting equilibria" is nonempty

Core Payoffs

Let $J \subseteq \{\text{bidders}\} \cup \{\textit{seller}\} \equiv \mathcal{N}$

Define coalitional value

$$w(J) = \begin{cases} 0 & \text{if seller } \notin J \\ \max_s \sum_{i \in J} g_i(s) & \text{if seller } \in J. \end{cases}$$

Define payoff vector $\pi \in \mathbb{R}^{M+1}$ to be in the *core* if

1. $\sum_{i \in \mathcal{N}} \pi_i \leq w(\mathcal{N})$ (feasibility)
2. $\nexists J \mid w(J) > \sum_{i \in J} \pi_i$ (no blocking coalition)

Note: $\sum_{i \in \mathcal{N}} \pi_i = w(\mathcal{N})$, i.e. core outcomes are always efficient.

Lemma: With one seller, the core is non-empty. Proof?

Bidder Optimal Core Payoffs

Definition: Core payoff π is *bidder optimal* if there is no other core payoff weakly preferred by every bidder and strictly by at least one.

The Main Result

Theorem: The bidder optimal core payoffs exactly coincide with the equilibrium payoffs of the profit-targeting equilibria.

Coalition-Proof Equilibria

Theorem: The set of profit-targeting equilibria coincide with the set of Coalition Proof eqa (except possibly off the eqm path.)

Pros and Cons

Pros:

- Simple strategies
- Efficient
- Robust to demand reduction
- Ex post stable payoffs (core payoffs)
- Robust to Collusion

Cons

- common knowledge assumption
- multiple equilibria
- no revelation of info (should we extend the model to common values)

Vickrey Auction

- Standard VCG mechanism - nothing special about multiple units.
- Players bid on packages; pay the externality they impose
- Idea: internalize the impact of announcement on others
- $\rightarrow$ Bidding true values is optimal
- $\rightarrow$ Outcome will be efficient

Formal Model

Each player i announces values $\tilde{g}_i(\cdot)$ (like announcing bids $b_i(\cdot)$)

$$\begin{aligned} \text{Outcome:} &\quad s^* \in \arg\max_s \textstyle\sum_j \tilde{g}_j(s) \\ i\text{'s payment:} &\quad \textstyle\sum_{j \neq i} \tilde{g}_j(s^*_{-i}) - \sum_{j \neq i} \tilde{g}_j(s^*) \end{aligned}$$

where $s^*_{-i} \in \arg\max_s \sum_{j \neq i} \tilde{g}_j(s)$.

Check: Announcing true values is (weakly) dominant strategy.

Vickrey Auction: Example

Two bidders with the following valuations:

	A	**B**	**AB**
1	0	0	12
2	10	10	10

- Goods assigned efficiently, so bidder 1 gets A and B.
- Bidder 1 pays 'opportunity value' of goods acquired. Without him, goods would be assigned to 2 for a value of 10. With him, 2 gets nothing. Hence, payment is 10.
- Losers pay 0
- $\pi = \langle 2, 0, 10 \rangle$
- Outcome is in the core

Vickrey Auction: Non-core outcomes

Problem: Vickrey auctions can lead to non-core outcomes with uncompetitively low seller revenue.

	A	**B**	**AB**
1	0	0	12
2	10	10	10
3	10	10	10

- 2 and 3 win the items at Vickrey price 2
- Seller revenue is just 4
- $\pi = \langle 0, 8, 8, 4 \rangle$ not in the core. (Why not?)

Vickrey Auctions and the Core

Theorem: If the Vickrey payoff vector v is not in the core, then for every core payoff vector π, we have $v_{seller} < \pi_{seller}$.

Vickrey Auction: Shill Bidders

Revenue Monotonicity Problem: Adding bidders can reduce seller revenue.

	A	**B**	**AB**
1	0	0	12
2	10	10	10
3	10	10	10

Adding bidder 3 reduces seller revenue from 10 to 4.

- Seller might seek to exclude bidder 3, or disqualify bid after it is made.
- bidder 2 could profitably sponsor a fake bidder 3
- In general, non-monotonicity is an unacceptable property

Vickrey Auction and Substitutes

Theorem: If goods are substitutes for all bidders, then Vickrey outcomes are core outcomes.

- Vickrey performs well when goods are substitutes
- Shill bidding also ruled out
- Converse theorems also exist (see Ausubel-Milgrom for details)

Simultaneous Ascending Auction with Package Bidding

The Model

- N types of items
- $M = (M_1, \ldots, M_N)$ = number of items of each type
- Special case: $M_i = 1$ for all i
- *Package* $z = (z_1, \ldots, z_N)$ is an N-vector of integers; $0 \leq z \leq M$
- L participants; single seller indexed by $l = 0$
- Each buyer l has valuation function $v_l(z)$

Assumptions about Preferences

1. *Private values*: Each bidder knows its own values v_l; it does not update upon learning values of others
2. *Quasilinear utility without externalities*
   (a) Bidder l who earns package z and pays $b_l(z)$ gets net payoff $v_l(z) - b_l(z)$
   (b) $v_l(0) = 0$
3. *Monotonicity/Free Disposal*: For all l and $z \leq z'$, $v_l(z) \leq v_l(z')$
4. *Zero Seller Value:* $v_0(z) = 0$ for all z.

Note: For assumption 2, can relax quasilinearity and maintain many of the results. For discussion of externalities and post game interaction, see Jehiel and Moldovanu (1996,2001).

Ausubel-Milgrom Ascending Proxy Auction

Auction Rules:

1. Bidders report maximum bids to a *proxy bidder*.
2. Auction initiates with bids of 0 by all bidders for all packages
3. Auctioneer holds most preferred feasible collection of bids
4. At each round
   - Bidders with bids held do nothing
   - For others, proxy bidders make the most "profitable" new bids, or no bid if none is profitable.
5. Bids accumulate; auctioneer may choose from all previously submitted bids.
6. Auction ends when there are no new bids

Proxy Auction Example

Values:

	A	**B**	**AB**
1	0	0	12
2	10	10	10

Time path of bids:

	Bidder 1	Bidder 2		
Round	AB	A	B	AB
1	1*	1	1	1
2	1	2*	2	2
3	2*	2	2	2
4	2	3*	3	3
...	...	...	...	...
19	10*	10	10	10

Matching with Contracts Framework

Observe that the auction is a type of deferred acceptance algorithm:

1. A contract corresponds to a package + bid
2. Set of contracts available to seller is growing (seller chooses from cumulative set of bids)
3. Set of contracts available to buyer is shrinking
4. Upon termination...

Algorithm Property

Theorem: The ascending proxy auction terminates at an efficient outcome and what is more, at a core allocation, both with respect to reported preferences.

Proof Sketch: Core $\Rightarrow$ efficiency, so just need to show core. Suppose upon termination, there is a blocking coalition.

- Every offer by every bidder in the coalition preferable to the termination outcome should have been made by the bidders.
- No feasible combination of these offers is preferred by the seller.

Hence, no blocking coalition.

SAA vs SAAPB

Several features of the SAAPB may seem peculiar...

1. Minimum bids can differ among bidders on any item or package.
2. Losing bids can later become winning bids (e.g. players may bid on complement)
3. Price of a package can increase or decrease. (e.g. high bid on a package no longer chosen b/c another bid from that bidder is used in another combination)

Proxy vs Direct Bidding

How restrictive is the use of a proxy?

In a direct bidding auction:

- If opponents are using complicated strategies, a non "proxy strategy" may be optimal.
- **Theorem:** If opponents are using proxy strategies, then it is optimal to use a proxy strategy.

Also, experiments have shown that players tend to use proxy strategies (perhaps due to their simplicity) and that these strategies do fairly well (Brewer Plott.)

Equilibria in the Proxy Auction

When Goods are Substitutes: Truthful Bidding

When items are viewed as substitutes, the proxy auction shares the efficiency and incentive properties of the Vickrey auction:

Theorem: Suppose the set of possible bidder valuations V includes all the purely additive valuations. Then these three statements are equivalent:

1. The set V includes only values for which goods are substitutes.
2. For every profile of bidder valuations drawn from V, truthful bidding is an ex-post Nash equilibrium.
3. For every profile of bidder valuations drawn from V, sincere bidding results in the Vickrey allocation and payments for all bidders.

Ex post equilibrium: After learning the other bids, no bidder could profit by changing her own bids

Full Information Case

Theorem: For every bidder optimal core payoff vector π, there is a full information Nash equilibrium with payoffs π at which the maximum bids reported to the proxy are identical to the coalition proof equilibrium bids in the menu auction.

- The strategies here are termed "semi sincere" or "profit target strategies." For each package, report to the proxy the value of the package, minus some fixed profit target π_i.

When Goods are not Substitutes...

When goods are not substitutes, the proxy algorithm still has many desirable properties. Example: Revenue Monotonicity.

Proof sketch: Follows from the fact that outcomes lie in the core.

$$\min \pi_0$$

subject to

$$\sum_{l \in S} \pi_l \geq v(S)$$

for every coalition S. More bidders increases the number of constraints, hence increasing π_0.

Comparing Auctions

Property	**Vickrey**	**SAAPB**
Sincere bidding is a Nash equilibrium	+	*
Equilibrium outcomes are in the core	*	+
No profitable shill bids	*	+
Revenue monotonicity	*	+
No profitable joint deviations for losers	*	+
Adaptable to limited budgets	No	+

+ means has the property generally

* means has the property when goods are substitutes

Implementation?

FCC Spectrum Auction 31

http://wireless.fcc.gov/auctions/default.htm?job=auction_factsheet&id=31