

Identifying Bidders' Groups in Collusive Auctions: Evidence from Average Bid Auctions

Timothy G. Conley and Francesco Decarolis

Premise:

The system of public procurement of works in Italy

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Introduction
AB_Collusion
Theory
Known Groups
Unknown Groups
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Various reforms since the '90s but the basic structure of procurement methods is the following:

- 1 Negotiations [19%; 8%]
- 2 Auctions, two possible awarding criteria:
 - 1 "Best Economic Value" (price, quality, etc....) [2%, 10%]
 - 2 "Lowest Price" (only price)
 - 1 First Price (FP) [2%, 33%]
 - 2 **Average Bid (AB) [77%, 49%]**

[x, y] are percentages respectively of the total number (115,000) and of the total value (€117 billion) of all public contracts for works procured in 2000-2007.

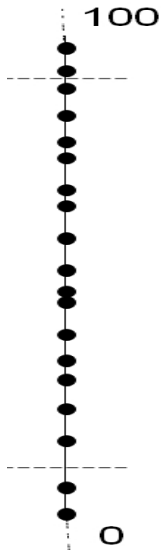
The Italian AB Auction (1/4): Sort the Discounts

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The Italian AB Auction (2/4):

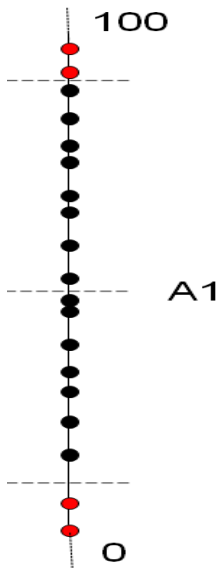
Calculate the Trim Mean A1

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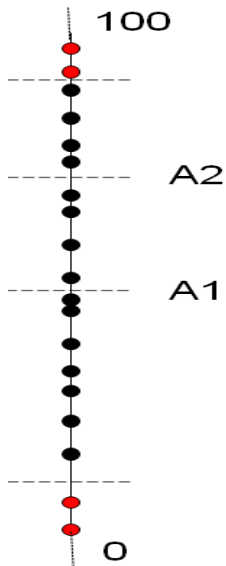
The Italian AB Auction (3/4): Calculate the Anomaly Threshold

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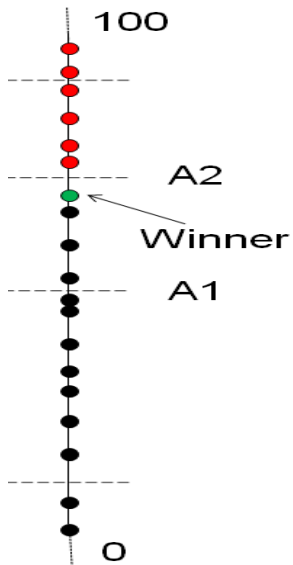
The Italian AB Auction (4/4): Select the Winner

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Rules Existing in Various Public Procurement Regulations to Deal with "Abnormal" Bids

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Automatic elimination

Chile

China

Colombia

Italy

Japan

Peru

Taiwan

USA -Florida DoT
-NYS Proc.Ag.

(Switzerland)

Only identification

Belgium

Brazil

Germany

Portugal

Romania

Spain

Turkey

UK

Non disclosed rule

Canada

USA (Most States)

- Most AB rules are "collusive": at least one equilibrium under competition identical to the case of full collusion.

Motivation of this Paper:

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- 1 Understand how bidders behave in AB auctions
- 2 Develop an empirical method to detect which firms respond to the incentive to coordinate bids and entry
- 3 Application of the method to the Italian case:
 - 1 AB rule particularly relevant in Italy
 - 2 The specific rule gives strong incentive to form groups
 - 3 Known cases of firms that formed groups ("cartels"?)

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- 1 Theory: competitive bidding Vs the incentive to collude
- 2 Two tests for collusion in AB auctions
- 3 Empirical results on collusion in the Italian auctions

Related Literature

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- Collusion in auction markets:
 - In a generic market: Klemperer, 2004
 - In the procurement or roadworks: Porter and Zona, 1993; Ishii (2007)
- Empirical studies of bidders' collusion:
 - Behavior of Known cartels: Porter and Zona, 1993 & 1999; Pesendorfer, 2000; Asker, 2008
 - Detection of possible cartels: Bajari and Ye, 2003; Abrantez-Metz and Bajari, 2010
- Average bid auctions:
 - Civil engineering: Ioannou and Leu, 1993; Liu and Lai, 2000
 - Economics: Albano, Bianchi and Spagnolo, 2006; Engel, Guanza, Hauk and Wambach, 2006; Burguet, Ganuza and Hauk, 2009; Chillemi and Mezzetti, 2009; Decarolis, 2010

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- 1 Theory: competitive bidding Vs the incentive to collude**
 - 1** Benchmark model of competitive bidding in AB
 - 2** The incentive to collude
- 2** Two tests for collusion in AB auctions
- 3** Empirical results on collusion in the Italian auctions

A Benchmark Model of Competitive Bidding in AB

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- General features of the auction game:
 - 1 There are $N > 4$ risk-neutral bidders (firms) bidding for one contract
 - 2 N is common knowledge
 - 3 Every bidder i draws his cost c_i from F_C (absolutely continuous) with support on $[\underline{c}, \bar{c}]$
 - 4 There is a reserve price, R , that is not binding (i.e. $R > \bar{c}$)
 - 5 The auction is sealed bid and bids are discounts over R
 - 6 The awarding rule is the Italian AB described before
 - 7 Ties are broken with a fair lottery

Results

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- **Theorem (Decarolis, 2010):** For any N , the strategy profile in which all firms bid according to the constant bidding function $b(c) = 0$ for every possible c is the unique symmetric BNE.
- **Remarks:**
 - 1 The previous result is robust to (some forms of) bidders' asymmetry and entry,
 - 2 The AB auction is both inefficient and "revenue minimizing",
 - 3 However, the data does not conform to this equilibrium:
 - 1 Winning bid: $B^w > 0$ (mean 13, sd 5, med 13)
 - 2 "Focal" bid for each auctioneer/contract type ($B^w = B^{w*} + \varepsilon$)
 - 3 Large number of bids (mean 57, sd 47, med 46) most concentrated close to B^{w*}

Alternative Model: Coordinated Bidding & Entry

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- Independent and groups' bidders: $N = \{I, N^1, \dots, N^G\}$
- For each group a mediator chooses the entry and the bid of each group's firm
- Independents decide whether to enter and, if so, their bid
- Independents do not observe N , groups, instead, observe it
- **Claim:** Unless all bidders belong to the same group or all groups are smaller than the minimum winning coalition $N^* = 2 + \text{integer}^+ \{(.10)|N|\}$, the strategy profile in which all bidders bid zero is not an equilibrium

Alternative Model: Coordinated Bidding & Entry

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Suppose bids distributed on $[b_l, b_h]$ with $b_h > b_h^l \geq b_l \geq 0$ and at least one group N^g of size $N^* \leq |N^g| < |N|$

■ Properties of bidding:

- 1 For N^g : ex post dominated not to shift A1; clustering of bids; mixing; only the group, not the independents, can bid b_h ; all the previous claims hold with multiple groups
- 2 Conjecture: there is an equilibrium in which groups cluster and mix on $[b_l, b_h]$ and independents mix on $[b_l, b_h^l]$

■ Properties of entry:

- 1 For N^g : coordinated entry, i.e., for each firm in the group entry is more likely if $N^* - 1$ other members enter
- 2 For the independents: entry is independent of each other and independent from the actual number of group bidders

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- 1 Theory: competitive bidding Vs the incentive to collude
- 2 **Two tests for collusion in AB auctions: Bid test & participation test**
 - 1 Test when the identity of the groups is known
 - 2 Test when groups are unknown but measures of similarity of firms exist
 - 3 Test when groups are unknown and only bids & identities are available
- 3 Empirical results on collusion in the Italian auctions

Known Groups: the Groups in Turin's AB Auctions

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- Between 1999 and 2002: **276 AB auctions** for road construction held by the County of Turin
- About 95 firms condemned for collusion in these auctions (Accusation: Jan 2003, Sentence: Apr 2008)

Data on the 8 Groups Active in Turin			
Group ID	No. Firms	No. Victories	No. Auctions
1 - Torinisti	18	83	247
2 - San Mauro	12	35	234
3 - Coop	18	73	240
4 - Pinerolesi	12	1	110
5 - Canavesani	11	7	155
6 - Settimo	6	10	220
7 - Provvisiero	7	11	73
8 - Tartara/Ritonnaro	11	1	62

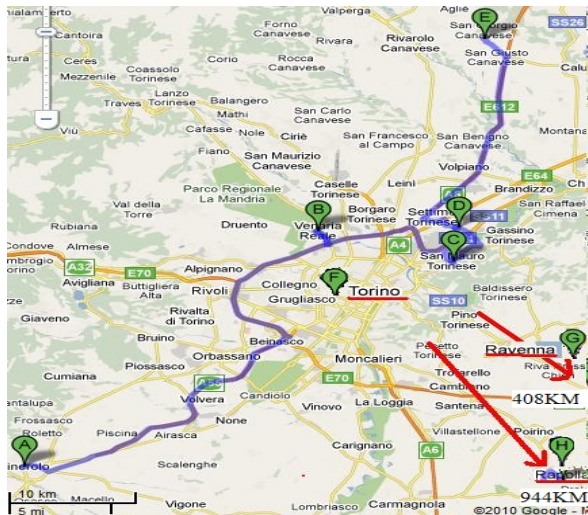
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Example of Bids' Coordination by Groups

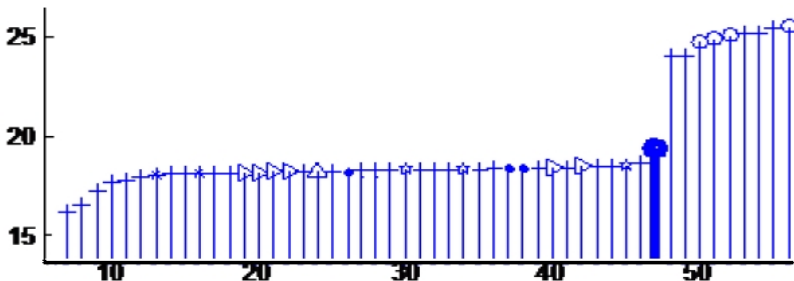
"Support Bids"

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All bidders (groups and independent) ordered by their bid

Example of Entry Coordination: by Groups

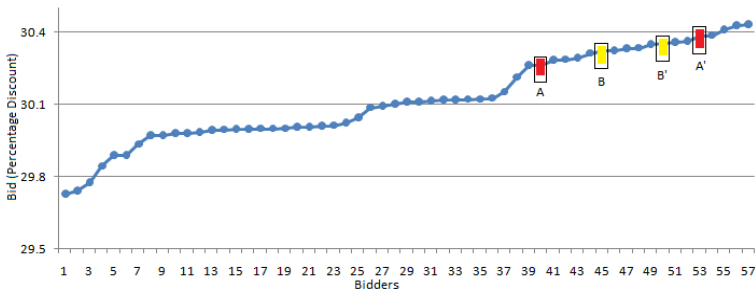
Sill Bidders

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A' and B' are shills of firms A and B respectively

Test for Coordinated Bids with Known Groups

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- Formal test of influence on the trim mean ($A1$): Bid Test
- Fix a group and then:
 - 1 Compute the trim mean without the bids of N^g firms ($A1^g$)
 - 2 Compute the trim mean without $|N^g|$ randomly picked bids, repeat T times ($A1_1^s, \dots, A1_T^s, \dots$).
 - 3 Reject independence of bids in favor of coordination if $A1^g$ falls in the tails of the empirical distribution of the $A1^s$
- Can be repeated for every auction.
- Groups that want to move $A1$ must coordinate in this way.

Test for Coordinated Bids with Known Groups

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- $B^{-g} = \{b_1^{-g}, \dots, b_{|N|-|N^g|}^{-g}\}$ is the ordered set of bids excluding group g 's bids; $N' = \text{integer}^+ \{(.10)|B^{-g}|\}$
- The trim mean without the group's bids is:

$$A1^g = \frac{1}{|B^{-g}| - 2N'} \sum_{i=N'+1}^{|B^{-g}|-N'-1} b_i^{-g}$$

- Same for T randomly picked groups s ($A1^s$)
- Hypothesis test: $H_0 : A1^g \sim A1^s$
- Reject the null if $A1^g \notin [P_{.025}^T, P_{.975}^T]$
- Extensions: many auctions; one tail; conditional on firms' observable characteristics.

Test for Coordinated Bids with Known Groups

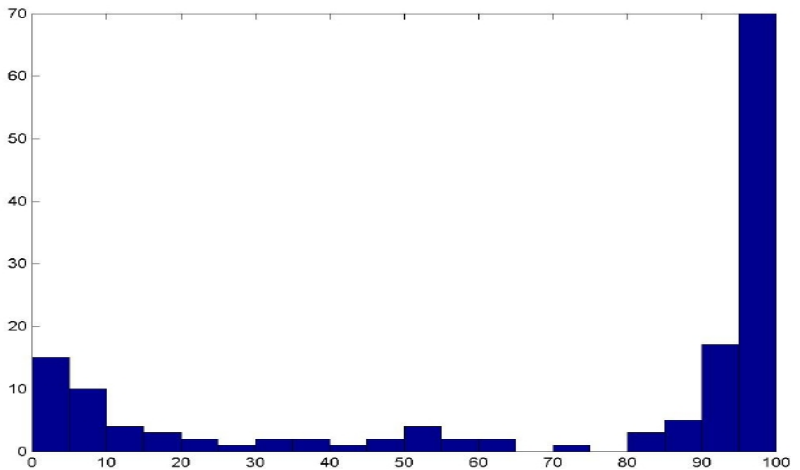
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■ Histogram of p-values for each auction for group 5



Test for Coordinated Bids with Known Groups

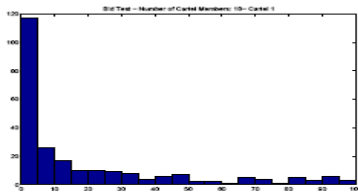
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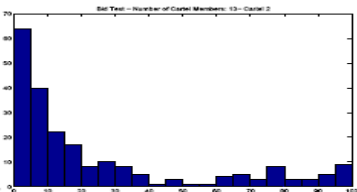
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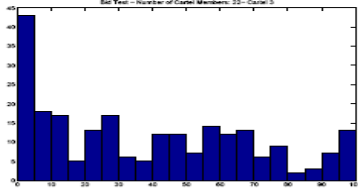
(a) Cartel 1 - Torinisti-Venaria



(b) Cartel 2 - San Mauro

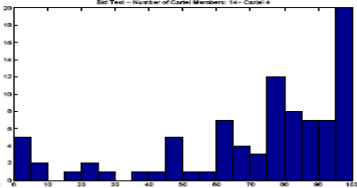


(a) Cartel 3 - Cooperative



(a) Cartel 3 - Cooperative

(b) Cartel 4 - Pinerolesi



(b) Cartel 4 - Pinerolesi

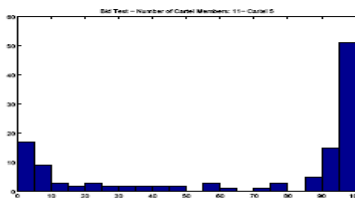
Test for Coordinated Bids with Known Groups

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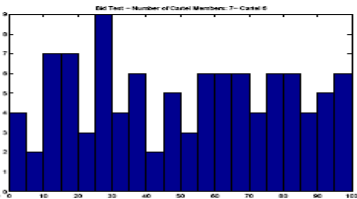
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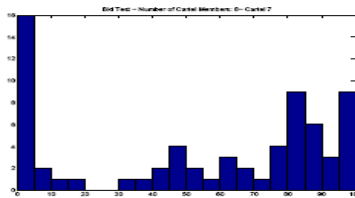
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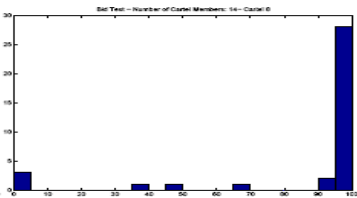
(a) Cartel 5 - Canavesani



(b) Cartel 6 - Settimo



(a) Cartel 7 - Provisiero



(b) Cartel 8 - Tartara-Ritonnaro

Test for Coordinated Participation with Known Groups

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- Formal test of coordinated participation: P_Test
- Fix a group and then:
 - 1 Count for every auction how many members of the group participated;
 - 2 Construct the pointwise confidence interval of the same quantity obtained by using randomly drawn groups of firms (of the same size of the initial group);
 - 3 Compare the two.

Test for Coordinated Participation with Known Groups

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- Count auctions entered ($d = 1$) together by all group members:

$$f^{N^g, |N^g|} = \sum_{r=1}^T \prod_{i=1}^{|N^g|} \{1 | d_{it} = 1\} \quad \text{for all } i \in N^g$$

- Do the same for T randomly picked groups h ($f^{h, |N^g|}$)
- Hypothesis test: $H_0 : f^{N^g, |N^g|} \sim f^{h, |N^g|}$
- Reject the null if $f^{N^g} > P_{.95}^T$
- Extensions: auctions attended by $|N^g| - i$ members ($|N^g| = 2$ is special); conditional on firms' observable heterogeneity; firm-by-firm test

Test for Coordinated Participation with Known Groups

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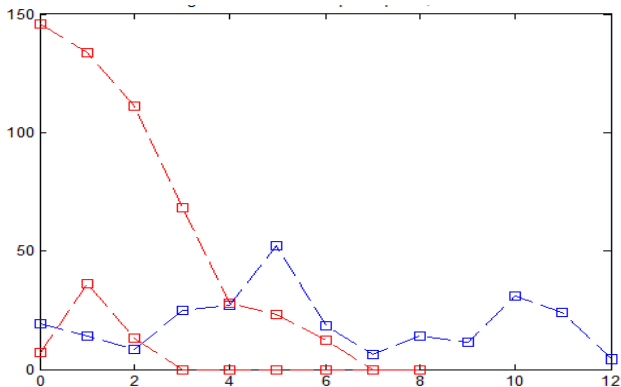
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■ The Participation Test for Group #1



Test for Coordinated Participation with Known Groups

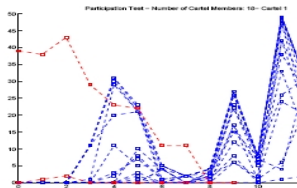
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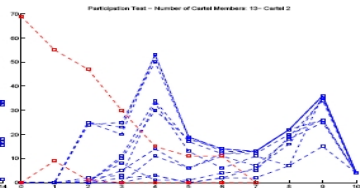
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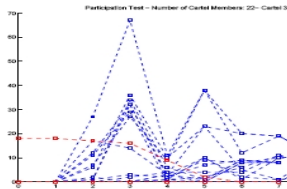
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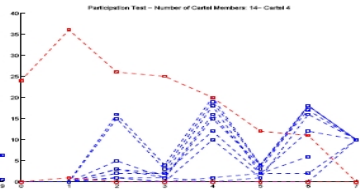
(b) Cartel 2 - San Mauro



(a) Cartel 3 - Cooperative



(b) Cartel 4 - Pinerolesi



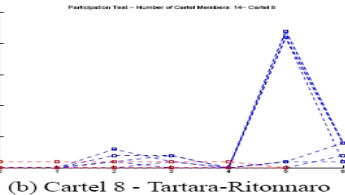
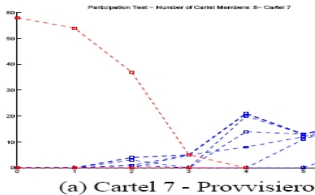
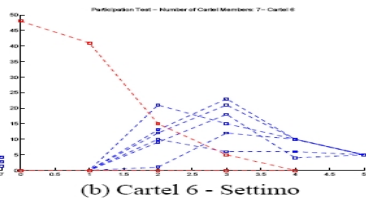
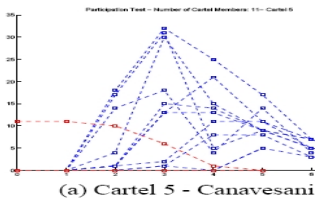
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- Both tests identify the groups
- Even Group 6 is a fine result
- Mixing of bids "in the market"
- The one tail version of the bid test would detect the more interesting groups
- Participation test most revealing for extreme sizes of groups
- The participation test needs to be run conditional on the firms' legal qualifications for the auctions

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PART II: UNKNOWN GROUPS

Detecting Coordination with the Tests

All the ingredients to suspect that collusion matters are present

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- Sample of 871 AB auctions in the North of Italy 2005-2010
- Same industry, rules, type of firms as in the Turin's case
- Same violations of the zero-bid equilibrium:
 - 1 Discounts are above zero and not all flat
 - 2 Many support bids and some suspicious skills
- Can we identify groups of bidders?
- Can we analyze their effect on the market outcomes?

Our Approach in Steps

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- 1 Select sets of "suspect" firms (like frequent winners)
- 2 Assign all suspect to groups (formed, for instance by clustering on some characteristics like: joint ownership, joint bids, location, subcontracts)
 - 1 **With data on firms' observable characteristics**
 - 2 With data only on bids and bidders' identity
- 3 Use the tests to check whether these groups coordinated their bids and participation

Case 1: Partial Info

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- 1 Get the links between every couple of firms in terms of joint ownership, joint bidding, exchange of subcontracts (both North and Turin's data)
 - 2 Run a probit for the probability of being together in the same group using the links as regressors (Turin's data)
 - 3 Obtain the predicted probability that a couple is in a group together using the links and the estimated probit coefficients (North data)
 - 4 Use the predicted probabilities to aggregate firms in groups through a hierarchical clustering algorithm
 - 5 Test the groups obtained
- Works well to detect Turin's groups (16 "ok" groups)

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- 1 Theory: competitive bidding Vs the incentive to collude
- 2 Two tests for collusion in AB auctions: Bid test & participation test
- 3 **Empirical results on groups in the Italian auctions: road construction contracts in the North, 2005-2010**
 - 1 Quantifying the extent of collusion
 - 2 Explaining the drop in participation associated with FP
 - 3 Measuring the effect of collusion on revenues and efficiency

Result 1: How Many Auctions Are Rigged?

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- Example using the Piedmont region: AB auctions for road construction, 2005-2010
- About 1,250 firms and 164 AB auctions
- Recover **20 groups** of potential cooperating firms (using clustering method)
- Fraction of auctions in which there was at least one group according to the Bid Test:

Significance level:	1%	5%	10%
Colluded auctions	21%	48%	52%

Drop in the Number of Bidders in FPA

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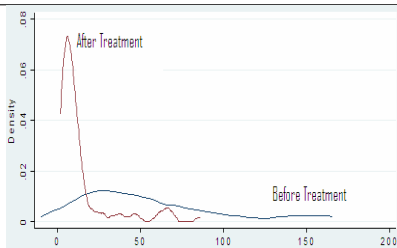
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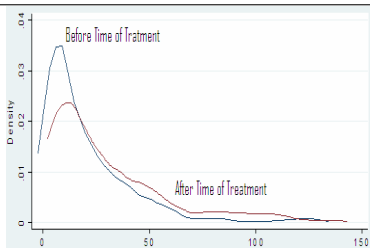
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Distribution of the number of bidders participating at auctions

Treated Group (Provincia & Comune Turin)



Untreated Group (All Other Piedmont Loc. Ad)



Drop in the number of bidders

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Number of Bidders Regressions				
	Turin Sample 00-07 ^o		North Sample 05-10	
	NEG.BIN [^]	Pred.Change	NEG.BIN [^]	Pred.Change
First Price	-1.84 (.15)***	-38.32	-1.87 (.18)***	-44.03
Observations	2,548		956	
P-Value Chi ²	.000		.000	

Pred.Change is the predicted discrete change of the number of bidders due to FP switching from 0 to 1.

^o Control group: all the PA with population > 500,000. Results are very close with the other control groups.

* significant at 10%; ** significant at 5%; *** significant at 1%. Clustered SE by administration and year.

[^]Log(contract value) and dummy variables for type and geographical location of the PA included.

Result 2: Explaining the Drop in Participation

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- Of the 1,254 firms active in Piedmont, define colluded those failing the bid test 30% of the times at the 10% level. Then:

	FP	Only AB
Colluded	68	220
Not Colluded	264	702

- Disappearing 702 not colluded firms: likely weak firms.
- Disappearing 220 colluded firms: cannot distinguish between skills and weak group members

Result 3: Revenues of PA & Other Bidders

A criminal offense but not a patrimonial damage

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■ Damages in the rigged auctions in Turin:

EFFECT FOR THE PA	Sum True Cost	Sum Reserve Price	Sum Cost W/O Collusion by C1	Sum Cost W/O Collusion by C5
Scenario 1 (cartel does not bid)	105,937,748	129,345,628	106,579,582	105,701,587
Scenario 2 (cartel members all bid 18%)	105,937,748	129,345,628	105,501,788	105,859,152
Scenario 3 (2/3 of the cartel members bids 18% and 1/3 does not bid)	105,937,748	129,345,628	106,163,857	105,759,604

EFFECT FOR THE OTHER BIDDERS (NOT CARTELS)	Expected revenues for a bidder attending every auction under competition	Actual revenues that accrued to the firms not colluded	Difference (i.e. Loss)
Scenario 1 (all cartel members are real and there are no shills)	1,873,343	996,051	877,291
Scenario 2 (1/3 of the cartel members are shills)	2,619,619	996,051	1,623,567

Ongoing Related Research:

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- Bidders' manipulation of the Low Income Subsidy and the changes in the cost of the Medicare Part D program
- Bidders' coordination in multiunit internet auctions: collusion in the generalized second price auction via SEM
- Contracts' renegotiations and bidders' integration: subcontracts Vs bidding consortia

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APPENDIX:

Theorems + details about the clustering method

[From Decarolis (2009)] Simple AB (S_AB)

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S_AB: the bidder closest to the average (above or below) wins and pays his price. Ties broken with a fair lottery.

Pure IPV: Every bidder i draws his value v_i from F_V (absolutely continuous) with support on $[0, \bar{v}]$.

Proposition: (i) For any N , the strategy profile in which all players bid according to the constant bidding function $b(v) = 0$ for every possible v is a symmetric BNE.

(ii) Four properties characterize any other symmetric BNE that might exist. The bidding function (1) is weakly increasing, (2) is flat at the top, (3) has all types greater than the lowest one bidding strictly less than their own value and (4) for any F_V and $\forall \varepsilon > 0$, $\exists N_{\varepsilon, F_V}^*$ such that $\forall N \geq N_{\varepsilon, F_V}^*$ the following is true: $\bar{v}_{\varepsilon, F_V} < \varepsilon$, where $\bar{v}_{\varepsilon, F_V}$ is defined by:

$$\boxed{1 - N \left(\frac{N-2}{N-1} \right) [F_V(\bar{v}_{\varepsilon, F_V}) (1 - F_V(\bar{v}_{\varepsilon, F_V}))^{-1}] = 0}$$

Sketch of the proof

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1 For any N $b(v) = v_{\min}$ is a BNE.

2 Any symmetric bidding function must be:

- Non decreasing;
- Flat at the top;
- Less than the 45' line in the (v,b) space;
- With the highest bid strictly less than the value \bar{v} ,
 $\bar{v} < v_{\max}$, such that

$$(1 - F(\bar{v}))^{N-1} - N\left(\frac{N-2}{N-1}\right)[F(\bar{v})(1 - F(\bar{v}))^{N-2}] = 0$$

(1): Non decreasing function

- Assume that the equilibrium bidding function, b , has a decreasing trait. Then we can take two types, v_1 and v_0 , with $v_1 > v_0$ such that $b(v_1) < b(v_0)$. Then by the assumption that b is equilibrium, it must follow that:

$$[v_1 - b(v_1)] \Pr(\text{win}|b(v_1)) \geq [v_1 - b(v_0)] \Pr(\text{win}|b(v_0)) \text{ and} \\ [v_0 - b(v_0)] \Pr(\text{win}|b(v_0)) \geq [v_0 - b(v_1)] \Pr(\text{win}|b(v_1)).$$

Therefore there must exist a solution to the system:

$$\begin{cases} \Pr(\text{win}|b(v_0)) \leq \{[v_1 - b(v_1)]/[v_1 - b(v_0)]\} \Pr(\text{win}|b(v_1)) \\ \Pr(\text{win}|b(v_0)) \geq \{[v_0 - b(v_1)]/[v_0 - b(v_0)]\} \Pr(\text{win}|b(v_1)) \end{cases}$$

However this requires:

$$[v_0 - v_1][b(v_0) - b(v_1)] \geq 0$$

impossible because with $v_1 > v_0$ and $b(v_1) < b(v_0)$.

(2)&(3): Flat top & Less than 45' line

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- Assume the equilibrium bidding function is strictly increasing at the top \rightarrow contradiction: it's not a symmetric eq. because we can construct a unilateral profitable deviation.
- Assume the equilibrium bidding function touches the 45' degree line in the (v,b) space for some v greater than the minimum one \rightarrow contradiction: it's not a symmetric eq. because we can construct a unilateral profitable deviation.

(4): Bound on the highest bid (1/3)

- Consider a symmetric equilibrium bidding strategy b^* such that:

$$b^* = \begin{cases} \bar{b} & \text{if } v \geq \bar{v} \\ b(v) < v & \text{if } v < \bar{v} \end{cases}$$

Where it is known that $b(v) < v$ for $v < \bar{v}$ is weakly increasing.

- Given the other $N-1$ players are using this strategy b^* , in equilibrium for an agent drawing \bar{v} it must be that:

$$\Pr(\text{win}|\bar{b})[\bar{v} - \bar{b}] \geq \Pr(\text{win}|b)[\bar{v} - b] \text{ for any } b \neq \bar{b}. (*)$$

(4): Bound on the highest bid (2/3)

- Define the following probabilities:

$$q_0 \equiv \Pr[(V_1 \geq \bar{v}) \cap (V_2 \geq \bar{v}) \cap \dots \cap (V_{N-1} \geq \bar{v})].$$

$$q_1 \equiv \Pr[(V_1 < \bar{v}) \cap (V_2 \geq \bar{v}) \cap (V_3 \geq \bar{v}) \cap \dots \cap (V_{N-1} \geq \bar{v})].$$

...

$$q_{N-2} \equiv \Pr[(V_1 < \bar{v}) \cap (V_2 < \bar{v}) \cap \dots \cap (V_{N-2} < \bar{v}) \cap (V_{N-1} \geq \bar{v})].$$

- Now define α_M to be the prob. \bar{b} is closer to the average than any other bid (conditional on all other players playing b^* and M of them drawing a valuation that is strictly less than \bar{v}). That is:

$$\alpha_M \equiv \Pr[|\bar{b} - \frac{1}{N} \sum_{r=1}^N b_r^*| < |b(v_j) - \frac{1}{N} \sum_{r=1}^N b_r^*| \text{ for any } v_j < \bar{v}$$

and $j = 1, 2, \dots, M \mid q_M = 1]$ where $M = 1, 2, \dots, N'$.

where N' is $\frac{N}{2}$, or the closest lower integer if N is odd.

(4): Bound on the highest bid (3/3)

- Then for (*) to hold it must be that:

$$q_0 > N[q_1(\frac{N-2}{N-1}) + q_2\alpha_2(\frac{N-3}{N-2}) + \dots + q_{N'}\alpha_{N'}(\frac{N-N'-1}{N-N'})].$$

- Hence, a necessary condition for the above to hold, is that:

$$q_0 > Nq_1(\frac{N-2}{N-1})$$

- Which can be rewritten using the definitions of q_0 and q_1 as:

$$(1 - F(\bar{v}))^{N-1} - N(\frac{N-2}{N-1})[F(\bar{v})(1 - F(\bar{v}))^{N-2}] > 0 \quad (**)$$

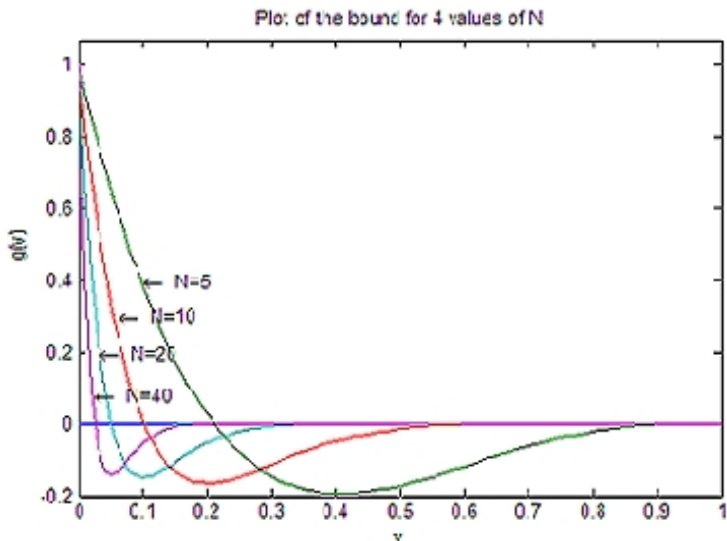
Tightness of the bound: values independently and uniformly distributed on $[0, 1]$

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Details about the Clustering

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CLUSTERING

Probit Regression: Turin's Collusion Data

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Marginal Effects Probit Regression

	Fixed at	
	Zero	One
Personal	.00056 (.0016)	.068 (.041)
Subcontract	.00056 (.0016)	.95 (.019)
Joint Bidding ("Authority Sample")	.00056 (.0016)	.049 (.033)
Joint Bidding (Turin Collusion)	.00056 (.0016)	.16 (.037)

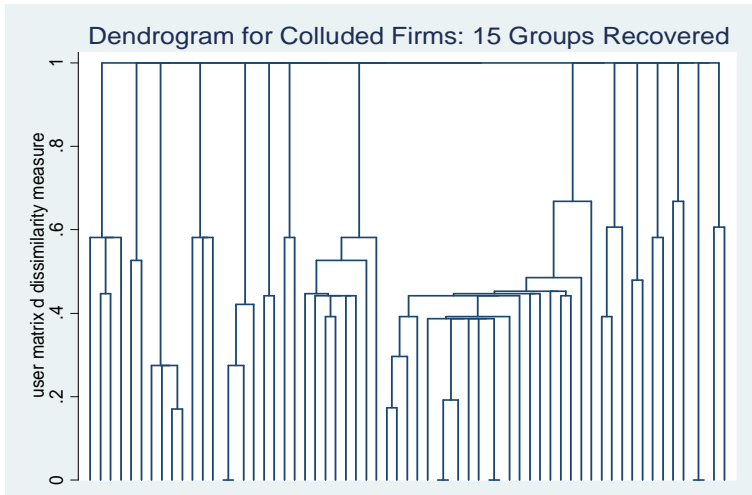
Hierarchical Clustering: How does it Create Groups?

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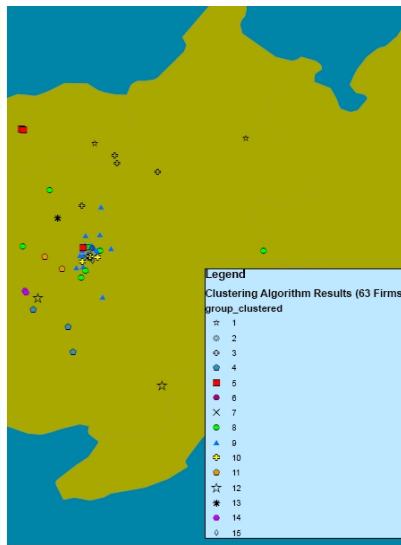
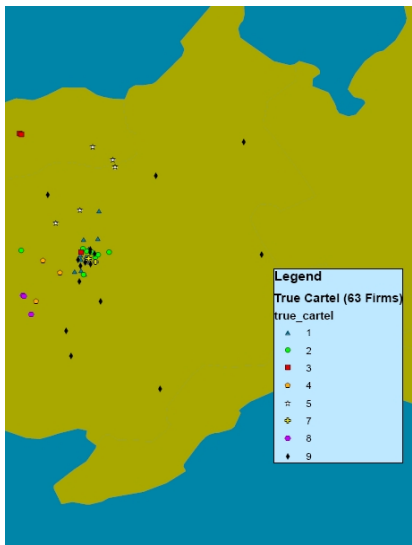
Good Performance with the Turin's Collusion Data:

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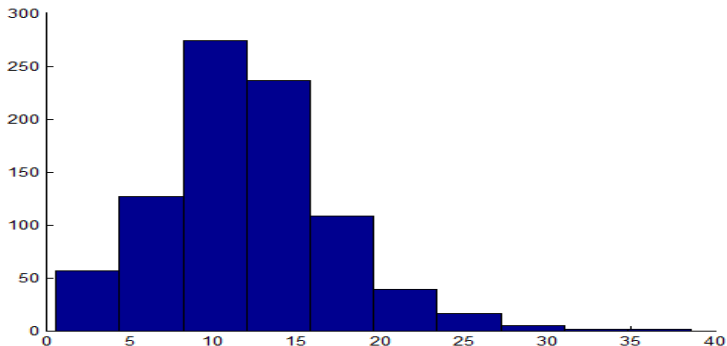
Bids' Range in AB Auctions

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Distribution of the within-auction range of the bids in the "IE Sample" of Decarolis (2009). Approximately 700 AB auctions.

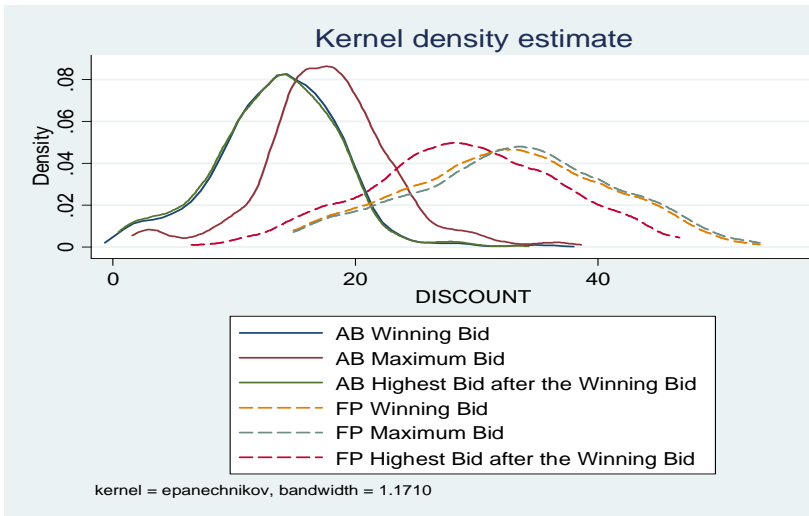
Violations: Discounts are Greater than Zero & not all Flat

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How do Firms Outside the Groups Bid?

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Bidding by Firms outside the Groups: Focal Points

$$B_i^w = \alpha X_i + \beta B_i^{w*} + \varepsilon_i$$

β	Robust S.E.	R ²	Obs.	Covariates
.997	(0.02)***	-	1,566	-
.82	(.04)***	.43	1,566	X=1
.72	(.05)***	.54	1,566	X=1+Z

Where:

B^w = winning discount

B^{w*} = avg. of past winning discounts (same auctioneer & type of work, max 1 year old, almost same reserve price)

Z = log reserve price, number of bidders, auctioneers' dummies