School Choice

Jacob Leshno (Harvard)
Today, at a class near you

Background

- In some public school systems students have the freedom to select their school
- School choice mechanisms are centralized clearinghouses for determining the student-school assignment
- Finding an assignment (matching) is a discrete allocation problem

Background

- Students have *strict* preferences over schools
- Schools have *weak* priorities over students, and a quota
  - Neighborhood
  - Sibling
  - *Many indifferences*

- A feasible assignment assigns each student to a school or being unmatched, such that the quota of each school is respected.

NYC – before the redesign

- Decentralized application and admission
- Over 90,000 students enter high school each year in NYC

- Each was invited to submit list of up to 5 choices
- Each student's choice list distributed to high schools on list, who independently make offers
- 3 rounds of offers

NYC – before the redesign

- *congested*: left 30,000 kids each year to be administratively assigned (while about 17,000 got multiple offers)
  - Waiting lists run by mail
  - Gaming by high schools; withholding of capacity
- Only approx. 40% of students receive initial offers, the rest put on waiting lists—3 rounds to move waiting lists…
- Approx. 30,000 students assigned to schools not on their choice list.

- Schools see rank orders
  - Gaming by students: first choice is important

Boston – before the redesign

- A central school system
  - Sets priorities for schools
  - Centralized clearing house
- Priorities:
  - Sibling
  - Walk zone
  - Random tie-breaking
- Mechanism: Priority Mechanism
  - Commonly known as "the Boston Mechanism"
  - Not Strategy Proof!

Costs of non-strategyproof mechanisms

- Given the revelation principle, so why should we care if a mechanism is truthful (or just equivalent to a truthful one)?

- A truthful mechanism would generate informative data
- Cost of figuring out the strategy
- Risk
- The mechanism is not "safe": students who fail to strategize are not hurt from the mechanism
  - Difference between parents who get strategic advice and those who don't

A Two Sided Market?

- In NYC incentives for schools matter
  - Schools conceal capacities
  - Some schools have different non-random priorities: some care about higher scores, others care about attendance records

- In Boston, less clear:
  - Yes – priorities are given for meaningful reasons
  - No – efficiency of allocation is paramount

Some theory

- A school choice system is defined by:
  - A set of schools $S = \{ a, b, \ldots, c\}$
  - A positive quota\capacity for each school $\{q_a\}_{a \in S}$
  - A set of students $I = \{\alpha, \beta, \ldots, \gamma\}$

  - School a has a (weak) priority over students R_a [strict part: P_a]
  - Student α has (strict) preferences over schools $_{\alpha}$
- An assignment\matching is a mapping $\mu: I \rightarrow S \cup \{\phi\}$ such that $| \mu^{-1}(a)| \leqq q_a$

Some theory

- μ is stable if BOTH:
  - It is individually rational: there is no student α such that $\phi \succ_{\alpha} \mu(\alpha)$
  - There is no pair (a, α) of school a and student α such that
    a $\;_{\alpha}\mu(\alpha)$
    and
    α P_a β for some $\beta \in \mu^{-1}$(a) [$\phi \in \mu^{-1}$(a) if $|\mu^{-1}(a)|<q_a$]

DA (student proposing)

- Step 0 : Randomly break all ties in schools preferences
- Step 1 : Each student applies to her favorite school that didn't reject her before.
- Step 2 : Each school a tentatively accepts the q_a students with highest priority, and rejects the rest.
  If any students were rejected, go to step 1. Otherwise, end and make assignments final.

DA - Incentives

- The outcome of DA is a stable matching (Gale and Shapely 1962)
- For DA with any tie breaking, it is a dominant strategy for students to report their true preferences (Dubins and Friedman 1981, Roth, 1982, 1985)

Boston

- Step 0 : Randomly break all ties in schools preferences
- Step 1 : Each student applies to her favorite school that didn't reject her before.
- Step 2 : Each school *a* *permanently* accepts the students with highest priority, up to the capacity left from previos steps, and rejects the rest.
  If any students were rejected, go to step 1. Otherwise, end and make assignments final.

Boston - Incentives

- The Boston mechanism is not strategy-proof
- 3 schools are commonly ranked by students as follows.

School	Best	Second	Third	No school
Seats	2	1	1	–
Payoff	100	67	25	0

- Two types of students: Top and Average.
  - Top always has priority over Average.
  - Within group, ties are broken by a lottery.
- 3 Tops and 2 Averages
- DA Outcome:

  Top: 2 get Best, 1 gets Second

  Average: 1 gets Third, 1 is unassigned
- What is the equilibrium under Boston?

Boston - Incentives

School	Best	Second	Third	No school
Seats	2	1	1	–
Payoff	100	67	25	0

Boston:

- Strategies:

  Top: (Best, Third, . . .) (skipping the middle)
  Average: (Second, ...) (skipping the top)

- Outcome:

  Top: 2 get Best, 1 gets Third
  Average: 1 gets Second, 1 is unassigned

Boston - Equilibrium

- Consider the full information game where students strategically report preferences to the Boston Mechanism
- The set of Nash Equilibrium outcomes of the full information game (after tie breaking – preferences are strict) is equal to the set of stable matchings under the true preferences (Ergin, Sonmez 2006)

- Is there a problem?

Boston – Advice

- Advice from the West Zone Parent's Group: (Introductory meeting minutes, 10/27/03)
  - "One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, OR, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a "safe" second choice."

Boston – Equilibrium?

- **Definition**: A school is overdemanded if the number of students who rank that school as their first choice is greater than the number of seats at the school.
- In Boston, of the 15,135 students in the analysis, 19% (2910) listed two overdemanded schools as their top two choices, and about 27% (782) of these ended up unassigned.

- *Students do not play the full information game*
- *Naïve students may get hurt*

Implementation

- Deferred acceptance was implemented in both NYC and Boston
- It is strategy proof, and therefore safe for students
- Under a stable mechanism schools have much less reason to withhold capacity (is DA strategy-proof for schools?)
- In NYC students can submit a rank order list of 12 schools (is it strategy-proof for students?)

Implementation

- Ranking only 12 schools – binds!

New Process: Average Number of Rankings Each Round

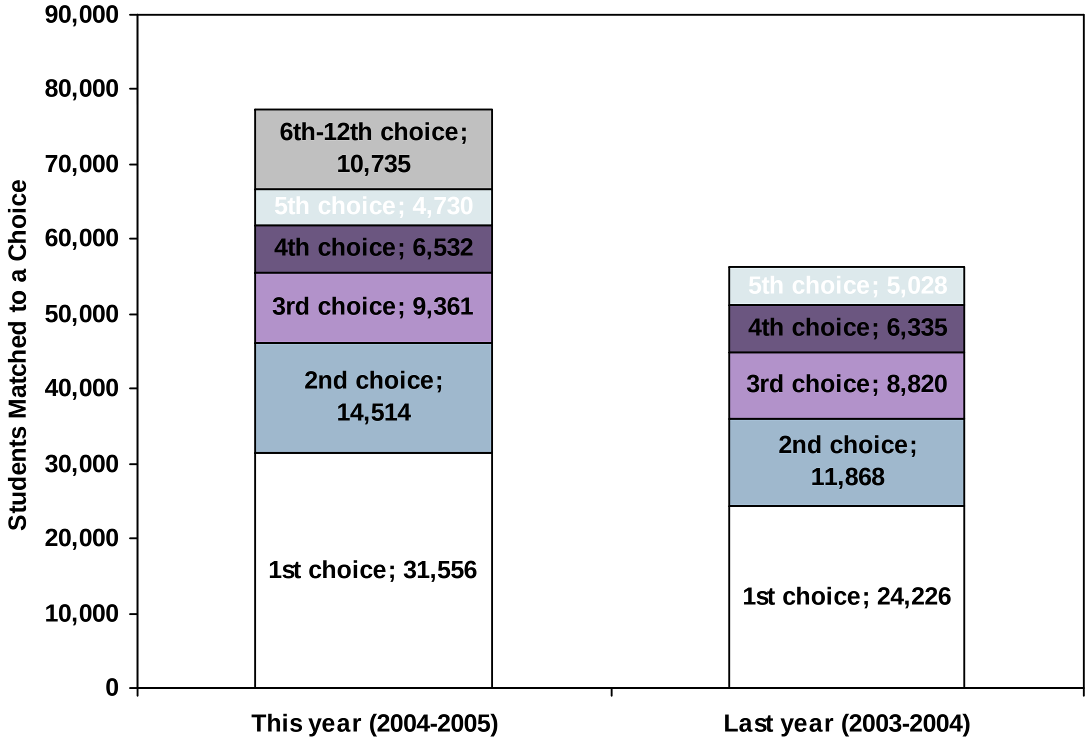
	Ranking											
Round	1	2	3	4	5	6	7	8	9	10	11	12
Round 1	91,286	84,554	79,646	73,398	66,724	59,911	53,466	47,939	42,684	37,897	31,934	22,629
	100%	93%	87%	80%	73%	66%	59%	53%	47%	42%	35%	25%
Round 2	87,810	81,234	76,470	70,529	64,224	57,803	51,684	46,293	41,071	35,940	29,211	18,323
	100%	93%	87%	80%	73%	66%	59%	53%	47%	41%	33%	21%
Round 3	8,672	8,139	7,671	7,025	6,310	5,668	5,032	4,568	4,187	3,882	3,562	3,194
	100%	94%	88%	81%	73%	65%	58%	53%	48%	45%	41%	37%

3,476 Specialized High Schools Students
91,286 Total students

Implementation

- Ranking only 12 schools – binds!
- But:
- ***<u>Proposition</u>*** *(Haeringer and Klijn, Lemma 8.1.)* In the student-proposing deferred acceptance mechanism where a student may only rank k schools,
  - if a student prefers fewer than k schools, then she can do no better than submitting her true rank order list,
  - if a student prefers more than k schools, then she can do no better than employing a strategy which selects k schools among the set of schools she prefers to being unassigned and ranking them according to her true preference ordering.

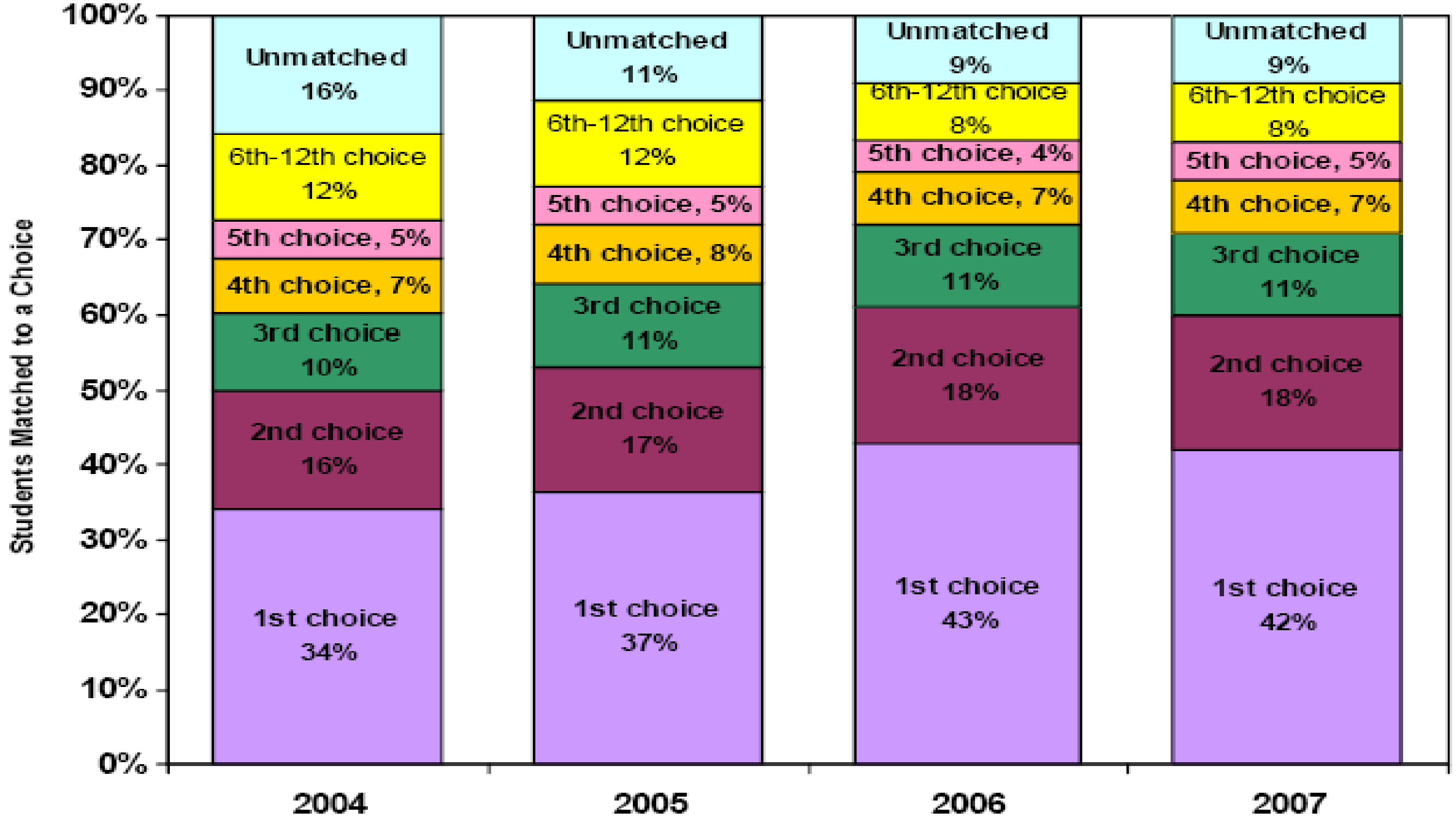
Results



- **21,000** more students matched to a school of their choice
- **7,000** more students receiving their first choice
- **10,000** more students receiving one of their top 5 choices

- Chart made by NYCDOE

Results



- NYC data
- Schools adjust

Results

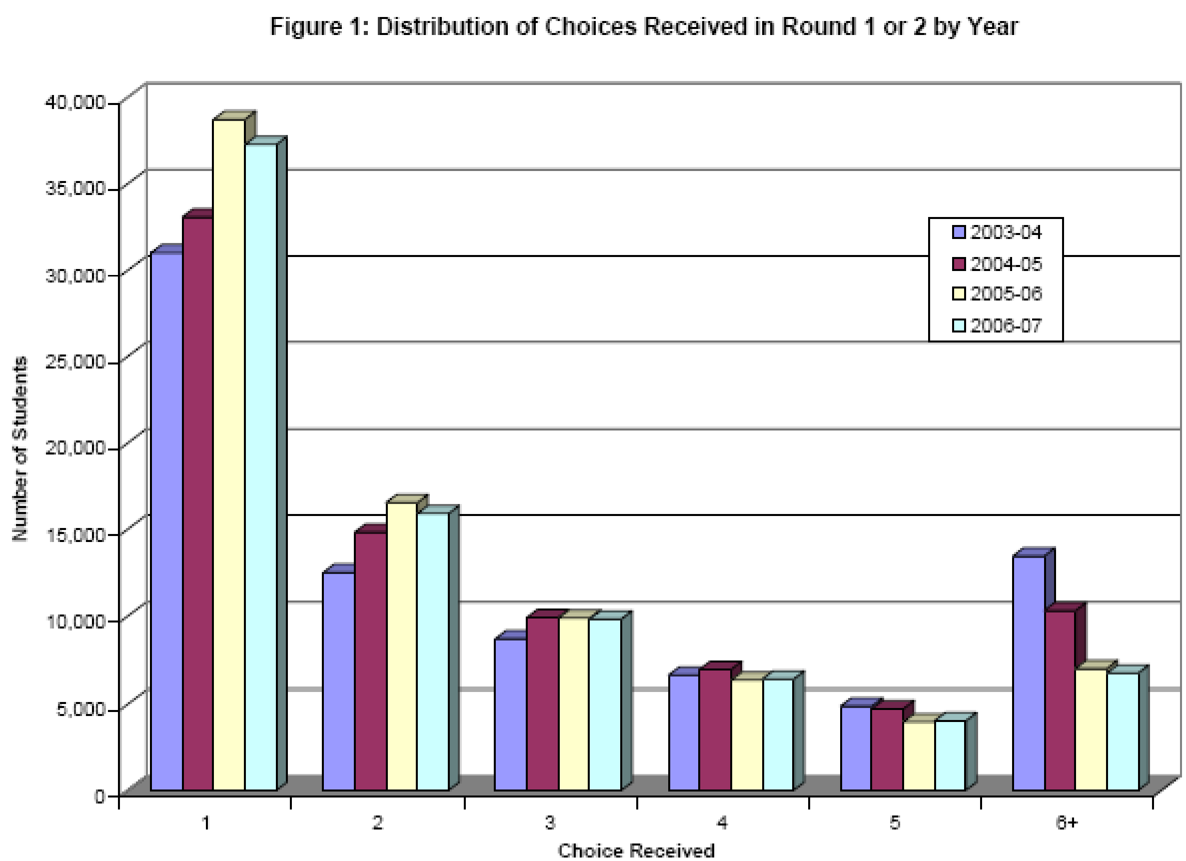


Figure 1: Distribution of Choices Received in Round 1 or 2 by Year
Number of Students
40,000
35,000
30,000
25,000
20,000
15,000
10,000
5,000
0
1
2
3
4
5
6+
Choice Received
2003-04
2004-05
2005-06
2006-07

Breaking ties

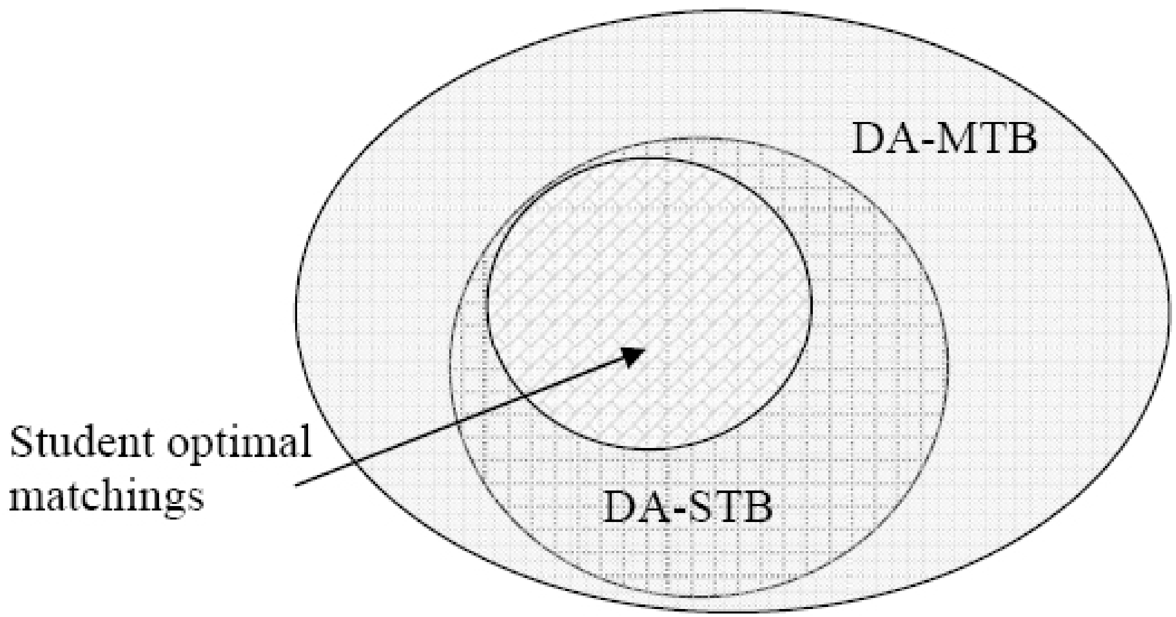
- How to break the ties?
  - Step 1: find a high tower
  - Step 2: take the ties to the tower
  - Step 3: drop the ties
  - Step 4: If some ties didn't break, repeat 2 and 3

Breaking ties

- Two main contestants:
  - Single tie breaking (DA-STB):
    For each student draw a single lottery number. Break ties in favor of the student with the higher lottery number *in every school*
  - Multiple tie breaking (DA-MTB):
    For each school, draw a lottery number for each student.
    Break ties in favor of the student with the higher lottery number.
    Draw new lottery numbers for the next school.

Breaking ties

- MTB sounds more equitable
- *But:* Any stable matching that can be produced with MTB, but not STB is not Pareto optimal for students (Abdulkadiroglu, Pathak, Roth 2009)



Breaking ties

- Sometimes, MTB is just bad:

	α_+	α_-	β_+	β-
	a	a	b	b
	b	b	a	a
Priority @	*b*	-	*a*	-
#	1	2	1	2

  - Both schools *a, b* have 2 seats.
  - The (sym) random allocation (for a large economy, A-L 2010):

	STB	MTB
α_+	½a, ½b	(1-√½) a, √½ b ≈ 0.3a, 0.7 b
α_-	½a, ½ϕ	(1-√½) a, (√½-½) b, ½ϕ ≈ 0.3a, 0.2 b, ½ϕ

Breaking ties

- Sometimes, STB vs. MTB is matter of transfers

	α	β
	a	b
	ϕ	a
Priority	-	-
#	2	2

  - Both schools a, b have 1 seat.
  - The random allocation (for a large economy, A-L 2010):

	STB	MTB
α	½ a	1/3 a
β	½ b	½ b, 1/6 a

Breaking ties – NYC data (g8 app, 06-07)

Choice	Deferred Acceptance Single Tie-Breaking DA-STB (1)	Deferred Acceptance Multiple Tie-Breaking DA-MTB (2)
1	32,105.3 (62.2)	29,849.9 (67.7)
2	14,296.0 (53.2)	14,562.3 (59.0)
3	9,279.4 (47.4)	9,859.7 (52.5)
4	6,112.8 (43.5)	6,653.3 (47.5)
5	3,988.2 (34.4)	4,386.8 (39.4)
6	2,628.8 (29.6)	2,910.1 (33.5)
7	1,732.7 (26.0)	1,919.1 (28.0)
8	1,099.1 (23.3)	1,212.2 (26.8)
9	761.9 (17.8)	817.1 (21.7)
10	526.4 (15.4)	548.4 (19.4)
11	348.0 (13.2)	353.2 (12.8)
12	236.0 (10.9)	229.3 (10.5)
unassigned	5,613.4 (26.5)	5,426.7 (21.4)

Breaking ties

- Now try explaining this to the school board…
- *"Through simple examples and simulations, we suggested that single tie breaking might have superior welfare properties to multiple tie breaking. The DOE remained unconvinced until student preferences had already been submitted, and computational experiments could be conducted comparing single and multiple tie breaking using actual data from the first round in 2003-04" (APR 2009)*

Tie breaking is inefficient

Choice	Deferred Acceptance Single Tie-Breaking DA-STB (1)	Deferred Acceptance Multiple Tie-Breaking DA-MTB (2)	Student-Optimal Stable Matching (3)	Improvement from DA-STB to Student-Optimal	Number of Students (4)
1	32,105.3 (62.2)	29,849.9 (67.7)	32,701.5 (58.4)	+1	633.2 (32.1)
2	14,296.0 (53.2)	14,562.3 (59.0)	14,382.6 (50.9)	+2	338.6 (22.0)
3	9,279.4 (47.4)	9,859.7 (52.5)	9,208.6 (46.0)	+3	198.3 (15.5)
4	6,112.8 (43.5)	6,653.3 (47.5)	5,999.8 (41.4)	+4	125.6 (11.0)
5	3,988.2 (34.4)	4,386.8 (39.4)	3,883.4 (33.8)	+5	79.4 (8.9)
6	2,628.8 (29.6)	2,910.1 (33.5)	2,519.5 (28.4)	+6	51.7 (6.9)
7	1,732.7 (26.0)	1,919.1 (28.0)	1,654.6 (24.1)	+7	26.9 (5.1)
8	1,099.1 (23.3)	1,212.2 (26.8)	1,034.8 (22.1)	+8	17.0 (4.1)
9	761.9 (17.8)	817.1 (21.7)	716.7 (17.4)	+9	10.2 (3.1)
10	526.4 (15.4)	548.4 (19.4)	485.6 (15.1)	+10	4.7 (2.0)
11	348.0 (13.2)	353.2 (12.8)	316.3 (12.3)	+11	2.0 (1.1)
12	236.0 (10.9)	229.3 (10.5)	211.2 (10.4)		
unassigned	5,613.4 (26.5)	5,426.7 (21.4)	5,613.4 (26.5)	Total:	1,487.5

DA vs. TTC

- What if we don't *really* need stability?
- Top Trading Cycles would produce a Pareto efficient assignment, and it is strategy-proof
- (If you don't know what is TTC, please look at the board that is hopefully in this room)
  - (You can also just check Wikipedia later)

Stability is not free

Choice	Student-Optimal Stable Matching (1)	Efficient Matching (2)	Improvement from Student-Optimal Stable Matching	Number (3)	k	Count of Students with k Blocking Pairs (4)
1	32,701.5 (58.4)	34,707.8 (50.5)	+1	1,819.7 (41.3)	1	22,287.5 (205.1)
2	14,382.6 (50.9)	14,511.4 (51.1)	+2	1,012.8 (26.4)	2	6,707.8 (117.9)
3	9,208.6 (46.0)	8,894.4 (41.2)	+3	592.0 (19.5)	3	2,991.0 (79.6)
4	5,999.8 (41.4)	5,582.1 (40.3)	+4	369.6 (16.0)	4	1,485.4 (56.5)
5	3,883.4 (33.8)	3,492.7 (31.4)	+5	212.5 (12.0)	5	716.6 (32.5)
6	2,519.5 (28.4)	2,222.9 (24.3)	+6	132.1 (9.1)	6	364.6 (22.9)
7	1,654.6 (24.1)	1,430.3 (22.4)	+7	77.0 (7.1)	7	183.1 (13.6)
8	1,034.8 (22.1)	860.5 (20.0)	+8	43.0 (5.6)	8	85.6 (10.9)
9	716.7 (17.4)	592.6 (16.0)	+9	26.3 (4.5)	9	44.7 (6.4)
10	485.6 (15.1)	395.6 (13.7)	+10	11.6 (2.8)	10	22.6 (4.9)
11	316.3 (12.3)	255.0 (10.8)	+11	4.8 (2.0)	11	9.9 (3.0)
12	211.2 (10.4)	169.2 (9.3)			12	3.2 (1.6)
unassigned	5,613.4 (26.5)	5,613.4 (26.5)	Total:	4,296.6		34,898.8

Nor does it grow on trees

Stability is cheap in Boston

Choice	Deferred Acceptance Single Tie-Breaking DA-STB (1)	Deferred Acceptance Multiple Tie-Breaking DA-MTB (2)	Student-Optimal Stable Matching (3)	Improvement from DA-STB to Student-Optimal	Number of Students (4)
1	2,251.8 (8.4)	2,157.3 (13.4)	2,256.6 (8.2)	+1	4.6 (2.6)
2	309.8 (10.3)	355.5 (12.0)	307.4 (10.0)	+2	1.2 (1.1)
3	154.9 (7.9)	189.3 (10.1)	154.0 (7.7)	+3	0.5 (0.7)
4	59.7 (5.5)	76.1 (7.0)	58.7 (5.5)	+4	0.3 (0.5)
5	27.4 (4.5)	34.1 (4.8)	27.0 (4.4)	+5	0.0 (0.1)
6	4.9 (1.9)	6.0 (2.5)	4.9 (1.9)	+6	0.0 (0.1)
7	2.6 (1.4)	2.8 (1.6)	2.5 (1.4)	+7	0.0 (0.1)
8	1.9 (1.2)	0.9 (0.9)	1.9 (1.2)	+8	0.0 (0.1)
9	1.2 (1.1)	0.4 (0.6)	1.2 (1.0)	+9	0.0 (0.0)
10	0.3 (0.6)	0.1 (0.2)	0.3 (0.5)		
unassigned	112.4 (4.6)	104.6 (4.5)	112.4 (4.6)	Total:	6.5

How Boston can do better

- 2 schools, Arts and Science
- 3 students, each an artists or scientists with prob ½
  - Both types want both schools. They prefer their school.
- Students submit preferences not knowing the identity of the other students, or the (single) tie-breaking

- In this uncorrelated environment truth-telling is an equilibrium

How Boston can do better

- DA:

Lottery rank	First choice	Second choice	No school
1	1	0	0
2 s	$^1/_2$ 1:a	$^1/_2$ 1:s	0
3	0	0	1
Average	$^1/_2$	$^1/_6$	$^1/_3$

- Boston:

Lottery rank	First choice	Second choice	No school
1	1	0	0
2 s	$^1/_2$ 1:a	$^1/_4$ 1:s; 3:s	$^1/_4$ 1:s; 3:a
3	$^1/_4$	0	$^3/_4$
Average	$^1/_2 + {}^1/_{12}$	$^1/_6 - {}^1/_{12}$	$^1/_3$

How Boston can do better

- DA is strategyproof, but it strips away intensity of preferences
- What about other mechanisms that consider cardinal preferences?