

Undeferred Acceptance in the Brown University A Cappella Market

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Outline

1. The A Cappella Market
2. Existing Research into Market Characteristics
3. The Underlying Model
4. Our Investigation
5. Our Model

Brown A Capella Market



A Cappella Groups at Brown

- All 11 groups participate
- Students can audition for several groups
- Those that get called back from any group are placed into a coordinated centralized matching market
- If students do not get called back, they are self-matched
- Students that get called back submit their strict preferences over groups

Matching Algorithm

1. Groups are given a random order
2. If a group is up, it selects a student to be evaluated by all groups
3. All groups that want to propose to that student simultaneously raise their hands
4. If one or more acceptable groups propose to that student, the student is matched with its most preferred group, and the student is removed from the market
5. If no acceptable groups raise their hand, the student is self-matched
6. Matching ends when no group wants to call up more students.

**What is compelling about
the market setup?**

Characteristics of the A Capella Market

- Stage 3 Market: transactions made through a centralized procedure (participation compelled)
- Swift market clearance
 - Groups forced to make decisions at same time
- Information about student's quality is known at the time of matching
- Incomplete information about selection order and other players' preferences
- Groups make binding decisions about students while more-preferred students may still be outstanding

Characteristics of the A Capella Market

- Strategy-proof for students:
 - When student is evaluated, matched to most preferred group that has hand raised
 - If false preferences given, either matched to same group or different one
 - If same, no benefit to misrepresenting preferences
 - If different group, new matching must necessarily pair them with a group that was lower on their truthful preference list otherwise it would have been the initial matching

What has already been analyzed?

What does previous research suggest?

Market Culture: How Rules Governing Exploding Offers Affect Market Performance

- M. Niederle and A. Roth

- Markets encounter difficulty maintaining a thick marketplace when transactions are made at a dispersed time.
 - Establish norms concerning when offers can be made, accepted and rejected
 - Difficult to establish a thick market at an efficient time with exploding and binding offers
 - Inefficient early contracting occurs
- A Cappella market is not currently unraveling

Jumping the Gun: Actions and institutions Related to the Timing of Market Transactions - A. Roth and X. Xing

- Incentives to "jump the gun" and arrange offers early but ultimately that leads to unraveling and ultimately market failure
 - Decisions are made before important information becomes available.
- Unraveling impeded if the consequences of hiring during uncertainty are large
 - In this market, student quality known

○ Turnaround Time and Bottlenecks in Market Clearing: Decentralized Matching in the Market for Clinical Psychologists - A. Roth and X. Xing

- Ability to hold onto multiple offers and deadline congests market
 - Firms can be rejected at the last second after all other acceptable candidates are already out of the market.
 - A capella market creates situations firms have to make binding decisions to accept students before they know who else will become available

Our Starting point: Deferred Acceptance Model in a Marriage Market

Assumptions we are making to fit models to market

Gale-Shapley Deferred Acceptance

- 2.8 - guarantees stable matching
- 4.7 - student-proposing mechanism makes truth-telling dominant for students
- 4.4 - in student-proposing, no way to make truth-telling dominant for groups
- Groups tentatively hold offers without binding decision until very end
- What happens without ability to hold?

Our Environment

Students: $S \{s_1, \dots, s_n\}$

Groups: $G \{g_1, \dots, g_m\}$

Assumptions:

- Marriage Market

- Strict preferences for students and groups

- $n > m$

What we are looking into

Proposed Model: Undeferred Acceptance

- 1. a:** each student simultaneously proposes to its most-preferred group that has not rejected them
 - b:** each group rejects any unacceptable proposals
 - c:** groups decide whether or not to reject acceptable proposals
 - d:** if a group decides to accept any proposal, it will accept its most preferred in that round
- 2.** accepted matchings become common information and are removed from the market
- 3.** step 1 is repeated for all unmatched students and groups
- 4.** matching ends when no more proposals can be made

Undeferred Acceptance

- Imperfect analogue

Model:

- Multiple students considered at once

- Students select order of proposal

- Accepting has $p = 1$ of matching to student

Market:

- One student at a time

- Groups select order

- Raising hand has $p < 1$ matching to student

- Focus: binding decision-making without knowledge of future prospects

Decentralized Job Matching (With Perfect Information)

•Decentralized job matching – G. Haeringer and M. Wooders

- Offering stage: According to the ordering given by their index numbers, each remaining [student] s_k , $k = 1, \dots, n$ offers its position to an acceptable [group] among the remaining [groups] who have not previously declined that firm, or exits the market if none of these [groups] are acceptable.

- Acceptance stage: According to the ordering given by their index numbers, each [group] g_k , $k = 1, \dots, m$ either accepts one of his offers (if he has any) or declines all his offers. A [group] cannot “hold” an offer and accept or decline it at a later stage. A [group] who has not received any offer waits for the next stage. If a [group], say w_i , accepts an offer from a [student], say s_j , then g_i and s_j are matched and exit.

- When [students] and [groups] act simultaneously during the offering and acceptance stages respectively, the sequential job market game can be seen as a decentralized version of the deferred acceptance algorithm.

Decentralized Job Matching (With Perfect Information)

- Firms as students, workers as groups
- There exists a group optimal stable matching μ_g found by running the deferred acceptance procedure

Sequential Group decisions:

- SPE strategy for g_i to reject all proposals except from $s_j = \mu_g(g_i)$
- Can strategically get μ_g

Results of Decentralized Job Matching

Simultaneous group decision-making --> expansion of SPE strategies:

- Any stable matching is SPE if students propose strategically
- Unstable matchings can be SPE

What Happens with Incomplete
Information about Preferences?

Imperfect Information about Students

$$S = \{s1, s2, s3\}$$

P(s1)	g1	g2	g3
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P(s2)	g1	?	?
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P(s3)	g2	g1	g3
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In round one, s3 proposes to g2

If P(s2) = g1 g2 g3

–g2 rejects s3 $\mu(g2) = s2$

–g2 accepts s3 $\mu(g2) = s3$

g2 does better by rejecting

$$G = \{g1, g2, g3\}$$

P(g1)	s1	s2	s3
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P(g2)	s2	s3	s1
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P(g3)	s1	s2	s3
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If P(s2) = g1 g3 g2

–g2 rejects s3 $\mu(g2) = g2$

–g2 accepts s3 $\mu(g2) = s3$

g2 does better by accepting

Possible Directions: Decision-Making Under Incomplete Information

- Equilibria:
 - Can we find an SPE strategy?
 - Is settling an SPE strategy?
 - Does settling come at a cost (i.e. assuming risk neutrality, can playing probabilistically increase expected utility)?
- Reason for participation in the market:
 - Not necessarily NE to make early offers
 - Potential social cost to not participating
- Stability:
 - Not as necessary to look at (draft market)

Relevant Literature

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Your Questions/Thoughts?

Considering simulating simple market.
Establishing utilities

Compare results of simple strategies

- Naive acceptance of best initial offer
- Truncate
- Probabilistic
 - Based on number of unmatched groups, number of more preferred students outstanding

Depend on size of market, how utilities are established