General Instructions for Students

1. The problem sets given in this handout are taken from old exams.
2. Exercises should NOT submitted (they will not be graded). However,
3. The best, and perhaps the only, way to ensure that you understand the material taught in class is to solve these exercises under "exam conditions" and only then check the proposed solution.
4. Solutions to all problems can be downloaded as a separate file.
5. Another advantage of solving these exercises is that they provide the best preparation for the exams. Most (but not all) exam questions will be based on variations of these exercises.

Set # 1: Concentration Measures

(a) The diaper industry is characterized by 8 firms producing identical diapers. Let s_i denote the percentage market share of firm i, $i = 1, 2, \ldots, 8$. It has been recently observed that the market shares are given by

Firm i	1	2	3	4	5	6	7	8
s_i	60%	10%	5%	5%	5%	5%	5%	5%

(i) Compute the concentration measures I_4 and I_{HH} for this industry

(ii) Suppose now firm 2 and firm 3 merge and become a single firm labeled firm $\overline{23}$. Compute the postmerger concentration measures values $\hat{I}_4$ and $\hat{I}_{\text{HH}}$.

(iii) Compute the change in concentration resulting from this merger, $\Delta I_4 \stackrel{\text{def}}{=} \hat{I}_4 - I_4$ and $\Delta I_{\text{HH}} \stackrel{\text{def}}{=} \hat{I}_{\text{HH}} - I_{\text{HH}}$.

(iv) Suppose now the merger between firm 2 and firm 3 did not work out, so the two firms remain separated. However, suppose firms 6, 7, and 8 now merge. Compute the postmerger concentration measures values $\bar{I}_4$ and $\bar{I}_{\text{HH}}$.

(v) Compute the change in concentration resulting from this merger, $\Delta I_4 \stackrel{\text{def}}{=} \bar{I}_4 - I_4$ and $\Delta I_{\text{HH}} \stackrel{\text{def}}{=} \bar{I}_{\text{HH}} - I_{\text{HH}}$.

(vi) The Merger Guidelines in the United States suggest that a merger should *not* be challenged if the postmerger and the change in the Herfindahl-Hirschman concentration index I_{HH} satisfies either

i. $I_{\text{HH}} < 1000$,

ii. $1000 \le I_{\text{HH}} < 1800$, and $\Delta I_{\text{HH}} < 100$, or

iii. $I_{\text{HH}} \geq 1800$ and $\Delta I_{\text{HH}} < 50$.

Use these guidelines to determine whether any of the above mergers is likely to be challenged by the DOJ.

(b) The diaper industry in Albania consists of 5 firms producing identical diapers. Similarly, the diaper industry in Bolivia consists of 6 firms. It has been recently observed that firms' market shares in each country are given by

Country	Firms						Concentration Index	
	1	2	3	4	5	6	I_4	I_{HH}
Albania	40%	15%	15%	15%	15%	0%		
Bolivia	45%	11%	11%	11%	11%	11%		

(i) Fill-in the missing items in the above table (show all your calculations). Then, conclude which industry is more concentrated (and according to which measure).

(ii) Suppose the distribution of market shares in the United States is the same as in Bolivia. Use the merger guidelines to conclude whether a merger between firms 5 and 6 is likely to be challenged by the FTC?

Set # 2: Normal-form Games

(a) The table below displays the profits from a price game between GM and FORD.

		FORD Low Price		High Price	
GM	Low	100	100	500	0
	High	0	300	200	250

Each firm can set either a high price, p^H, or a low price, p^L, where $p^H > p^L > 0$. Solve the following problems.

(i) Write down GM's best-response function. Also, write down FORD's best-response function.

(ii) Conclude which outcomes of the game constitute a Nash equilibrium.

(iii) Are there outcomes in this game which Pareto dominate the Nash equilibrium outcomes that you found in the previous problem? Prove your result.

(iv) Suppose GM and FORD form a cartel. Which outcome maximizes joint industry profit?

(b) Firms A and B can choose to adopt a new technology (N) or to adhere to their old technology (O). Formally, firms' action sets are: $t_A \in \{N, O\}$ and $t_B \in \{N, O\}$. The table below exhibits the profit made by each firm under different technology choices.

		Firm B			
		NEW TECHNOLOGY (N)		OLD TECHNOLOGY (O)	
Firm A	NEW	200	0	0	200
	OLD	50	100	100	50

Write down the best-response functions of firms A and B, $t_A = R_A(t_B)$ and $t_B = R_B(t_A)$, and conclude which outcome(s) is a Nash equilibrium.

Set # 3: Extensive-form Games

(a) Firms A and B can choose to adhere to their old technology or to adopt a new technology. Table 1 below exhibits the profit made by each firm under different technology choices. Roughly, Table 1 roughly shows that both firms earn higher profits when they choose to

		Firm B			
		NEW TECHNOLOGY (N)		OLD TECHNOLOGY (O)	
Firm A	NEW	6	4	2	2
	OLD	1	0	4	5

Table 1: The standardization extensive-form game for Set # 3:.
Note: in each entry, profits are listed in the order $\langle \pi_A, \pi_B \rangle$.

adopt the same technology. Let t_A denote firm A's technology choice and t_B denote firm B's technology choice, where $t_A \in \{N, O\}$ and $t_B \in \{N, O\}$. Answer the following questions.

(i) Write down the best-response functions of firms A and B assuming that they play a normal-form game. Formally, solve $t_A = R_A(t_B)$ and $t_B = R_B(t_A)$.

(ii) Conclude which outcomes constitute a Nash equilibrium of the normal-form game.

(iii) Now suppose a two-stage extensive-form game is played in which firm A sets its technology standard before firm B does. Write down the subgame-perfect equilibrium strategy for firm A and for firm B.

(iv) Suppose now that firm B sets its technology standard before firm A does. Write down the subgame-perfect equilibrium (SPE) strategy for firm A and for firm B.

(b) Firms A and B can choose to adopt a new technology (N) or to adhere to their old technology (O). Formally, firms' action sets are: $t_A \in \{N, O\}$ and $t_B \in \{N, O\}$. The table below exhibits the profit made by each firm under different technology choices.

		Firm B			
		New Technology (N)		Old Technology (O)	
Firm A	New	200	0	0	200
	Old	50	100	100	50

(i) Write down the best-response functions of firms A and B, $t_A = R_A(t_B)$ and $t_B = R_B(t_A)$

(ii) Draw the tree of a two-stage extensive-form game in which firm A chooses its technology t_A in stage I, and Firm B chooses its t_B in stage II (after observing the choice made by firm A). Make sure that you indicate firms' profits at the termination points on the tree. Solve for the subgame-perfect equilibrium of this game. Provide a short proof or an explanation justifying your answer.

(iii) Draw the tree of a two-stage extensive-form game in which firm B chooses its technology t_B in stage I and Firm A chooses its t_A in stage II (after observing the choice made by firm B). Solve for the subgame-perfect equilibrium of this game. Provide a short proof or an explanation justifying your answer.

(iv) Compare the equilibrium firms' profit levels of the games played in (ii) and in (iii). Conclude under which game firm A earns a higher profit. Briefly explain your answer.

(c) The table below displays the profits from a price game between GM and FORD.

		FORD					
		Low Price		Mid Price		High Price	
GM	Low	100	100	150	50	200	0
	Mid	50	150	200	200	350	250
	High	0	200	250	350	300	300

Each firm can set either a high price, p^H, a mid price, p^M, or a low price, p^L, where $p^H > p^M > p^L > 0$. Solve the following problems.

(i) Write down Ford's and GM's price best-response function, and conclude which pairs of prices constitute Nash equilibria.

(ii) Is the outcome $\langle p_G, p_F \rangle = \langle p^H, p^M \rangle$ *Pareto superior* to $\langle p^H, p^H \rangle$? Prove your answer.

(iii) Solve for a subgame-perfect equilibrium price strategies of this two-stage game in which Ford announces its price before GM does.

(d) The table below displays the profits from a price game between GM and FORD.

		FORD					
		LOW PRICE		MID PRICE		HIGH PRICE	
	LOW	100	100	150	50	200	0
GM	MID	50	150	200	200	350	250
	HIGH	0	200	250	350	300	300

Each firm can set either a high price, p^H, a mid price, p^M, or a low price, p^L, where $p^H > p^M > p^L > 0$. Solve the following problems.

(i) Find which pairs of prices (if any) constitute an equilibrium in *dominant* actions. Prove your answer!

(ii) Is the outcome $\langle p_G, p_F \rangle = \langle p^H, p^H \rangle$ a *Nash* equilibrium? Prove your answer!

(iii) Suppose the FTC prohibits both firms from setting high prices. That is, setting p^H becomes illegal. Which outcome(s) constitute a *Nash* equilibrium in the restricted game? Prove your answer.

(iv) Assuming again that firms are not allowed to set p^H, solve for a subgame-perfect equilibrium price strategies of a two-stage game in which Ford announces its price before GM does.

Set # 4: Simple Monopoly

(a) In Ann Barber (a small town in the Midwest) there is only one fortune teller who acts as a monopoly. The inverse demand function for this service is given by

$$p = 12 - \frac{Q}{2},$$

where p denotes the price charged per visit, and Q the quantity demanded for fortune telling. Answer the following questions.

(i) Suppose the cost function of this fortune teller is given by

$$C(Q) = 4 + 2Q.$$

That is, the marginal cost is $c = \$2$ (consisting of her value time and other "communication" expenses), and the fixed cost is $F = \$4$ (say, monthly rent on her office space).

Compute and draw the fortune teller's marginal cost and average functions, as well as the marginal revenue function.

(ii) Algebraically compute the fortune teller's profit-maximizing output, price, and profit.

(iii) Compute the price elasticity at the profit-maximizing output.

(iv) Suppose now that the cost function of this fortune teller has changed and is now given by

$$C(Q) = 4 + Q^2.$$

Compute and draw the fortune teller's new marginal and average cost functions as well as the marginal revenue function.

(v) Algebraically compute the fortune teller's profit-maximizing output, price, and profit based on her new cost function.

(vi) We now look at a small town located across the border in Canada. The demand function faced by a monopoly fortune teller has a constant elasticity and is given by $Q = 120p^{-2}$. The cost function of the Canadian fortune teller is $C(Q) = 4 + Q/2$. Compute the monopoly price, output level, and profit made by the Canadian monopoly fortune teller.

(b) The market demand function for Marzipan in Frankenmuth Michigan has a constant elasticity of -3. More precisely the actual daily demand was estimated to be $Q = 34560\, p^{-3}$, where p is the price per pound. Each pound costs $c = \$8$ to produce. Frankenmuth is served by a local monopoly producer. Compute the monopoly's profit-maximizing *price* and the monopoly's *profit* level. Show your computations.

(c) Each *u*Phone costs $c = \$100$ to produce. The producer owns exclusive patent rights which prevents competition in this market. On the demand side, there are:
$n^H = 1000$ consumers who are willing to pay a maximum amount of $V^H = \$500$ for a *u*Phone,
$n^M = 3000$ consumers who are willing to pay up to $V^M = \$300$ for a *u*Phone, and
$n^L = 5000$ consumers who are willing to pay a maximum amount of $V^L = \$200$ for a *u*Phone.
Each consumer chooses whether to buy one unit or not to buy at all. Compute the profit-maximizing price of this monopoly *u*Phone producer.

Set # 5: Discriminating Monopoly

(a) The demand function for concert tickets to be played by the Ann Arbor symphony orchestra varies between nonstudents (N) and students (S). Formally, the two inverse demand demand functions of the two consumer groups are given by

$$p_N = 12 - q_N \quad \text{and} \quad p_S = 6 - q_S.$$

Thus, at any given consumption level nonstudents are willing to pay a higher price than students.

Assume that the orchestra's total cost function is $C(Q) = 10 + 2Q$ where $Q = q_N + q_S$ is to total number of tickets sold. Solve the following problems.

(i) Suppose the orchestra is able to price discriminate between the two consumer groups by asking students to present their student ID cards to be eligible for a student discount. Compute the profit-maximizing prices p_N and p_S, the number of tickets sold to each group of consumers, and total monopoly profit.

(ii) Suppose now the local mafia has distributed a large number of fake student ID cards, so basically every resident has a student ID card regardless of whether the resident is a student or not. Compute the profit-maximizing price, the number of tickets sold to each group of consumers, and total profit assuming that the monopoly orchestra is unable to price discriminate.

(iii) By how much the orchestra enhances its profit from the introduction student discounted tickets compared with the profit generated from selling a single uniform ticket price to both consumer groups.

(b) Consider the market for the G-Jeans (the latest fashion among people in their late thirties). G-Jeans are sold by a single firm that carries the patent for the design. On the demand side, there are $n^H = 200$ high-income consumers who are willing to pay a maximum amount of $V^H = \$20$ for a pair of G-Jeans, and $n^L = 300$ low-income consumers who are willing to pay a maximum amount of $V^L = \$10$ for a pair of G-Jeans. Each consumer chooses whether to buy one pair of jeans or not to buy at all.

(i) Draw the market aggregate-demand curve facing the monopoly.

(ii) The monopoly can produce each unit at a cost of $c = \$5$. Suppose that the G-Jeans monopoly cannot price discriminate and is therefore constrained to set a uniform market price. Find the profit-maximizing price set by G-Jeans, and the profit earned by this monopoly.

(iii) Compute the profit level made by this monopoly assuming now that this monopoly can price discriminate between the two consumer populations. Does the monopoly benefit from price discrimination. Prove your result!

(c) The demand function for concert tickets to be played by the Ann Arbor symphony orchestra varies between nonstudents (N) and students (S). Formally, the two demand functions of the two consumer groups are given by

$$q_N = 240(p_N)^{-2} \quad \text{and} \quad q_S = 540(p_S)^{-3}.$$

Assume that the orchestra's total cost function is $C(Q) = 2Q$ where $Q = q_N + q_S$ is to total number of tickets sold. Compute the concert ticket prices set by this monopoly orchestra, and the resulting ticket sales, assuming that the orchestra can price discriminate between the two consumer groups.

(d) A brewery is allowed to sell beer in two domestic markets called market 1 and market 2. Since both markets are located nearby each other, the brewery *cannot* price discriminate, and therefore must set a uniform price p in both markets.

The inverse demand function in market 1 is $p_1 = 60 - q_1$. The demand in market 2 is $p_2 = 50 - q_2$, where q_1 and q_2 are quantity demanded in each market (measured in cans), and the price is measured in cents. The cost of producing each can of beer is $c = 30$. Solve the following problems:

(i) Compute the monopoly's profit-maximizing uniform price, the quantity of beer sold in markets 1 and 2, and total profit.

(ii) Suppose now the brewery is allowed to sell beer to a nearby town (call it market 3) located just across the border. The inverse demand function in this town is $p_3 = 40 - q_3$. Due to the proximity of the three markets, the brewery cannot price discriminate and must set again a uniform price of p in all three markets. Compute the profit-maximizing price, the quantity sold in each market, and total profit.

(e) Discuss whether it is illegal to price discriminate according to the U.S. Law. Explain which section of the law deals with price discrimination, and how this section should be interpreted.

(f) The demand function for concert tickets to be played by the Ann Arbor symphony orchestra varies between nonstudents (N) and students (S). Formally, the two demand functions of the two consumer groups are given by

$$q_N = 7290(p_N)^{-3} \quad \text{and} \quad q_S = 40960(p_S)^{-4}.$$

Assume that the orchestra's total cost function is $TC(Q) = 6Q$, where $Q = q_N + q_S$ is to total number of tickets sold. Compute the concert ticket prices set by this monopoly orchestra, and the resulting ticket sales, assuming that the orchestra can price discriminate between the two consumer groups, say by requiring students to submit their student ID cards.

(g) *Impel*™ is the sole producer of memory chips for supercomputers. Each chip costs $c = 30$ to produce. This monopoly can sell in two markets with the following inverse demand functions:

$$p_1 = 120 - q_1 \quad \text{and} \quad p_2 = 120 - \frac{q_2}{3}.$$

(i) Compute the monopoly's profit-maximizing prices in each market, p_1 and p_2, sales levels q_1 and q_2, and the monopoly's total profit assuming that *Impel*™ can *price discriminate* between the two markets.

(ii) Now, due to a fire that nearly destroyed its factory, this monopoly cannot produce and sell more than 160 units. In other words, assume that the production capacity of *Impel*™ is limited to no more than 160 chips. Compute the monopoly's profit-maximizing prices in each market, p_1 and p_2, sales levels q_1 and q_2, and the monopoly's total profit.

(h) A monopoly sells in three markets with the following inverse demand functions:

$$p_1 = 36 - q_1, \quad p_2 = 24 - \frac{q_2}{2}, \quad \text{and} \quad p_3 = 12 - \frac{q_3}{2}.$$

For simplicity, assume that production is costless ($c = 0$). Also, assume that the monopoly is unable to price discriminate, hence it must charge the same price in all three markets, $p = p_1 = p_2 = p_3$. Compute the monopoly's profit-maximizing price, p, aggregate sales, and total profit.

Set # 6: Cournot Quantity Competition (Static)

(a) In Ann Barber there are two suppliers of distilled water, labeled firm A and firm B. Distilled water is considered to be a homogenous good (well, all water taste the same, anyway). Let p denote the price per gallon, q_A quantity sold by firm A, and q_B the quantity sold by firm B.

Firm A is located nearby a spring and therefore bears a production cost of $c_A = \$1$ per one gallon of water. Firm B is not located near a spring, and thus bears a cost of $c_B = \$2$ per gallon.

Ann Barber's inverse demand function for distilled water is given by

$$p = 120 - \frac{1}{2}Q = 120 - \frac{q_A + q_B}{2},$$

where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Ann Barber.

Solve the following problems:

(i) Formulate the profit-maximization problem of firm A.

(ii) Solve for firm A's best-response function, $q_A = R_A(q_B)$.

(iii) Formulate the profit-maximization problem of firm B.

(iv) Solve for firm B's best-response function, $q_B = R_B(q_A)$.

(v) Draw the two best-response functions. Denote the vertical axis by q_A, and the horizontal axis by q_B.

(vi) Solve for the Cournot equilibrium output levels q_A^c and q_B^c. State which firm sells more water and why.

(vii) Solve for the aggregate industry supply and the equilibrium price of distilled water in Ann Barber.

(viii) Solve for the profit level made by each firm, and for the aggregate industry profit. Which firm earns a higher profit and why?

(b) In Waterville there are two suppliers of distilled water, labeled as firm A and firm B. Distilled water is considered to be a homogenous good. Let p denote the price per gallon, q_A quantity sold by firm A, and q_B the quantity sold by firm B.

Both firms are located close to a spring so the only production cost is the cost of bottling. Formally, each firm bears a production cost of $c_A = c_B = \$3$ per one gallon of water. Waterville's aggregate inverse demand function for distilled water is given by $p = 12 - Q = 12 - q_A - q_B$, where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Waterville. Solve the following problems:

(i) Solve for firm A's best-response function, $q_A = R_A(q_B)$. Also solve for firm B's best-response function, $q_B = R_B(q_A)$. Show your derivations.

(ii) Solve for the Cournot equilibrium output levels q_A^c and q_B^c. State which firm sells more water (if any) and why.

(iii) Solve for the aggregate industry supply and the equilibrium price of distilled water in Waterville.

(iv) Solve for the profit level made by each firm, and for the aggregate industry profit. Which firm earns a higher profit and why?

(c) Solve for the Cournot equilibrium for the following market: $p = 120 - Q$, $c_A = \$10$, $c_B = \$20$.

(d) Solve for the Cournot equilibrium for the following market: $p = 240 - Q/2$, $c_A = \$10$, $c_B = \$20$.

Set # 7: Sequential Moves (Quantity Game)

(a) In Ann Barber there are two suppliers of distilled water, labeled firm A and firm B. Distilled water is considered to be a homogenous good (well, all water taste the same, anyway). Let p denote the price per gallon, q_A quantity sold by firm A, and q_B the quantity sold by firm B.

Firm A is located nearby a spring and therefore bears a production cost of $c_A = \$1$ per one gallon of water. Firm B is not located near a spring, and thus bears a cost of $c_B = \$2$ per gallon.

Ann Barber's inverse demand function for distilled water is given by

$$p = 120 - \frac{1}{2}Q = 120 - \frac{q_A + q_B}{2},$$

where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Ann Barber.

Suppose firm A sets its quantity produced q_A, before firm B does. That is, firm B sets its production level q_B, only after observing the quantity produced by firm A. Solve the following problems.

(i) Derive firm B's (the follower) output best-response as a function of firm A's output level, $q_B = R_B(q_A)$.

(ii) Formulate and solve firm A's (the leader) output profit-maximization problem.

(iii) Compute the profit-maximizing output level produced by firm B (the follower).

(iv) Compute the aggregate industry supply of distilled water in Ann Barber and the equilibrium price.

(v) Compute the equilibrium profit level of each firm.

(vi) Compare the output and profit levels of firm A as a leader in a sequential-move equilibrium to the output and profit levels in the Cournot equilibrium which you computed in Set # 6:(a).

(vii) Compare the output and profit levels of firm B as a follower in a sequential-move equilibrium to the output and profit levels in the Cournot equilibrium which you computed in Set # 6:(a).

(viii) Compare *aggregate* industry output, aggregate profit levels and the price level under a sequential-move equilibrium to those under the Cournot equilibrium.

(b) Consider the following market: $p = 12 - Q/2$, and unit costs $c_A = \$1$ and $c_B = \$2$.

(i) Solve for the sequential-move equilibrium assuming that firm A is the leader and B is the follower.

(ii) Solve for the sequential-move equilibrium assuming that firm B is the leader and A is the follower.

(c) Consider the following market: $p = 12 - Q$, and unit costs $c_A = \$1$ and $c_B = \$2$.

(i) Solve for the sequential-move equilibrium assuming that firm A is the leader and B is the follower.

(ii) Solve for the sequential-move equilibrium assuming that firm B is the leader and A is the follower.

(d) In Ben Barber there are two suppliers of distilled water, labeled firm A and firm B. Distilled water is considered to be a homogenous good. Let p denote the price per gallon, q_A quantity sold by firm A, and q_B the quantity sold by firm B. Firm A and firm B bear a production cost of $c_A = c_B = \$2$ per one gallon of water. Ann Barber's inverse demand function for distilled water is given by

$$p = 12 - \frac{1}{3}Q = 12 - \frac{q_A + q_B}{3},$$

where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Ben Barber. Solve the following problems:

(i) Suppose the firms compete in quantities (production levels). Compute each firm's quantity best response function, and conclude how much each firm produces in a Cournot-Nash equilibrium.

(ii) Compute the price and the profit of each firm in a Cournot-Nash equilibrium.

(iii) Compute the quantity produced by each firm in a sequential (Stackelberg) game in which firm A sets its output level before firm B.

(iv) Compute the price and the profit of each firm in a Stackelberg equilibrium.

(e) The market inverse demand function for internet connection is given by $p = 120 - Q$, where Q is the number of subscribers. There are three firms labeled as firm 1, firm 2, and firm 3. Assume that internet connection is costless to provide. Formally, assume that the marginal and fixed costs satisfy $c_1 = c_2 = c_3 = 0$ and that $F_1 = F_2 = F_3 = 0$

Consider the following three-stage game. In stage $t = 1$ firm 1 sets its output level, q_1. In stage $t = 2$ firm 2 sets its output level, q_2 after observing q_1. In stage $t = 3$ firm 3 sets its output level, q_3, observing q_1 and q_2. Compute the firms' output and profit levels in a subgame perfect equilibrium. .

Set # 8: Bertrand Price Competition (Static and Sequential)

(a) Two firms have technologies for producing identical paper clips. Assume that all paper clips are sold in boxes containing 100 paper clips. Firm A can produce each box at unit cost of $c_A = \$6$ whereas firm B (less efficient) at a unit cost of $c_B = \$8$.

(i) Suppose that the aggregate market demand for boxes of paper clips is $p = 12 - Q/2$, where p is the price per box and Q is the number of boxes sold. Solve for the Nash-Bertrand equilibrium prices p^b_A and p^b_B, and the equilibrium profits π^b_A and π^b_B. Explain your reasoning!

(ii) Answer the previous question assuming that firm A has developed a cheaper production technology so its unit cost is now given by $c_A = \$2$.

(b) In Ben Barber there are two suppliers of distilled water, labeled firm A and firm B. Distilled water is considered to be a homogenous good. Let p denote the price per gallon, q_A quantity sold by firm A, and q_B the quantity sold by firm B. Firm A and firm B bear a production cost of $c_A = c_B = \$2$ per one gallon of water. Ann Barber's inverse demand function for distilled water is given by

$$p = 12 - \frac{1}{3}Q = 12 - \frac{q_A + q_B}{3},$$

where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Ben Barber. Solve the following problems assuming that the firms compete in prices, p_A and p_B.

(i) Write down each firm's price best response function, and solve for the price each firm sets in a Bertrand-Nash equilibrium.

(ii) Compute the quantity produced and the profit of each firm in a Bertrand-Nash equilibrium.

(iii) Suppose now that firms set their prices in sequence. In stage I, firm A sets p_A. In stage II, after observing p_A, firm B sets p_B. Write down the firms' price *strategies* in a subgame-perfect equilibrium (SPE).

(c) In Ben Barber there are two suppliers of distilled water, called firm A and firm B. Distilled water is considered to be a homogenous good. Let p_A and p_B denote the price per gallon, and q_A and q_B the quantity sold by firms A and B, respectively. Suppose that the Ben Barber municipality provides all the water for free, so firms don't bear any production cost. Formally, assume that $c_A = c_B = 0$.

Ben Barber's inverse demand function for distilled water is given by

$$p = 12 - \frac{1}{3}Q = 12 - \frac{q_A + q_B}{3},$$

where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Ben Barber.

(i) Suppose the firms compete in quantities (production levels). Assume that firm A sets its output level q_A first. Then, firm B observes q_A and sets its output level q_B. Compute the quantity produced by each firm in this two-stage game. Also, compute the resulting market price, p, and the firms' equilibrium profit levels, π_A and π_B.

(ii) Suppose the firms compete in prices. Assume that firm A sets its price p_A first. Then, firm B observes p_A and sets its price p_B to maximize profit. Solve for the Subgame-perfect equilibrium price strategies of this game.

(iii) Answer the previous question (two-stage price game) assuming that the firms' unit costs are $c_A = 0$ and $c_B = \$4$.

Set # 9: Self-enforcing Collusion

(a) The table below displays the profits from a price game between GM and FORD which you have already analyzed in Set # 2:.

		FORD			
		Low Price		High Price	
GM	Low	100	100	500	0
	High	0	300	200	250

Suppose this game is repeated indefinitely in each period $t = 0, 1, 2, \ldots$. Let ρ, where $0 \le \rho \le 1$, denote the firms' common time discount factor. Solve the following problems.

(i) Formulate the trigger strategy of each firm under which both firms collude on setting high prices.

(ii) Compute the infinite discounted sum of the stream of profits earned by each firm assuming that both firms set the collusive price p^H in each period.

(iii) Suppose GM deviates from the collusive price in period $t = 0$, while FORD keeps maintaining the collusive price p^H. From the table we infer the what is the profit GM earns during period $t = 0$.

(iv) Suppose instead that FORD deviates from the collusive price in period $t = 0$, while GM keeps maintaining the collusive price p^H. From the table we infer what is the profit FORD earns during period $t = 0$.

(v) Compute the infinite sum of the stream of profits made by GM assuming that only GM deviates from the collusive price in period $t = 0$, but FORD follows its trigger-price strategy from period $t = 1$ and on.

(vi) Compute the infinite sum of the stream of profits made by FORD assuming that only FORD deviates from the collusive price in period $t = 0$, but GM follows its trigger-price strategy from period $t = 1$ and on.

(vii) For each firm separately, compute the minimum threshold value of ρ that would make it unprofitable for the firm to unilaterally deviate from the collusive outcome assuming that the competing firm adheres to its trigger-price strategy.

(b) Consider an infinitely-repeated price competition game between GM and FORD. Each firm can set a high price or a low price in each period $t = 0, 1, 2, \ldots$. The profit of each outcome are given in the following matrix:

		FORD			
		Low Price (p^L)		High Price (p^H)	
GM	Low (p^L)	4	3	5	1
	High (p^H)	1	6	5	4

Suppose that each firm adopts a trigger-price strategy under which the firms may be able to implicitly collude on setting the high price. Let ρ $(0 < \rho < 1)$ denote the time discount factor. Solve the following problems:

(i) Compute the minimum threshold value of ρ which would ensure that GM sets p^H in every period t. Show and explain your derivations.

(ii) Compute the minimum threshold value of ρ which would ensure that FORD sets p^H in every period t. Show and explain your derivations.

(c) Two firms, labeled firm A and firm B, compete in prices in a market for a homogeneous product. In this market there are $N > 0$ consumers; each buys one unit if the price of the product does not exceed \$10, and nothing otherwise. Consumers buy from the firm selling at a lowest price. In case both firms charge the same price, assume that $N/2$ consumers buy from each firm.

Assume the unit production of firm A and firm B are given by $c_A = c_B = \$2$. Solve two problems on the next page.

(i) Find the Bertrand equilibrium prices for a single-shot game, assuming that the firms choose their prices simultaneously.

(ii) Now suppose that the game is repeated infinitely many times, $t = 0, 1, 2 \ldots$. Let ρ $(0 < \rho < 1)$ denote the time-discount parameter. Propose trigger price strategies for each firm yielding the collusive prices of $(10, 10)$ each period, assuming that each firm reverts to the Bertrand equilibrium price if any firm deviates from its collusive price. Calculate the minimal value of ρ that would enforce the collusive prices under the trigger price strategies you proposed.

(d) In Ben Barber there are two suppliers of distilled water, labeled firm A and firm B. Distilled water is considered to be a homogenous good. Let p denote the price per gallon, q_A quantity sold by firm A, and q_B the quantity sold by firm B. Firm A and firm B bear a production cost of $c_A = c_B = \$20$ per one gallon of water. Ann Barber's inverse demand function for distilled water is given by

$$p = 140 - 2Q = 140 - 2(q_A + q_B),$$

where $Q = q_A + q_B$ denotes the aggregate industry supply of distilled water in Ben Barber. Solve the two problems on the next page assuming that the firms compete in prices, p_A and p_B.

(i) Write down each firm's price best response function, and solve for the price each firm sets in a Bertrand-Nash equilibrium.

(ii) Now, suppose this game is repeated indefinitely in each period $t = 0, 1, 2, \ldots$. Let ρ, where $0 \le \rho \le 1$, denote the firms' common time discount factor. Compute the minimum threshold value of ρ that would make it <u>unprofitable</u> for each firm to unilaterally deviate from the collusive outcome assuming that the each firm i adheres to its trigger-price strategy given by

$$p_i(\tau) = \begin{cases} \$80 & \text{if } p_A(t) = p_B(t) = \$80 \text{ in each period } t = 0, 1, 2, \ldots, \tau - 1 \\ \$20 & \text{otherwise.} \end{cases}$$

Set # 10: Differentiated Brands

(a) Car producers A and B produce and sell substitute cars (differentiated goods). The inverse demand functions for cars A and B, respectively, are given by

$$p_A = 60 - \frac{3}{2}\, q_A - q_B \quad \text{and} \quad p_B = 60 - \frac{3}{2}\, q_B - q_A.$$

Assuming that firms do not bear any production costs ($c_A = c_B = 0$), solve the following problems:

(i) Suppose the firms compete in *quantities*, where firm A sets q_A and firm B sets q_B, simultaneously. Formulate the profit function of each firm as a function of the quantity supplied by both firms. Formally, write down and spell out the exact equations of each firm's maximization problem, $\max_{q_A} \pi_A(q_A, q_B)$ and $\max_{q_B} \pi_B(q_A, q_B)$.

(ii) Solve for the firms' quantity best-response functions $q_A = R_A(q_B)$ and $q_B = R_B(q_A)$. Plot both best-response functions where you denote the vertical axis by q_A and the horizontal axis by q_B. Indicate whether these best-response functions are upward or downward sloping.

(iii) Solve for the Nash equilibrium quantity levels, q_A and q_B, the corresponding prices, p_A and p_B, as well as the equilibrium profit levels, π_A and π_B.

(iv) Solve for the two direct demand functions from the above-given system of two inverse demand functions. Formally, compute the parameters a, b, and c of the direct demand functions $q_1 = a - bp_1 + cp_2$ and $q_2 = a - bp_2 + cp_1$, which are consistent with the above-given inverse demand functions.

(v) Suppose now that the firms compete in *prices*, where firm A sets p_A and firm B sets p_B, simultaneously. Formulate the profit function of each firm as a function of both prices. Formally, write down and spell out the exact equations of each firm's maximization problem, $\max_{p_A} \pi_A(p_A, p_B)$ and $\max_{p_B} \pi_B(p_A, p_B)$.

(vi) Solve for the firms' price best-response functions $p_A = R_A(p_B)$ and $p_B = R_B(p_A)$. Plot both best-response functions where you denote the vertical axis by p_A and the horizontal axis by p_B. Indicate whether these best-response functions are upward or downward sloping.

(vii) Solve for the Nash equilibrium prices, p_A and p_B, the corresponding quantities, q_A and q_B, as well as the equilibrium profit levels, π_A and π_B.

(viii) Compare with Nash equilibrium of the quantity game to the Nash equilibrium of the price game with respect to the quantity produced, prices, and profit levels. Explain these differences.

(b) Aike (Brand A) and Beebok (Brand B) are leading brand names of fitness shoes. The direct demand functions facing each producer are given by

$$q_A(p_A, p_B) = 180 - 2p_A + p_B \quad \text{and} \quad q_B(p_A, p_B) = 120 - 2p_B + p_A.$$

Assume zero production cost ($c_A = c_B = 0$), and solve the following problems:

(i) Derive the price best-response function of firm A as a function of the price set by firm B, $p_A = BR_A(p_B)$. Show your derivations, and draw the graph associated with this function.

(ii) Derive the price best-response function of firm B as a function of the price set by firm A, $p_B = BR_B(p_A)$. Show your derivations, and draw the graph associated with this function.

(iii) Solve for the Nash-Bertrand equilibrium prices, $\langle p_A^b, p_B^b \rangle$. Then, compute the equilibrium output levels $\langle q_A^b, q_B^b \rangle$, the equilibrium profits $\langle \pi_A^b, \pi_B^b \rangle$, and aggregate industry profit $\Pi^b = \pi_A^b + \pi_B^b$.

(iv) Suppose now that the two producers hold secret meetings in which they discuss fixing the price of shoes to a uniform (brand-independent) level of $p = p_A = p_B$. Compute the price p which maximizes joint industry profit, $\pi_A + \pi_B$. Then, compute aggregate industry profit and compare it to the aggregate industry profit made under Bertrand competition which you computed in part (iii).

(v) Suppose now that the two firms merge. However, they decide to keep selling the two brands separately and charge, possibly, different prices. Compute the prices p_A and p_B which maximize joint industry profit, $\pi_A + \pi_B$. Then, compute aggregate industry profit and compare it to the aggregate industry profit made under Bertrand competition which you have already computed under separate ownership.

(c) Consider the following system of inverse demand functions:

$$p_A = 120 - 2q_A - q_B \quad \text{and} \quad p_B = 120 - 2q_B - q_A.$$

Assume that firms A and B do not bear any production costs (that is, $c_A = c_B = 0$). Solve the following problems:

(i) Solve for the Nash-Cournot equilibrium quantity levels q_A^c and q_B^c. Also, compute the resulting equilibrium market prices p_A^c and p_B^c, and profits π_A^c and π_B^c.

(ii) Invert the above system of inverse demand functions to obtain the direct demand functions which map quantities to prices (instead of prices as functions of quantities sold). That is compute the coefficients a, b and c of the equations given by $q_A = a - bp_A + cp_B$ and $q_B = a - bp_B + cp_A$.

(iii) Solve for the Nash-Bertrand equilibrium price levels p_A^b and p_B^b. Also, compute the resulting equilibrium quantity levels q_A^b and q_B^b, and profits π_A^b and π_B^b.

(iv) Compare the equilibrium quantities sold, prices, and profit level under the Cournot game to the levels obtained under the Bertrand game. Explain the differences.

(d) Consider the following system of inverse demand functions:

$$p_A = 120 - 4q_A - 2q_B \quad \text{and} \quad p_B = 120 - 4q_B - 2q_A.$$

Assume that firms A and B do not bear any production costs (that is, $c_A = c_B = 0$). Solve the following problems:

(i) Solve for the Nash-Cournot equilibrium quantity levels q_A^c and q_B^c. Also, compute the resulting equilibrium market prices p_A^c and p_B^c, and profits π_A^c and π_B^c.

(ii) Invert the above system of inverse demand functions to obtain the direct demand functions which map quantities to prices (instead of prices as functions of quantities sold). That is compute the coefficients a, b and c of the equations given by $q_A = a - bp_A + cp_B$ and $q_B = a - bp_B + cp_A$.

(iii) Solve for the Nash-Bertrand equilibrium price levels p_A^b and p_B^b. Also, compute the resulting equilibrium quantity levels q_A^b and q_B^b, and profits π_A^b and π_B^b.

(iv) Compare the equilibrium quantities sold, prices, and profit level under the Cournot game to the levels obtained under the Bertrand game. Explain the differences.

(e) Aike (Brand A) and Beebok (Brand B) are leading brand names of fitness shoes. The inverse demand functions for these brands are

$$p_A = 80 - \frac{3}{2}\,q_A - q_B \quad \text{and} \quad p_B = 80 - \frac{3}{2}\,q_B - q_A.$$

Assume that firms A and B do not bear any production costs (that is, $c_A = c_B = 0$). Solve the following problems:

(i) Solve for the Nash-Cournot equilibrium quantity levels q_A^c and q_B^c. Also, compute the resulting equilibrium market prices p_A^c and p_B^c, and profits π_A^c and π_B^c.

(ii) Invert the above system of inverse demand functions to obtain the direct demand functions which map prices to quantities (instead of prices as functions of quantities sold). That is compute the coefficients a, b and c of the equations given by $q_A = a - bp_A + cp_B$ and $q_B = a - bp_B + cp_A$.

(iii) Solve for the Nash-Bertrand equilibrium price levels p_A^b and p_B^b. Also, compute the resulting equilibrium quantity levels q_A^b and q_B^b, and profits π_A^b and π_B^b. Compare the equilibrium quantities sold, prices, and profit level under the Cournot game to the levels obtained under the Bertrand game. Explain the differences.

(f) The inverse demand functions for orange juice (O) and grape juice (G) are given by: $q_O = 24 - 2p_O + p_G$ and $q_G = 12 - 2p_G + p_O$, where p_O and p_G denote the price of orange juice and grape juice, respectively.

JUICIANA company is the sole producer and seller of both orange and grape juice. JUICIANA has no cost of production ($c_O = c_G = 0$), and maximizes the monopoly profit from selling in both markets, $\Pi = \pi_O + \pi_G$.

(i) Suppose that JUICIANA is restricted to setting a single uniform price for all juices, so that $p = p_O = p_G$. Compute JUICIANA's profit-maximizing price p, quantities sold q_O and q_G, and the resulting profit.

(ii) Answer the above question assuming that JUICIANA is now free set different prices $p_O \neq p_G$ for the juices it sells. Which pricing policy (uniform versus nonuniform) yields a higher profit for JUICIANA?

Set # 11: Location Models

(a) Ann Arbor and Ypsilanti are very similar cities, because each city has exactly one McDonald's. Ann Arbor has $N_A = 120$ residents and Ypsilanti has $N_Y = 120$ residents. Each demands one hamburger. A resident of Ann Arbor who wishes to buy a hamburger in Ypsilanti must bear a transportation cost of $T_A = \$1$. Similarly, a resident of Ypsilanti who wishes to buy a hamburger in Ann Arbor must bear a transportation cost of $T_Y = \$2$. Suppose that McDonald's has the technology of producing hamburgers at no cost. Answer the following questions.

(i) Suppose that the Ypsilanti store charges p_Y for a hamburger. Derive the demand function facing the Ann Arbor store as a function of its price, p_A. *Hint*: See equation (7.35) on p.159.

(ii) Suppose that the Ann Arbor store charges p_A for a hamburger. Derive the demand function facing the Ypsilanti store as a function of its price, p_Y.

(iii) Suppose that the Ypsilanti store charges p_Y. Compute the profit of the Ann Arbor store when this store undercuts the price set by the Ypsilanti (so that all consumers in the area purchase from the Ann Arbor Store.

(iv) Suppose that the Ann Arbor store charges p_A. Computes the profit of the Ypsilanti store when this store undercuts the price set by the Ann Arbor (so that all consumers in the area purchase from the Ypsilanti Store.

(v) Compute the Undercut-proof equilibrium prices p_A and p_Y. Explain why in equilibrium store A charges lower price than store Y.

(vi) Compute the equilibrium profit levels π_A and π_Y.

(b) Ann Arbor and Ypsilanti are very similar cities, because each city has exactly one McDonald's. Ann Arbor has $N_A = 200$ residents and Ypsilanti has $N_Y = 200$ residents. Each resident demands one hamburger. A resident of Ann Arbor who wishes to buy a hamburger in Ypsilanti must bear a transportation cost of $T_A = \$3$. Similarly, a resident of Ypsilanti who wishes to buy a hamburger in Ann Arbor must bear a transportation cost of $T_Y = \$3$. Solve the following problems:

   (i) Solve for the undercut-proof equilibrium prices p_A^U and p_Y^U and profit levels π_A^U and π_Y^U assuming that McDonald's has the technology for producing hamburgers at no cost. Show your derivation.

   (ii) Answer the previous question assuming now that McDonald's in Ann Arbor bears a cost of $\$1$ of producing each hamburger, whereas McDonald's in Ypsilanti bears a cost of $\$4$ of producing each hamburger. Show your derivation.

Set # 12: Choice of Location

All the problems below are variations of the example given in Section 7.3.3 of the textbook.

(a) Suppose there are only 2 firms (instead of three firms as in Section 7.3.3 of the textbook). Consider the following two stage game. In stage $t = 1$, firm 1 chooses it location, x_1, on the linear street $[0, 1]$. In stage $t = 2$, firm 2 chooses its location x_2 after observing and taking x_1 as given. Compute the subgame perfect equilibrium location strategies of the two firms, and the equilibrium market shares, π_1 and π_2.

(b) Suppose there are 3 firms as in Section 7.3.3. However, let firm 1 be located at $x_1 = 1/3$. Solve for the SPE locations of firms 2 and 3 and the equilibrium market shares of all three firms, π_1, π_2, and π_3 .

(c) Same problem as the above, but assume that $x_1 = 1/2$.

(d) Same problem as the above, but assume that $x_1 = 0$.

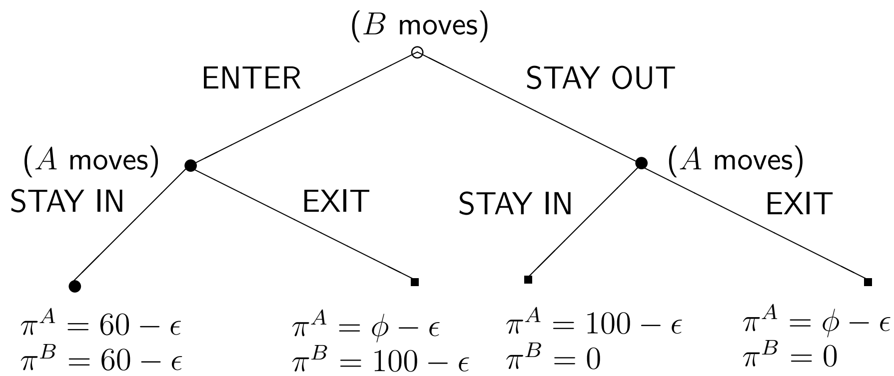
(e) Same problem as the above, but assume that $x_1 = 1$.

Set # 13: Mergers and Entry Barriers

(a) Describe the three main classifications of mergers.

(b) Demonstrate a case in which a *vertical* merger can reduce or even eliminate competition in the market for the final good. Draw a picture explaining graphically what some economists fear may be a consequence of a *vertical* merger.

(c) Consider the entry-exit two-stage game in which firm A is the incumbent firm that faces a potential entrant firm B. In stage I, firm B decides whether to enter into A's market or whether to stay out. The cost of entry is denoted by ϵ. In stage II, the established firm, firm A, decides whether to stay in the market or exit.



The game tree reveals that firm A can recover some of its sunk entry cost by selling its capital for the price ϕ, where $0 \le \phi \le \epsilon$. Solve the two problems:

(i) Compute the subgame-perfect equilibrium strategies of firms B and A assuming that $\epsilon < 60$. Prove your answer.

(ii) Answer the above assuming that $60 < \phi \le \epsilon < 100$.

(d) Consider two differentiated brands, A and B, with the following direct demand functions:

$$q_A = 120 - 2p_A + p_B \quad \text{and} \quad q_B = 120 - 2p_B + p_A$$

Suppose both firms initially charge $p_A = p_B = \$60$. Using the SSNIP test, solve the following problems:

(i) Suppose unit production costs are $c_A = c_B = \$40$. Find whether the market for brand A should be considered as the "relevant" market or whether the markets for brand A and B combined should be considered as the "relevant" market.

(ii) Answer the above assuming that $c_A = c_B = \$30$.

(e) The direct demand functions for orange juice (O), grape juice (G), and tomato juice (T) are given by

$$q_O = 90 - 2p_O + p_G + p_T, \quad q_G = 100 - 2p_G + p_O + p_T, \quad \text{and} \quad q_T = 120 - 2p_T + p_O + p_G.$$

where p_O, p_G, and p_T denote the price of orange juice, grape juice, and tomato juice, respectively. The cost of producing one unit of orange juice, grape juice, and tomato juice, are the same and given by $c_O = c_G = c_T = 20$.

Assume that the ORANGADA company is the sole producer and seller of orange juice, and that the following market prices are observed: $p_O = 60$, $p_G = p_T = 40$.

Use the SSNIP test to determine which of the following markets should be considered as the "relevant market" for the ORANGADA company: (i) Orange juice only, (ii) Orange and grape juice, (iii) Orange, grape, and tomato juice, or (iv) broader market (more juices should be included).

(f) The cost function of providing car rental services is given by $TC(Q) = 40 + 10Q$ where Q is the number of customers served (who each rents one car). All existing and potential firms have the same technology (hence, the same cost function).

The demand for car rental service in Inchilanti (a small town somewhere in the Midwest) is: $p = 22 - 0.5Q$. There is only one incumbent operator and many potential entrants. The incumbent firm would like to avoid competition from potential entrants. Compute the price charged by the incumbent firm and the quantity of service in a *contestable market equilibrium*.

Set # 14: Innovation & Patent Races

(a) The inverse market demand function for 20 Amps vacuum cleaners is given by $p = 120 - 0.5Q$. Initially, firm A and firm B produce at equal unit cost, $c_0 = \$80$. After heavy investment in R&D, firm A has managed to reduce its unit production cost to $c_1 < \$80$. For which values of c_1, firm A's innovation can be classified as drastic (or major), and for which values of c_1 the innovation is classified as minor. Prove your result using the definition.

(b) The inverse market demand function for MP3 players is given by $p = 240 - 2Q$. Initially, firm A and firm B produce at equal unit cost, c_0. After investing heavily in R&D, firm A has managed to reduce its unit production cost to $c_1 = \$40 < c_0$. For which values of c_0, firm A's innovation can be classified as drastic (major), and for which values of c_0 the innovation is classified as nondrastic (minor). Prove your result using the definition.

(c) The value of the patent for producing a pill for curing the Michigan Flu is estimated to be around $V = \$150$. Suppose that three firms, labeled $i = 1, 2, 3$, engage in an innovation race for this pill. If only one firm discovers, it is expected to earn $V = \$150$. If two firms discover at the same firm, the value of the patent is shared, so each firm earns $V/2 = \$75$. If all the three firms discover at the same, each firm earns $V/3 = \$50$.

The firms are privately and separately owned. Each firm has a probability of $1/3$ discovering the cure provided that it invests an amount of $I = \$40$ in constructing research lab. If a firm does not invest, the probability of discovery is zero. Solve the following problems.

(i) Compute the expected profit of firm 1 assuming that firm 1 is the only firm to invest in a lab.

(ii) Compute the expected profit of firm 1 assuming that only firm 1 and firm 2 engage in R&D.

(iii) Compute the expected profit of firm 1 assuming that all three firms engage in R&D.

(iv) Conclude how many firms will invest in a lab and engage in R&D.

(v) Compute the level of social welfare assuming that only one firm engages in R&D.

(vi) Compute the level of social welfare assuming that exactly two firms engage in R&D.

(vii) Compute the level of social welfare assuming that all three firms engage in R&D.

(viii) Conclude what is the socially optimal number of firms that should engage in R&D. Is there a market failure?

(d) The value of the patent for producing a pill which can cure the Michigan Flu is estimated to be around $V = \$16$ (all numbers are in millions of dollars). There is only one company in the entire world that invests in R&D in order to discover this pill. This company has to choose between two options:

Option A: Investing in **two** independent (expensive) labs. Each lab costs \$2 to operate. The probability that each lab independently discovers the pill is 0.75.

Option B: Investing in **three** independent (cheap) labs. Each lab costs \$1. The probability that each lab independently discovers the pill is 0.5.

Compute which option maximizes the expected profit of this firm?

(e) Three separate labs consider engaging in R&D for developing an anti-laziness pill (intended to used mainly by students). The value of the patent on this pill is estimated to be $V = \$240$. If more than one firm discovers the patent, the firms equally share the prize ($240/2 = \$120$ or $240/3 = \$80$).

Lab A**:** Cost of the lab is $I_A = \$40$, probability of discovery is $1/3$.

Lab B**:** Cost of the lab is $I_B = \$60$, probability of discovery is $1/3$.

Lab C**:** Cost of the lab is $I_C = \$70$, probability of discovery is $1/3$.

Solve the following problems:

(i) Does there exist an equilibrium in which all three labs enter the R&D competition? Prove your answer!

(ii) Does there exist an equilibrium in which exactly two labs enter the race while a third lab finds it profitable to stay out? How many such equilibria exist? Prove!

(iii) Does there exist an equilibrium in which only one lab enters the race?

(iv) Suppose now that Google has purchased all the 3 labs so all three labs are now under a single ownership. Compute how many and which lab(s) will be operated and which will be closed down.

(f) Two separate labs consider engaging in R&D for developing an anti-laziness pill (intended to used mainly by students). The value of the patent on this pill is estimated to be $V = \$240$. If two labs discover the patent, the firms equally share the prize so each earns $240/2 = \$120$.

Lab A**:** Cost of the lab is $I_A = \$40$, probability of discovery is $1/4$.

Lab B**:** Cost of the lab is $I_B = \$60$, probability of discovery is $1/3$.

Solve the following problems:

(i) Solve for the equilibrium number of labs which enter the R&D race.

(ii) Suppose that both labs are now under a single ownership (or have been nationalized by the local government). Which lab(s) will be operated under this joint ownership?

(g) Three separate labs consider engaging in R&D for developing an anti-laziness pill (intended to used mainly by students). The value of the patent on this pill is estimated to be $V = \$640$. If more than one firm discovers the patent, the firms equally share the prize ($640/2$ or $640/3$).

These profit-maximizing labs are privately and separately owned. Each lab has a probability $1/4$ of discovering the cure provided that it invests an amount of $I = \$120$ in constructing a research lab. If a firm does not invest, the probability of discovery is zero. Solve the following problems.

(i) Compute the equilibrium number of labs engaging in R&D.

(ii) Now suppose that all three labs have been purchased by a single investor. How many labs will be operated under the new single ownership?

(h) Lazy Labs Inc. conducts medical research on a certain anti-laziness pill. It is estimated that the lifetime value of the patent on this pill would be $V = \$1024$ (in millions). The company can invest in many separate identical labs. Each lab costs $\$16$ million to operate, and each has a probability $\alpha = 0.5$ of discovery. Find the profit-maximizing number of separate labs that the company should be investing in. Prove your result.

(i) From your reading of patent law and class discussion, list the four types of patents on inventions that can be granted. Also, list the three requirements in order for an innovation to be qualified for a patent

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Set # 15: Subsidies to R&D

(a) The table exhibits the profit levels of Airbus and Boeing under the four possible market outcomes.

BOEING \ AIRBUS	Produce		Don't Produce	
Produce	-10	-10	50	0
Don't Produce	0	50	0	0

Solve the following problems:

(i) Calculate the minimal subsidy to Airbus that will ensure that Airbus will develop the mega-carrier. Explain!

(ii) Suppose that the EC provides Airbus with 15 units of money as a subsidy. Which subsidy by the US government to Boeing would guarantee that Boeing will develop this mega-carrier?

(iii) Suppose that the EC provides Airbus with 15 units of money as a subsidy. Is there any level of subsidy given by the US government that would deter Airbus from developing this airplane.

(iv) From you answer to the previous question, conclude whether the world benefits by having both governments subsidizing their own aircraft manufacturing firms. Explain!

(b) Suppose that there only two civilian aircraft manufacturers in the entire world, and that the world consists of two countries, the US and the EC. Suppose further that each firm is a profit maximizer and has a binary choice: develop (and produce) or Don't develop (and don't produce). The following table demonstrates the profit levels of each firm under the four possible market outcomes. Suppose that the EC government promises Airbus

BOEING \ AIRBUS	Produce		Don't Produce	
Produce	-10	-15	50	0
Don't Produce	0	50	0	0

to 'contribute' 16 ($ bil) if Airbus develops and produces the new airplane. Then, there exists a certain amount of US subsidy to Boeing that would deter Airbus from developing and producing the new airplane.

(c) Suppose that there only two firms capable of making TV sets in the entire world, and that the world consists of two countries, the US and Japan. Suppose that the US manufacturer

		ZENITH	
		Produce	Don't Produce
SHARP	Produce	2 3	50 0
	Don't Produce	0 50	0 0

is called ZENITH, and the Japanese firm is called SHARP. Each firm is considering developing the future High-Definition-TV (HDTV) having a resolution of 1275 lines (compared with the 575 lines of the current TV standard (called NTSC)).

Suppose further that each firm has a binary choice: develop (and produce) or Don't develop (and don't produce). The Table below demonstrates the profit levels of each firm under the four possible market outcomes.

What level of subsidy should MITI (Japan's Ministry of Intern'l Trade and Industry) provide SHARP in order to deter ZENITH from developing its HDTV (Can MITI succeed in deterring ZENITH?).

(d) The European aircraft producer Airbus and the American producer Boeing consider developing a new air-to-air refueling tanker. The table below exhibits the profit levels of Airbus and Boeing under the four possible market outcomes.

		AIRBUS	
		Develop	Don't Develop
BOEING	Develop	2 -10	50 0
	Don't Develop	0 20	0 0

Is there any level of R&D subsidy that the EU government can provide Airbus that would *deter* Boeing from developing this tanker? Prove your answer.

Set # 16: Advertising

(a) Compute the profit-maximizing advertising budget for a monopoly firm using the following three important pieces of information:

(i) The company is expected to sell $50 million worth of the product.

(ii) It is estimated that a 1% increase in the advertising budget would increase the quantity sold by 0.04%.

(iii) It is also estimated that a 1% increase in the product's price would reduce quantity sold by 0.2%.

(b) A monopoly selling internet services spends 20% of its sales revenue on persuasive advertising. The price elasticity is $\epsilon_p = -2$. Using the Dorfman-Steiner condition, determine what is the advertising elasticity of demand in this market.

Set # 17: Monopoly & Durability

(a) Assume that there are two technologies for producing car batteries (for large trucks):

Long-lasting blades: Each battery lasts for 40 months. Each costs $c_L = \$240$ to produce.

Short-lasting blades: Each battery lasts for 30 months. Each costs $c_S = \$180$ to produce.

Suppose that all truck drivers are identical. Each truck driver is willing to pay no more than $v = \$20$ for a one-month services obtained from a car battery.

When a battery needs to be replaced, the driver must drive to the shop and spend some time there to get it replaced. The cost to the driver associated with this loss of time (loss of business) is $T = \$120$. Solve the following problems:

(i) Which type of battery will be produced by a monopoly seller? *Hint*: For each type of battery, first compute the maximum price per use that the monopoly can charge consumers, taking into account the time cost associated with replacing a battery. Then, compare the profit per use made by this monopoly manufacturer, either on a per month basis, or any other common time denominator.

(ii) Which type of battery will be produced and sold in a competitive industry?

(b) Assume that there are two technologies for producing batteries for heavy trucks:
Long-lasting batteries: Each battery lasts for 60 months. Each costs $c_L = \$120$ to produce.
Short-lasting batteries: Each battery lasts for 40 months. Each costs $c_S = \$80$ to produce.

Suppose that all car owners are identical. Each truck owner is willing to pay no more than $v = \$30$ for a one-month service obtained from a car battery. Assume that truck owners do NOT bear any cost of time and transportation when they go to the shop to replace a battery. Solve the following problems:

(i) Which type of battery will be produced by a monopoly seller?

(ii) Which type of battery will be produced and sold by a competitive industry?

Set # 18: Warranties

(a) A monopoly offers a product for sale. The product costs $c = \$60$ to produce. The product may fail with probability 0.2, hence it is fully operative with probability $\rho = 0.8$. This probability is public information in the sense that it is known to the seller and all buyers.

The product can either be fully functioning or totally defective. Consumers are willing to pay up to $V = \$120$ for a fully-functioning product. If the product is found to be defective, consumers do not gain any utility. Solve the following problems.

(i) What is the maximum price a consumer will be willing to pay for a product with no warranty.

(ii) Determine the profit made from the sale of each product assuming that the monopoly sells without any warranty.

(iii) Suppose now the monopoly provide a full-replacement warranty. That is, the monopoly seller replaces the product if it is found to be defective. If the replacement product is also found to be defective, it also gets replaced, and so on. Compute the monopoly's expected total production cost under a full-replacement warranty

(iv) Determine the profit made from the sale of each product under the full-replacement warranty.

(v) Suppose now the monopoly provides a one-time replacement warranty. That is, the monopoly seller replaces the product if it is found to be defective. However, if the replacement product also fails, the seller does not replace it. Compute the profit the monopoly makes from the sale of each product. *Hint*: First determine consumers' maximum willingness to pay for a product with a one-time replacement warranty. Second, determine the monopoly's expected total production cost under this type of warranty.

(vi) Determine which type of warranty is most profitable to the monopoly seller.

(b) A monopoly offers a product for sale. The product costs $c = \$10$ to produce. The product may fail with probability 0.2, hence it is fully operative with probability $\rho = 0.8$. This probability is public information in the sense that it is known to the seller and all buyers.

The product can either be fully functioning or totally defective. Consumers are willing to pay up to $V = \$40$ for a fully-functioning product. If the product is found to be defective, consumers do not gain any utility. Solve the following problems:

(i) Compute the monopoly price p^{NW} and profit π^{NW} assuming that the monopoly does not provide any warranty to customers.

(ii) Compute the monopoly price p^{FW} and profit π^{FW} assuming that the monopoly provides a full-replacement warranty.

(iii) Compute the monopoly price p^{PW} and profit π^{PW} assuming that the monopoly provides a partial cash-back warranty which states that the seller will pay $\phi = \$20$ to the consumer if the product is found to be defective.

(iv) Compute the monopoly price p^{MW} and profit π^{MW} assuming that the monopoly provides a full money-back warranty which states that the seller will return the full amount p^{MW} paid by the consumer if the product is found to be defective.

(c) A monopoly offers a product for sale. The product costs $c = \$60$ to produce. The product may fail with probability $1/4$, hence it is fully operative with probability $\rho = 3/4$. This probability is public information in the sense that it is known to the seller and to all buyers.

The product can be either fully functioning or totally defective. Consumers are willing to pay up to $V = \$120$ for a fully-functioning product. If the product is found to be defective, consumers do not gain any utility. Solve the following problems.

(i) Compute the monopoly's price and profit level assuming that the monopoly does not provide any warranty.

(ii) Suppose now that the monopoly provides a repair warranty for the case where the product is found to be defective after purchase. The cost of a repair (borne by the monopoly) is $R = \$40$. Compute the monopoly price and profit level under this repair warranty. Assume that a repaired product becomes a fully-functioning product (which cannot break).

(d) A monopoly offers a product for sale. The product costs $c = \$60$ to produce. The product may fail with probability 0.5, hence it is fully operative with probability $\rho = 0.5$. This probability is public information in the sense that it is known to the seller and all buyers.

The product can either be fully functioning or totally defective. Consumers are willing to pay up to $V = \$240$ for a fully-functioning product. If the product is found to be defective, consumers do not gain any utility. Solve the following problems.

(i) The monopoly provides a "twice-replacement" warranty. That is, if the original purchase is found to be defective, the consumer can have the product replaced free of charge. If the replacement product is also found to be defective, it also gets replaced free of charged. However, the monopoly will not replace the replacement of the replacement product if it also found to be defective. Compute monopoly's profit-maximizing price and the resulting expected profit.

(ii) Now suppose the monopoly provides a money-back guarantee (instead of the twice-replacement warranty). Compute monopoly's profit-maximizing price and the resulting expected profit.

Set # 19: Peak-load pricing

(a) Congratulations, you have been appointed the CEO of LuftMama Airlines. The passengers' inverse demand functions facing LuftMama during summer and winter are $p_S = 12 - q_S/2$ and $p_W = 24 - 2q_W$, respectively. There are no fixed costs, but the marginal capacity cost and the marginal operating costs are given by $r = c = \$2$. Solve the following problems:

(i) Compute the summer and winter airfares assuming that LuftMama implements a peak-load pricing structure.

(ii) During an election campaign, the transportation minister in your country declares that if her party gets reelected, she will require all airlines to fix their airfares during the entire year, so $p_{S,W} \stackrel{\text{def}}{=} p_S = p_W$. Compute the profit-maximizing season independent price $p_{S,W}$, and compare the resulting profit level to the profit generated by peak-load pricing.

(b) The passengers' inverse demand functions facing an airline company during summer and winter are $p_S = 36 - q_S/2$ and $p_W = 36 - q_W$, respectively. The marginal operating cost is $c = \$2$, and the marginal capacity costs is $r = \$4$. Solve the following problems:

(i) Compute the summer and winter airfares assuming that this airlines implements a peak-load pricing structure.

(ii) Suppose this airlines is not allowed to price discriminate between seasons and must fix the airfare throughout the entire year, so $p_{S,W} \stackrel{\text{def}}{=} p_S = p_W$. Compute the profit-maximizing season independent price $p_{S,W}$.

(c) Electata is the sole provider of electricity in a remote island near Africa. The company debates how much to invest in electricity generation capacity (measured in Kilowatt/hour). There are no operating costs ($c = 0$) because Electata's generators are installed on river dams. However, the capacity cost is $r = 4$ per Kilowatt/hour generation capacity.

The daytime and nighttime inverse demand functions for electricity are $p_D = 12 - 0.5\,q_D$ and $p_N = 24 - 2q_N$, respectively. Compute Electata's profit-maximizing investment level in capacity K, the daytime price of electricity p_D, and nighttime price, p_N.

(d) Congratulations, you have been appointed the CEO of LuftMama Airlines. The passengers' inverse demand functions facing LuftMama during summer and winter are $p_S = 12 - q_S/2$ and $p_W = 24 - 2q_W$, respectively. There are no fixed costs, but the marginal capacity cost is $r = \$3$ and the marginal operating costs is $c = \$4$.

Compute the summer and winter airfares (and the resulting profit over a cycle of two seasons) assuming that LuftMama implements a peak-load pricing structure.

Set # 20: Tying

(a) Congratulations, you have been appointed the general manager of the Paradise Hotel, which is the only hotel on Paradise Island, located somewhere in the Pacific Ocean. This hotel also owns the only restaurant in town that serves breakfast. As the hotel manager, your first responsibility is to decide whether to include breakfast in the standard hotel rate or to charge extra for breakfast. The table below shows the willingness to pay of type 1 and type 2 hotel guests for hotel room (R) and for breakfast (B), as well as the expected number of guests of each type and the hotel's marginal cost of providing each service.

Guest Type	Hotel Room (R)	Breakfast (B)	Expected # guests
Type 1	$\$100$	$\$5$	200
Type 2	$\$60$	$\$10$	800
Marginal Cost	$\mu_R = \$40$	$\mu_B = \$2$	

Solve the following problems:

(i) Compute the hotel's profit-maximizing room rate p_R, breakfast price p_B, and resulting profit π^{NT}, given that both services are sold separately (untied).

(ii) Suppose now that the hotel rents a room together with breakfast. Compute the package's profit-maximizing price p_{RB} and the corresponding profit level π^{PT}. Conclude whether the hotel should tie the two services in a single package or sell them separately.

(iii) Solve part (a) assuming that the expected number of type 2 guests falls from 800 to $N_2 = 200$ guests, whereas the expected number of type 1 guests remains $N_1 = 200$.

(iv) Solve part (b) under the modification made in part (c).

(b) Consider a monopoly cable TV operator with unit costs and potential subscribers described in the following table.

Consumer Type	CNN	BBC	HIS	# Subscribers
Type 1	$\$11$	$\$2$	$\$3$	$N_1 = 100$
Type 2	$\$11$	$\$2$	$\$6$	$N_2 = 100$
Type 3	$\$2$	$\$11$	$\$3$	$N_3 = 100$
Type 4	$\$2$	$\$11$	$\$6$	$N_4 = 100$
Marginal Cost	$\mu_C = \$1$	$\mu_B = \$1$	$\mu_H = \$1$	

The marginal cost of a channel is the fee that the cable operator must pay to the content provider of this channel for each consumer subscribed to this channel. Solve the following problems:

(i) Compute the operator's profit-maximizing subscription rates p_C, p_B, and p_H and the resulting profit π^{NT}, given that each channel is sold separately (no tying).

(ii) Compute the profit-maximizing subscription rate p_{CBH} and the corresponding profit level π^{PT}, assuming that the operator offers only subscriptions for a single package composed of all three channels (pure tying).

(iii) Can you find alternative packages that would generate a higher profit than that achieved by pure tying and no tying?

(c) The problem facing the manager of the PARADISE Hotel is whether to tie breakfast and a visit to the gym with the room rental (pure tying) or whether to sell the three services separately (no tying). The guests' willingness to pay for each service and the hotel's marginal cost of providing each service are given in the Table below.

Type	Room (R)	Breakfast (B)	Gym (G)	# Guests
Type 1	$\$100$	$\$5$	$\$10$	200
Type 2	$\$60$	$\$10$	$\$10$	800
Marginal Cost	$\mu_R = \$40$	$\mu_B = \$2$	$\mu_G = \$0$	

Solve the two problems:

(i) Compute the hotel's profit-maximizing room rate p_R, breakfast price p_B, gym entrance fee p_G, and resulting profit π^{NT}, given that each service is sold separately (no tying).

(ii) Suppose now that the hotel sells all three services in one package (pure tying). Compute the package's profit-maximizing price p_{RBG} and the corresponding profit level π^{PT}. Conclude whether the hotel should tie the three services in a single package or sell them separately.

Set # 21: Dealerships

(a) The inverse market demand for HUMMERs in NYC is $p = 120 - 2Q$. The manufacturer licenses a single dealer to sell HUMMERs in NYC. Therefore, the dealer acts as a monopoly in NYC in this market. The manufacturer sells each HUMMER to the dealer for $\$d \geq c$, where $c = \$40$ is the cost of producing one HUMMER. In addition, the manufacturer levies a fixed fee of $\$\phi$ on the dealership.

Consider a two-stage game in which in Stage I the HUMMER manufacturer sets the per-unit price charged to the dealer d, and the fixed fee ϕ. In Stage II the dealer determines the amount of HUMMERs to sell to maximize the dealership's profit.

Solve the following problems:

(i) Suppose the unit price the manufacturer charges the dealer d is given (determined in the first stage of the game). Also, suppose that the manufacturer does not charge the dealership any fixed fee, that is $\phi = 0$. Compute the dealer's profit-maximizing sales of HUMMERs as a function of d.

(ii) Compute the manufacturer's profit-maximizing price d it charges the dealer for each HUMMER sold.

(iii) Compute quantity of HUMMERs sold by the dealer, and the price p charged to buyers.

(iv) Compute the profit made by the dealer and the manufacturer.

(v) Can you find a price d and a fixed fee ϕ which the manufacturer charges the dealer so that, both, the dealer and the manufacturer earn higher profit compared with the profits you computed in the previous question?

(b) The inverse market demand for TOASTER-PHONEs in NYC is $p = 36 - Q$. The manufacturer licenses a single dealer to sell this brand in NYC. Therefore, the dealer acts as a monopoly in the NYC market. The manufacturer sells each TOASTER-PHONE to the dealer for $\$d \geq c$, where $c = \$20$ is the cost of producing one TOASTER-PHONE. In addition, the manufacturer may levy a fixed fee of $\$\phi$ on the dealership.

Consider a two-stage game in which in Stage I the manufacturer sets the per-unit price charged to the dealer d, and the fixed fee ϕ. In Stage II the dealer determines the quantity sold as to maximize the dealership's profit.

(i) Compute the dealer's price p, quantity sold Q, and profit π^d as a function of d and ϕ.

(ii) Suppose the manufacturer does not charge the dealer any fixed fee, $\phi = 0$. Compute the dealer's price d which maximizes the manufacturer's profit. Then, compute the equilibrium consumer price p, and the profits made by the manufacturer π^m and the dealer π^d.

(iii) Can the manufacturer set a different contract with the dealer so that both, the manufacturer and the dealer, make a higher profit. Formally, find d and ϕ which generates higher profit levels, π^m and π^d, compared to the levels you computed in (7b).

THE END

