1) [15 points] In a certain country over some period of time, the annual growth rates of output per worker, physical capital per worker, human capital per worker, and technology were as follows:

\[ \hat{y} = 4\% \]
\[ \hat{k} = 5\% \]
\[ \hat{h} = 2\% \]
\[ \hat{T} = 2\% \]

The production function was

\[ y = A k^\alpha h^{1-\alpha} \]

where \( \alpha = \frac{1}{3} \)

What was the growth rate of efficiency?

2) [15 points] Technological progress in industries in which the price elasticity of demand is low contributes _______ to aggregate economic growth than does technological progress in industries in which the price elasticity of demand is high.

Should the word filling in the blank be “more” or “less”? Explain in three or four sentences.
3) [30 points] Consider the two country model of technological progress from Chapter 8. Country 1 and Country 2 have labor forces of equal size. Assume for convenience that the labor force in each country is equal to one. The parameters of the model are

Cost of invention: $\mu_i = 10$

R&D in country 1: $\gamma_{A,1} = 0.2$

R&D in country 2: $\gamma_{A,2} = 0.1$

The cost of copying function has the usual shape, but I have intentionally not given you an exact specification of that equation.

The two countries are in a steady state in terms of their relative levels of technology. Suddenly, the value of $\gamma_{A,2}$ triples, while nothing happens to the value of $\gamma_{A,1}$.

A) [15 points] What will the growth rate of technology in Country 2 be immediately after this change in R&D?

B) [15 points] What will the growth rate of technology in Country 2 be once a new steady state has been reached?

4) [15 points] In the country of Oz, there are only two goods that are produced and consumed: peanut butter and jelly. These goods are always consumed in a ratio of 1-to-1, that is, one tablespoon peanut butter with one tablespoon of jelly. The production functions for these goods are

$$Y_{jelly} = L_{jelly}$$

$$Y_{peanutbutter} = 2 \times L_{peanutbutter}$$

Oz is a very small country. It has two very large neighbors, Freedonia and Sylvania. Oz is thinking of establishing free trade with one of these countries (the two countries do not trade with each other). In Freedonia, the price of peanut butter is equal to the price of jelly. In Sylvania, the price of peanut butter is one quarter of the price of jelly. There is no cost to transporting either peanut butter or jelly between countries.

Which country would it be better for Oz to establish free trade with? You must explain your answer, including any appropriate calculations.
5) [25 points] Consider a country in which there are two sectors, called Sector 1 and Sector 2. The production functions in the two sectors are:

\[ Y_1 = 24 \times L_1^{1/2} \]

\[ Y_2 = 24 \times L_2^{1/2} \]

Workers in both sectors are paid their marginal products. The total number of workers in the economy is \( L = L_1 + L_2 = 25 \).

You are a powerful bureaucrat. You have the power to issue licenses that give workers the ability to work in sector 1. Without a license, a worker cannot work in this sector. Of course, being corrupt, you have decided to sell these licenses and keep the revenue from the sales for yourself. Let \( X \) be the price that you charge for one license.

A) [10 points] Draw a picture showing the allocation of labor that will result from this action, and how it relates to the optimal allocation of labor between sectors. Also show on this picture the revenue that this scheme will generate for you.

B) [10 points] Write the equation that relates the number of people who will end up working in sector 1 (i.e. the value of \( L_1 \)) to the size of the license fee, \( X \). This will be a nasty looking equation, and you should not attempt to solve it (in other words, you don’t have to get all the value of \( L_1 \) to one side of the equation or anything like that).

C) [5 points] Confirm that the following values solve the equation that you just derived: \( X=1 \), \( L_1 = 9 \).