Econ 1560 First Midterm Exam

Instructions: Please answer all questions in the blue books. You may not use notes, books, or calculators. Please show your work. There are a total of six questions, for 100 points. Questions vary in their level of difficulty. Partial credit will be given for partially correct answers. Good luck!

1) [10 points] In the blue book, list the words that should go in the blanks in the paragraph below. You should also include two or three sentences explaining the statement. Note that by “the average age of mothers” I mean the age at which women give birth.

*If the NRR in a country is greater than one, then the growth rate is population will be higher, the __________ is the average age of mothers. However, if the NRR is less than one, then the population growth rate is higher, the __________ is the average age of mothers.*

2) [15 points] In a certain country, output is produced using only labor and land. The production function is

\[ Y = AX^\alpha L^{1-\alpha} \]

where \( Y \) is output, \( X \) is the quantity of land, and \( L \) is the quantity of labor. Factors of production are paid their marginal products. In this country there are 100 acres of land and 200 workers. Land earns a rent of $1,000 per acre per year. Workers are paid a wage of $1 per hour, and work 2,000 hours per year.

What is the value of \( \alpha \)?
3) [20 points] A certain country is described by the Malthusian model. Output is produced using only land and labor, so that the higher is population, the lower is income per capita. The relationship between these quantities is given by the equation

\[ y = 5000 \times L^{-1/2} \]

In this country, mortality takes place only before age five or in old age. Some fraction of children die between birth and age five. If people survive to age five, then they live to be 100 years old, at which point they die. The “under five mortality rate” (U5M) is the fraction of children who do not live to their fifth birthday.

The under five mortality rate depends on the level of income per capita. Specifically:

\[
U5M = 1 - \frac{y}{1000} \quad \text{if} \quad y < 1000
\]

\[
U5M = 0 \quad \text{if} \quad y \geq 1000
\]

The Total Fertility Rate (TFR) in the country is 4. There are equal numbers of boys and girls born.

What is the size of the population in the Malthusian steady state? Show how you got your answer.
4) [20 points] A country is described by the Solow model with population growth. The production function in per worker terms is

\[ y = Ak^{1/2} \]

The parameters describing the economy are:

Productivity: \( A = 10 \)

Depreciation rate: \( \delta = .05 \)

Population growth rate: \( n = .02 \)

Investment rate \( \gamma = .20 \)

At a certain point in time, the value of the capital stock per worker is \( k = 100 \). At that time, an economist conducts a study of investment in the country. She divides investment into three categories:

i) Investment to make up for depreciation of capital

ii) Investment to give the new workers who are being added to the labor force the same level of capital per worker as existing workers.

iii) Investment to raise the level of capital per worker.

A. [15 points] What are the fractions of investment that fall into each of these three categories? Show how you got your answers.

B. [5 points] What are the fractions of investment that will fall into each of these three categories once the economy has reached steady state?
5) [15 points] In a country, there are three population age groups: young, old, and working age. All working age people work, while no one in the other age groups works. The numbers of people in each age group in the years 2015 and 2050 are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>2015</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young</td>
<td>180</td>
<td>100</td>
</tr>
<tr>
<td>Working Age</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>Old</td>
<td>20</td>
<td>50</td>
</tr>
</tbody>
</table>

GDP per worker grows at a rate of 1% per year over the period 2015-2050. What is the rate of growth of GDP per capita over this period? [Needless to say, you may not use a calculator on this problem, and the solution does not involve difficult calculation.]

6) [20 points] A country is described by the Solow model with no population growth. The production function (in per worker terms) is

\[ y = k^{1/2} \]

The rate of depreciation is 5% per year. The investment rate (\( \gamma \)) is determined based on the level of income per capita. Specifically,

\[ \gamma = 0.20 \quad \text{if} \quad y < 3 \]
\[ \gamma = 0.40 \quad \text{if} \quad y \geq 3 \]

A country initially has capital per worker of 4.

Your goal in this question is to draw a graph that has time along the horizontal axis and the growth rate of output per worker in the country on the vertical axis. To draw this picture, you will almost certainly want to also draw a version of the standard Solow model diagram that has \( k \) on the horizontal axis and output, investment, and depreciation on the vertical axis.

Using these two diagrams, you should describe qualitatively and quantitatively the path followed by the growth rate of output. A complete quantitative description of the growth path would require tools beyond what we use in this course, but there are some quantitative dimensions of the growth path that you should be able to describe pretty easily.