Economic Growth - Fall 2012, Midterm 2

October 28th

1 Problem 1

Wagner’s law states that the size of the government, typically measured as government spending as a percentage of GDP, will increase as the government gets larger due to greater complexity of regulation and demand for public goods.

2 Problem 2

Progressive taxation can increase equality by transferring resources to the relatively less wealthy, however it may come at a cost to efficiency if the tax distorts behavior. If labor supply is very inelastic, namely the slope of the labor supply curve is steep, then a decrease in the private return to labor via taxation will have only a small impact on labor supplied and the trade off between equity and efficiency is minimal. On the other hand, if labor supply is elastic, then taxation may discourage the wealthy from working and thus make increasing equity very costly to efficiency.

3 Problem 3

a) The Lorenz curve shows the fraction of wealth held by people below a given percentile of income. As the poorest 2/3rds hold no wealth, the curve

*If you find an error or typo please send an e-mail to david.glancy@brown.edu. Thank you.
takes the value of 0 for percentiles from 0 to 67. It is then increasing linearly and hits 100 at 100 because \( \forall p \in \left[ \frac{2}{3}, 1 \right] \) the amount of income held for poorest fraction \( p \) of the population will be \( 0 \ast 200 + 1 \ast 3 \ast (p - \frac{2}{3}) \ast 100 \) and the cumulative household income is 100, making the curve \( L(p) = 3p - 2 \).

b) The gini coefficient is the fraction of the area under the line \( y=x \) which falls between the Lorenz curve and the line \( y=x \). The area under \( y=x \) is \( \frac{1}{2} \ast 1 \ast 1 = \frac{1}{2} \). The area under the first part of the Lorenz curve is 0, the area under the second is \( \frac{1}{2} \ast \frac{1}{3} = \frac{1}{6} \). Subtracting the area under \( y=x \) (the line of perfect equality) from the area under the Lorenz curve gives the area between the curves, which is \( \frac{1}{3} \). As the total area under the line is \( \frac{1}{2} \), \( \frac{2}{3} \) rds of the area is between the lines, making the gini coefficient \( \frac{2}{3} \).

4 Problem 4

Dividing the production function from 1600 and 1740, noting the land is unchanged, plugging in that population quadrupled, and productivity into technology and efficiency gives us:

\[
\frac{Y_2}{Y_1} = \frac{A_2 X_2^{\frac{1}{2}} L_2^{\frac{1}{3}}}{A_1 X_1^{\frac{1}{2}} L_1^{\frac{1}{3}}} = \frac{T_2 E_2}{T_1 E_1}
\]

Since living standards are unchanged and \( L \) quadrupled, we know that \( Y \) quadrupled too. Thus we have that \( TE \) doubled over the course of 140 years. By the rule of 70, \( TE \) grew at a rate of .5% per year. As \( T \dot{E} = \dot{T} + \dot{E} \), and \( \dot{T} = .3\% \), we have that \( E \) grew at a rate of .2% per year.

5 Problem 5

a) Capital flows to wherever it earns the highest return, thus the marginal product of capital is equalized in the two countries:

\[
\frac{1}{2} A_1 k_1^{\frac{1}{2}} = \frac{1}{2} A_2 k_2^{\frac{1}{2}} \implies \frac{k_1}{k_2} = \left( \frac{A_1}{A_2} \right)^2 = 4
\]

\(^{1}1 \) is the income of each green person, \( 3 \ast (p - \frac{2}{3}) \) is the fraction of greens in the lower \( p \)th percentile, and 100 is the number of greens
b) The development accounting equation is
\[
\frac{y_1}{y_2} = \frac{A_1}{A_2} \left( \frac{k_1}{k_2} \right)^{1/2}
\]
The ratio of technologies is 2, and the square root of the capital ratio is also 2. Thus the accounting exercise concludes that half of the difference in incomes is due to capital per worker. In reality the only difference is due to technology, but since higher technology induces more capital, we find that capital differences are a contributor to income differences.

6 Problem 6

In the initial steady state, all countries grow at 1%, as this is the technology growth induced by country 1’s investment in technology growth. There will be a discrete change at the year 2050 when 2 invests more in R+D. Their growth rate will jump up because they still have the low cost of imitation from backwardness, but now are devoting more resources to imitating the technology of 1. 1 and 3’s paths are unchanged as 1 is still the leader in technology and investments don’t change.

Thus 1 and 3 grow at 1% while 2 grows faster. As 2 grows faster, they catch up to 1 and thus imitation becomes more costly, making growth slow. However, since 2 now devotes more than 1 to research, they will eventually overtake 1. At this point growth in 2 will be flat as they are devoting a constant share of labor to innovation and being the leader the cost of innovation doesn’t change.

However, now that 1 is no longer the leader, growth increases in 1 and 3. Previously they had been growing a 1% based on the cost of innovation and cost of adoption given \( \frac{A_1}{M_1} \) respectively. However, once the other countries become followers, they are now able to imitate or more cheaply imitate bringing higher growth given technology investment. Growth will increase slowly at as backwardness is only marginally higher, but it will increase as 2 gets further and further ahead. Growth will then asymptote to the growth level determined by the investment in 2.