1 Problem 1

We have Cobb Douglas production with land and labor: \( Y = AX^{\frac{1}{3}}L^{\frac{2}{3}} \). Since we are given the growth rate of output per worker we divide by \( L \) to put this in per worker terms.

\[
y = AX^{\frac{1}{3}}L^{-\frac{1}{3}}
\]

Taking the natural log of each side and differentiating with respect to \( t \) gives the equation in terms of growth rates.

\[
\hat{y} = \hat{A} + \frac{1}{3}\hat{X} - \frac{1}{3}\hat{L}
\]

Since land doesn’t grow, income per worker grows at 2% per year and labor at 1% per year.

\[
.02 = \hat{A} - \frac{1}{3}\cdot.01 \implies \hat{A} = \frac{7}{3} \%
\]

2 Problem 2

1 devotes two-thirds of their labor force to R&D compared to one-third for country 2, but the two have the same output. This means that:

\[
(1 - \frac{2}{3})A_1 = (1 - \frac{1}{3})A_2 \implies A_1 = 2A_2
\]
The number of years that 2 will take to catch-up to the current technology in 1 is thus the number of years it takes productivity to double. Since steady state technology growth is 1%, by the rule of 70 it is 70 years behind.

3 Problem 3

The college premium is higher than the return to college, because the return to college is the increase in wages an individual would get from going to college, while the college premium also includes the selection effect that those who attend college tend to be higher ability, harder working, better connected, etc..

3.1 B.1

If admission starts to reflect variables unrelated to ability, then college attendance ceases to be as useful a signal of ability. Thus the college premium will get closer to simply being a return to college. Thus the randomness causes the gap to fall.

3.2 B.2

Note: This answer is sensitive to some assumptions as to how the robots change the functioning of the labor market. Professor Weil and I both came up with something along the lines of the following answer, but if you made reasonable assumptions and a logical argument as to their implications but got a different answer you could still get full credit.

If robots start replacing many jobs of college graduates, the return to college and the college premium both will fall as the skills learned in college will do less to increase your earnings. However, it doesn’t change the fact that more able people are going to sort into colleges. If you assume that the return to ability is unaffected by the robots, then the gap will remain the same.

4 Problem 4

The premise of the argument that we should let the patents of bankrupt companies go into the public domain is that the patent is of no value to the
inventor after a bankruptcy, so there is no reason to continue to allow the inefficiencies from monopoly power to persist if it doesn’t incentive innovation.

The flaw with this argument is the ability to keep the patent in the event of a bankruptcy is valuable to the firm, thus the enacting of the law would reduce the value of innovation and reduce growth. If the firm is reorganized following a bankruptcy, being able to continue to hold the patents increases the likelihood that the firm will be able to eventually become successful.

Even in the event that the company gets liquidated in a bankruptcy, and owners get wiped out, the ability to sell the patent instead of having it become public will increase the payout to creditors. This increases the expected payout to creditors, and may enable the firm to have better access to credit to fund the initial R&D activities. Thus even if the law did no harm to owners conditional of them being bankrupt, it still benefits them and other innovators prior to bankruptcy.

5 Problem 5

Since the economies are open to capital flows, capital will go to where it is most productive. In equilibrium the marginal product of capital must be equal to some $r_w$ in every country.

$$MPK_i = \frac{1}{2} A_i K_i^{\frac{-1}{2}} L_i^{\frac{1}{2}} = r_w, \ i = 1, 2$$

Dividing the equation for each country:

$$\frac{A_1 K_1^{\frac{-1}{2}} L_1^{\frac{1}{2}}}{A_2 K_2^{\frac{-1}{2}} L_2^{\frac{1}{2}}} = 1$$

Since $L_1 = L_2$ and $\frac{A_1}{A_2} = 2$:

$$\frac{K_1}{K_2} = (\frac{A_1}{A_2})^2 = 4$$

Since laborers are paid the marginal product of labor:

$$w_i = \frac{1}{2} A_i K_i^{\frac{1}{2}} L_i^{\frac{-1}{2}}$$
Making the ratio of wages

\[
\frac{w_1}{w_2} = \frac{1}{2} A_1 K_1^{\frac{1}{2}} L_1^{\frac{1}{2}}
\]

\[
= \frac{1}{2} A_2 K_2^{\frac{1}{2}} L_2^{\frac{1}{2}}
\]

Since again L is the same, A differs by a factor of 2, and K by a factor of 4:

\[
\frac{w_1}{w_2} = \frac{A_1}{A_2} \left( \frac{K_1}{K_2} \right)^{\frac{1}{2}} = 4
\]

6 Problem 6

Since 1 is the leader, their growth rate will be \( \frac{\gamma_{A,1} L}{\mu} \) regardless of how far behind 1 and 2 are. The cost of innovation in the other two countries is \( \max \{ \mu_i, \frac{\mu}{A} \} \) making their growth rate:

\[\hat{A}_f = \min \left\{ \frac{2 \gamma_{A,1} L}{\mu_i}, \frac{\gamma_{A,1} L A_1}{\mu_i} \right\}\]

Country 2 invests two-thirds as much in R&D as 1 so when they are far enough behind, they grow faster than 1, when they are close enough they grow slower. There is a steady state where \( \hat{A}_1 = \hat{A}_2 \implies \frac{\gamma_{A,1} L}{\mu_i} = \frac{\gamma_{A,1} L}{\mu_i} \).

This gives an equilibrium ratio:

\[\frac{A_1}{A_2} = \frac{\gamma_{A,1}}{\gamma_{A,2}} = \frac{3}{2}\]

At this ratio, each country grows at the same rate. For 3 on the other hand can never grow faster than \( \frac{2 \gamma_{A,2} L}{\mu_i} = \frac{2 \gamma_{A,1} L}{\mu_i} < \hat{A}_1 \). Thus 3 has a lower steady state growth rate at this level, only two-thirds of that in 1. Since 1 and 2 grow faster the ratio of technology in 1 to 3 will go to infinity.