When we build models to theorize about the world, we can’t chase realism.

The tradeoff we face as modelers is how much detail or realistic complications to include in our model versus how broadly applicable we want the story to be.

In this section, we’ll consider a model that has huge scope, and so we must sacrifice a lot of detail.

This model will try to tell a story about the whole world.
Lost in a model

We must be careful not to lose ourselves in our models

- An analogy of “uncanny valley” (Masahiro Mori 1970) to economics: a model that is not the world, but close enough to be creepy
- Do we want a model like that?
- The trap is Jean Baudrillard’s “desert of the real” (Simulacra and Simulation 1981) where the model replaces a reality from which it is untethered
- (cf. The Matrix)
- The world is not the model, and the model is not the world
- We seek to predict and to understand, not to explain

Scale and scope

- When we build models to theorize about the world, we can’t chase realism
- The tradeoff we face as modelers is how much detail or realistic complications to include in our model versus how broadly applicable we want the story to be
- In this section, we’ll consider a model that has huge scope, and so we must sacrifice a lot of detail
- This model will try to tell a story about the whole world

General equilibrium

- Partial equilibrium analysis looks at one good in isolation
- For example: we can derive demand curves that relate the price of a good to the quantity demanded, and supply curves that relate the price to quantity supplied
- But things are interrelated
  - Cash is only a medium: at the bottom must be resources and stuff
  - The value of something that I can offer determines my income and so in turn my demand for other goods
  - We want a model that can untangle this web
- General equilibrium analysis looks at the interaction of all markets at once: supply and demand of all things together, as two sides of the same coin

Endowments and allocations

- General equilibrium involves many-dimensional math, since it is concerned with many goods and many people.
- However, we can illustrate ideas in a two-good model, which lets us depict the economy graphically.
- First we will look at a pure exchange economy without production. We will build a model, and alongside these notes we will solve side-by-side a numerical example to illustrate concretely the moving parts.
- Let’s have two consumers, A and B, and two goods, 1 and 2.
- A starts with an endowment of \( \omega_A = (\omega_A^1, \omega_A^2) \), and B starts with \( \omega_B = (\omega_B^1, \omega_B^2) \).
Endowments and allocations

- We’re interested in seeing how these initial resources might ultimately be distributed.
- Call the amount $A$ ultimately consumes $x_A = (x_A^1, x_A^2)$, and similarly $x_B = (x_B^1, x_B^2)$.
- Each distribution is an allocation: $(\omega_A, \omega_B)$ is the initial allocation, and $(x_A, x_B)$ is the final allocation.
- Clearly the consumers can’t end up with more than the total stock of a particular good:
  $x_A^1 + x_B^1 \leq \omega_A^1 + \omega_B^1$  \hspace{1cm} (1) \\
  $x_A^2 + x_B^2 \leq \omega_A^2 + \omega_B^2$  \hspace{1cm} (2)

These are the market clearing conditions.

Edgeworth boxes

- The Edgeworth box is a way to depict this economy.
- Each point inside the box represents a feasible allocation for the economy, a division of the total amount of goods between our two consumers.
- The width of the box is $\omega_A^1 + \omega_B^1$, the total amount of good 1 that is available.
- The height of the box is $\omega_A^2 + \omega_B^2$, the total amount of good 2 that is available.

An Edgeworth box

- We read $A$’s share of the total amount of stuff upward from an origin at the south-west corner.
- At the north-east corner is the allocation in which $A$ has everything; thus we read $B$’s share downward from an origin at the north-east.
- The initial allocation (the endowment) is just some point in the box - the division of goods that our consumers start with.
To talk about what allocations we would expect to see in this model, we need a way of capturing how much each consumer likes each allocation.

As in standard consumer theory, we assume that each consumer has preferences: given any two allocations, the consumer can rank them.

As usual, indifference curves connect all allocations among which a consumer is indifferent.

What do indifference curves look like in the Edgeworth box?
Edgeworth boxes

- If consumer \( A \) has monotonic preferences (more is better), she prefers points to the north-east.
- If consumer \( B \) has monotonic preferences, he prefers points to the south-west.
- (Remember our definitions of ‘goods’ in economics are entirely flexible, so ‘selfish’ means something different than it does in natural language.)
- The indifference map for each consumer is just the same as it would be in a single-consumer choice problem, except for \( B \) it’s flipped \( 180^\circ \), since \( B \)’s origin is at the top-right.

Mutually beneficial trade

- Consider consumer \( A \). We can divide the box into two regions: allocations that \( A \) prefers to initial allocation, and allocations that \( A \) likes less than the initial allocation.
- The allocations \( A \) prefers to \( \omega_A \) are those that lie on a higher indifference curve.
- We can do the same for \( B \); if there is some region that both consumers prefer to the initial allocation, moving into that region would be a mutually beneficial trade.
Pareto efficiency

A point from which no mutually beneficial trade exists is *Pareto efficient*. At such an allocation, there is no other allocation that both consumers would prefer.

**Definition**

An allocation of resources (call it $x$) is Pareto *inefficient* if there exists some alternative allocation $x'$ such that

- at least one person prefers $x'$ to $x$, and
- no person prefers $x$ to $x'$.

**Definition**

Allocation $x'$ is a *Pareto improvement* over allocation $x$ if

- at least one person prefers $x'$ to $x$, and
- no person prefers $x$ to $x'$.

**Applying Pareto efficiency**

What value judgments do we imply if we use Pareto efficiency in a normative argument for the ‘quality’ of an allocation?

- “It doesn’t matter how an allocation was arrived at, only what it yields.”
- “Allocations should be judged in reference to individuals’ own preferences, not in reference to what we think they should do or like.”
- “All else equal, an increase in utility for a very rich person, with everyone else just as well off as before, is an improvement.”
Applying Pareto efficiency

“An allocation in which I have everything and no-one else has anything is Pareto efficient, since to make someone else better off requires taking something away from me.”

- This is not necessarily true: it makes a hidden assumption on preferences.
- Pareto efficiency is definitional for given preferences.
- For example: “I have all the goods in the world. I am also ascetic and thus get more satisfaction from having less goods. You always like more goods. Therefore the outcome in which I have all the goods in the world is Pareto inefficient.”
- We’ll return to Pareto efficiency when we study welfare metrics more generally later in the course.

Pareto efficiency in the Edgeworth box

- In our Edgeworth box, Pareto efficient points are characterized by tangency between an indifference curve for consumer A and an indifference curve for consumer B.
- Geometrically: the region that A prefers to a given Pareto efficient point x does not overlap with the region that B prefers to x.
- The two consumers’ marginal rates of substitution are equal are a Pareto efficient point.
- There will generally be many Pareto efficient allocations.
- The contract curve joins all the Pareto efficient allocations in the economy.
- For purely ‘selfish’ consumers it passes through the origins, but this need not be true.

Mutually beneficial trades

No further mutually beneficial trades

The contract curve

Contract curve linking Pareto efficient points
So far we have a model that has put structure on an exchange setting. We might expect that consumers who found themselves in a situation like the one we’ve built to engage in mutually beneficial trades when they exist.

But this is very effort-intensive for consumers, especially if there are lots of them: economies that run entirely on barter are not particularly realistic.

Let’s build an unrealistic institution that introduces prices, and doesn’t require our two consumers to get together and negotiate.

This is a decentralized institution.

Imagine that there is an auctioneer. Our two consumers come along with their endowments, and the auctioneer calls out prices for each good, \( p_1 \) and \( p_2 \).

Each consumer’s endowment has a given value at those prices, which we’ll call ‘income’, \( m \):

\[
\begin{align*}
  m_A &= p_1 \omega_A^1 + p_2 \omega_A^2 \\
  m_B &= p_1 \omega_B^1 + p_2 \omega_B^2
\end{align*}
\]

This defines a budget constraint for each consumer. Just as in a regular consumer choice problem, we’ll ask what amount of each good each consumer would like to consume, given the possibilities afforded by the budget constraint.

Since the endowment point lies on the budget constraint of both consumers, in the Edgeworth box the budget line for \( A \) coincides with the budget line for \( B \).

The line has a slope \(-\frac{p_1}{p_2}\).

The budget set for \( A \) is everything under the line, and the budget set for \( B \) everything over the line.
Utility maximization

- When the auctioneer announces prices, we imagine each consumer solving her own utility maximization problem: what is my most-preferred point in my budget set?
- Remember from consumer theory that optimal choices are characterized in general by tangency between the consumer’s indifference curve and her budget line: her marginal rate of substitution is equal to the price ratio.

General equilibrium

- At given prices, each consumer will have some gross demand for each good, the amount of that good she would like to choose.
- This will have an associated excess demand, the difference between the gross demand and the amount of that good that was in the consumer’s endowment:
  \[ x_A^1(p_1, p_2) - \omega_A^1 \]  
- If the auctioneer announces prices and the two consumers’ excess demands for a good don’t balance, the amount that A wants to buy doesn’t match the amount that B wants to sell.
- This is disequilibrium.
Excess demand

A’s excess demand

Disequilibrium

Disequilibrium in the Edgeworth box

Competitive equilibrium

Definition

A *competitive equilibrium* in an exchange economy is an allocation \( x \) and prices \( p \) such that

- \( x \) is utility-maximizing for each consumer given his budget constraint defined by \( p \), and
- the total demand for each good is no more than the total endowment.

Also sometimes known as a *Walrasian equilibrium* or even just a *general equilibrium*.

Competitive equilibrium

- In competitive equilibrium, excess demands balance: at those prices, the amount of each good that one consumer wants to buy or sell mirrors the amount that the other consumer wants to buy or sell.
- In the Edgeworth box, at these prices, the solution to each consumer’s private utility maximization problem coincides.
- Both consumers’ marginal rate of substitution is equal to the price ratio, and *markets clear*: the two consumers’ choices together use up the available supply of each good.
- To find this mathematically, we combine three things:
  - Consumer A’s demand function
  - Consumer B’s demand function
  - The market clearing condition
Competitive equilibrium

1. Competitive equilibrium is a postulate, a definition.
2. It is a definition that is not at all built on a mechanism of interaction between our consumers; one consumer’s relationship to the equilibrium allocation $x$ is entirely through prices, and independent of anything to do with the other consumer.
3. Viewing competitive equilibrium as a descriptive mechanism is therefore risky. In any realistic exchange economy with two consumers, there are surely strategic considerations.
4. In particular, our postulated ‘prices’ are not very much like natural-language prices at all. Prices in a real economy have to come from somewhere; all we have is a property that a particular system of prices would have, in conjunction with a particular allocation.
5. We’ll return extensively to the relationship between competitive equilibrium prices and real-world prices later in the course.

Walras’ Law

- The market clearing condition relates to Walras’ Law.
- Define aggregate excess demand for good 1 at prices $p$ as

\[ z_1(p_1, p_2) = (x_A^1(p_1, p_2) - \omega_A^1) + (x_B^1(p_1, p_2) - \omega_B^1) \]  

excess demand by A  

excess demand by B

- Walras’ Law says that the value of aggregate excess demand is zero, at any set of prices:

\[ p_1z_1 + p_2z_2 = 0 \]  

This comes from the fact that each consumer’s net ‘income’ from selling one good must balance her net ‘expenditure’ on the other.

- Walras’ Law guarantees that if the market clears for one good (so that aggregate excess demand for that good is zero), the market clears for the other by necessity.

Walras’ Law says that the value of aggregate excess demand is zero, at any set of prices:

\[ p_1z_1 + p_2z_2 = 0 \]  

Where partial equilibrium asked how a price could clear a single market, competitive equilibrium postulates a set of prices that clear all markets.

- Of course, the auctioneer is not a realistic institution, so the formation of prices is still somewhat mysterious.
- And similarly, as always, we are proposing a model designed to capture the interesting features of an economy in a tractable way: we clearly do not believe that we have formulae for each consumers’ demand for all goods.
- The existence of equilibrium is also an issue. One obvious way our model could fail to apply is if a competitive equilibrium doesn’t exist even in an abstract model.
Existence of equilibrium

- Existence depends on continuity of the excess demand function: if we change price just a little, aggregate demand should change just a little.
- This will be easier if each consumer is small relative to the size of the economy as a whole.
- The existence of a competitive equilibrium here demonstrates the conditions of possibility for an institution (prices and the auctioneer) to exist that requires only a private decision by each consumer, yet manages to balance their interests (market clearing).

Competitive equilibrium

- We have looked at a model of general equilibrium in an exchange economy, to see how a relative price can exist to clear many interrelated markets simultaneously.
- But ‘supply’ here was just an endowment of the same goods the consumers ended up consuming.
- Later in our course, after we’ve learned about producer theory, we’ll add the idea of production to our general equilibrium model: the idea that we can turn stuff into other stuff.

The First Theorem of Welfare Economics

The following Theorem applies the Pareto efficiency metric to the prediction of our general equilibrium model.

FTWE

If
- preferences are locally nonsatiated,
- a market exists for all commodities which enter into production and utility functions, and
- all markets are competitive with prices publicly known,

then every general equilibrium involves a Pareto efficient allocation.

This combines our model of general equilibrium and the Pareto efficiency metric as an argument in favour of organizing an economy with a decentralized market mechanism.

If the conditions of the theorem hold, then any allocation that can be supported as a competitive equilibrium is on the contract curve.

Sketch of FTWE proof

A sketch of a proof by contradiction: take an equilibrium \((x^*, y^*)\) and some proposed Pareto superior \((x, y)\).

- At any feasible allocation, total cost of all consumption bundles at prices \(p\) must equal social wealth at prices \(p\).
- Local nonsatiation implies that if \((x, y)\) dominates the equilibrium \((x^*, y^*)\), the cost of consumption bundles \(x\) (and thus social wealth) must exceed the total cost of the equilibrium allocation.
- But since profits were maximized at the equilibrium allocation, no technologically feasible production levels attain social wealth in excess of that cost.
An informal graphical 'proof' for the exchange economy:

A proposed non-efficient equilibrium

Say there exists a non-efficient general equilibrium allocation.
- By definition it does not lie on the contract curve.
- Both consumers must be maximizing utility given prices by definition of general equilibrium.
- But since we aren’t on the contract curve, the proposed allocation cannot be utility-maximizing for both consumers.
- Equivalently, at prices that make the allocation just affordable to each consumer, A’s optimal choice does not coincide with B’s optimal choice.
- Markets don’t clear; the original allocation could not have been a general equilibrium.
Local satiation

B’s preferences violate local nonsatiation: a non-efficient equilibrium

Interpreting FTWE

- FTWE seems innocuous, given the definition of general equilibrium.
- But the conditions of the theorem demonstrate that it is not innocuous.
- For example, it states not that “a market economy is Pareto efficient” but that “a complete set of competitive markets is Pareto efficient”.
- A non-exhaustive list of conditions that render FTWE not applicable:
  - Some agents not being price-takers
  - There being no market for some good (including intertemporally)
  - Markets being in disequilibrium
- And of course we can criticize the relevance of Pareto efficiency as a metric. For some preferences, very inequitable allocations can be Pareto efficient, since the definition says nothing about equity per se.

The Second Theorem of Welfare Economics

This result is a partial converse of the FTWE:

STWE

If

i. all consumers have convex preferences, and
ii. all firms have convex production possibility sets,

then any Pareto efficient allocation can be achieved as the equilibrium of a complete set of competitive markets, given some suitable reallocation of the endowment.

- This again combines the model of general equilibrium with the Pareto efficiency metric, now as an argument in favour of equity-driven intervention by redistribution of initial resources rather than price manipulation.
- If the conditions of the theorem hold, then any allocation on the contract curve can be supported as a competitive equilibrium, for some reallocation of the endowment.

Illustrating STWE

Equilibrium at x from a given endowment
Illustrating STWE

Redistribution of endowment to achieve equilibrium at a different Pareto efficient allocation

Non-convexity and STWE

Why are convex preferences required for STWE to hold?

Interpreting STWE

- STWE seems to offer an escape route from the criticism that Pareto efficiency is too weak a metric: it implies that we can pick our favourite Pareto efficient allocation and implement it by a decentralized process.
- But transferring endowments is almost certainly impossible. How can we transfer “time”, or “ability”, or “knowledge”? 
- Transferring income is not the same thing: it is the product of an agent’s decisions, not the endowment they start with.
- This makes real-world redistribution much more tricky.
- A different interpretation of STWE: “prices should be reserved as allocative signals, and shouldn’t be used to achieve distributional goals”?

FTWE and STWE

- If the conditions of both theorems hold, the contract curve coincides with the set of allocations supportable as competitive equilibria.
- Together these theorems outline the conditions under which a market economy is Pareto efficient and redistribution of endowments can select among the efficient allocations.
- Note that FTWE is, in a way, actually weaker than STWE. The first theorem says “if a general equilibrium exists, it is Pareto efficient under these conditions.” The second theorem says that “a general equilibrium does exist at every Pareto efficient allocation.”
- Neither theorem is realistic. The conditions for FTWE to apply and that costless reallocation of endowments is possible represent extremely strong assumptions.
- In the next section of the course we will ask to what extent markets fail: what if the FTWE conditions don’t hold? Can we ‘improve’ on the market outcome then?
Beyond efficiency

So far we’ve been using Pareto efficiency as a metric, but it doesn’t explicitly concern fairness or equity. Can we find a metric that adds something like this?

- A simple equal division of goods isn’t enough, since it doesn’t acknowledge different preferences.
- Instead, let’s say an allocation is equitable if no agent prefers any other agent’s bundle of goods to her own.
- And call an allocation fair if it is both equitable and Pareto efficient.

Fair allocations

If FTWE holds, a competitive equilibrium from an endowment of an equal division allocation must be a fair allocation.

- If we trade from an equal division, each agent is maximizing utility given the same wealth and facing the same prices. By FTWE the equilibrium that results from this trade is Pareto efficient.
- If at equilibrium allocation, A envies B’s bundle, then since A’s bundle is the best she can afford, it must be that B’s bundle costs more.
- But they have the same wealth, since they started from an equal division.
- So it can’t be that A envies B: the equilibrium allocation must be fair.

Measurement

- The three big welfare results—the two welfare theorems, and the fairness result—are theoretical welfare results.
- They show conditions under which the market mechanism is ‘good’, according to theoretical metrics that are based on preferences.
- When we want to actually evaluate an allocation’s quality, the metrics of Pareto efficiency and fairness are difficult to apply—we don’t know everyone’s preferences.
- So we need more general metrics that can at least in theory be tangible when we analyze real-world problems; later in our course we’ll return to this issue.