Today we start to build a model of a system of competitive markets. This extends our exchange economy model into a world with production to think about the connectedness of all markets, for input and output goods.

In the exchange economy we showed that we might be able to have a competitive equilibrium in which prices balance supply and demand across all markets at once.

Can prices achieve the same thing in an economy with production, in which the total amount of goods in the economy is not ordained by an exogenous endowment, but comes from making and combining inputs?

Robinson Crusoe

- The simplest possible model is the *Robinson Crusoe economy*.
- There are two goods, produced by one firm and consumed by one consumer.
- Robinson will be playing the role of both the firm and the consumer.
  - This helps us to emphasize that in some sense we are all producers and consumers simultaneously
  - The complication of course is that what we are personally capable of producing is not necessarily what we want to consume
  - And the production process is often complicated, so we sell our productive services in the market to be part of a larger production process
- In this model, we will avoid these complexities: the two goods are ‘leisure’ and ‘coconuts’. Coconuts are produced using a single input, ‘labor’, which is the inverse of leisure.

Production possibilities

- A production function links the amount of coconuts gathered to the amount of labor input.
- For example, say that it’s pretty easy to gather the low-hanging fruit (hence the phrase), so that the first hour of labor yields a lot of coconuts, but it gets harder and harder to get extra coconuts, so that each hour yields successively fewer.
- As we know from producer theory, this represents a technology with *decreasing returns to labor*, the *marginal product of labor* is decreasing.
- Remember that in this one-input case, the slope of the production function at a given point measured the input’s marginal product: the rate at which the technology allows leisure to be swapped for coconuts.
Now turn to Robinson the consumer. Let’s say he likes coconuts and also likes lounging on the beach, i.e. leisure, i.e. not working.

Let’s also make the typical assumption that his preferences are convex, so that he values an extra coconut more when he has few coconuts, and similarly for labor.

We can draw a few indifference curves in the same space as our production function.

This sets up a very standard utility maximization problem for Robinson.

He has a set of bundles that the resources at his disposal (his time) allow him to consume, and as per our canonical model of consumer choice, we predict that he will choose his most preferred bundle of these.

As usual, since preferences are monotonic (positive marginal utility for both leisure and coconuts at all points) it will be on the boundary of the production possibility set, or else there would be an available bundle he would like better.
Optimal choice

Interior bundles aren’t optimal: this is exactly like a consumer choice problem

The optimal choice is characterized (again in general) by tangency between the production possibility frontier and an indifference curve for Robinson.

This means that the marginal product of labor at the optimal choice is equal to the marginal rate of substitution.

Robinson goes to get coconuts while the rate at which he can gather them justifies giving up leisure; at the optimal choice, he prefers to stop working than work another hour to get a few extra coconuts.

Adding prices

- This brings us to the same point as in the exchange economy, when we exhausted mutually beneficial trade by having our two consumers interact.
- Here this is even less satisfying: we have a consumer and producer who reach an optimal choice by some process of balancing \( MRS \) and \( MP(l) \). This is OK for Robinson, but not in general.
- As in the exchange economy, we want to see if the hypothetical construct of prices—again called by the auctioneer—would accomplish the same in a decentralized fashion.
- Let’s say there’s a wage rate \( w \) in this economy. This is the price of leisure: to take an hour of leisure, Robinson gives up earning \( w \).
- Let’s make the price of coconuts equal to 1. This is our numeraire good.
- This will be an ownership economy: the profits from production side accrue to consumers; in this case the single consumer, Robinson.
Let’s think about Robinson the producer. As usual our canonical model of firm behavior is *profit maximization*. We already have a production possibility set that shows the bundles the producer can feasibly make with the resources available.

Given the prices $w$ for leisure and 1 for coconuts, at some level of production $(L, C)$ the firm gets profits of

$$\pi = C - wL.$$  

(1)

Familiar from producer theory, we can draw *isoprofit lines* that join all production points that yield the same profit. The equation of an isoprofit line associated with some profit level $\bar{\pi}$ is:

$$C = \bar{\pi} + wL.$$  

(2)

Geometrically, maximizing profit means choosing that point in the production possibility set that is on the highest isoprofit line.

The producer has chosen a production plan to maximize profit given the production technology and the prevailing prices.

The profit $\pi^*$ accrues to our consumer, Robinson.
The consumer

- Now Robinson the consumer. He is now in possession of $\pi^*$, which at the prevailing price 1 will buy an amount $\pi^*$ of coconuts.
- This is the endowment: $\pi^*$ coconuts and 0 labor. But Robinson the consumer doesn’t have to consume his endowment: we have postulated a wage rate $w$. He can choose to add some labor (give up leisure), getting $w$ per hour, which he can use to buy $w$ extra coconuts.
- This defines a budget line for our consumer.
- Then we have a standard utility maximization problem again.

Equilibrium

- Does there exist a wage rate that balances the amount of labor our consumer wants to supply with the amount of labor our firm wants to use?
- If this is not true at some wage rate, these quantities do not balance: the market for labor does not clear, and so by Walras’ Law neither does the market for the consumption good.
- Why? The solution to the consumer’s problem does not coincide with that for the firm. Say the wage is ‘too low’: the firm wants to use a lot of cheap labor to produce the relatively valuable coconuts, but the wage is not enough to induce the consumer to give up much leisure.
- This is in some sense reinforcing too: when the consumer doesn’t work much, his demand for coconuts is low.
The opposite would be true if the wage was ‘too high’ relative to the price of the consumption good. In equilibrium, the relative price clears the factor and output markets simultaneously.

A competitive equilibrium in an economy with production is an allocation $x$, a production plan $y$ and prices $p$ such that:
- each consumer maximizes utility subject to his budget constraint,
- each firm maximizes profits subject to production possibilities, and
- the market for each good (input and output) clears.

In competitive equilibrium in this economy with production, the system of prices does double duty: they at once are a signal of how consumers value each good, and their marginal contribution to the firms’ profitability.
- They are thus capturing technological scarcity—the marginal product of labor—and consumption scarcity—the marginal rate of substitution.
- We still have the Walrasian auctioneer problem from the exchange economy: we have the conditions of possibility for an equilibrating price system, but we remain fuzzy on where prices come from and how the adjustment process works.
Production technology

- We assumed a technology with decreasing returns to scale. As we consider successively higher isoprofit lines, we eventually reach one that is unattainable given the technological constraint.
- If we assume a technology constant returns to scale or increasing returns to scale, we can’t say the same thing.
- This is a problem. Even if there is a point at which Robinson’s marginal rate of substitution coincided with the technological marginal product of labor, we would not be able to support this with a decentralized price mechanism.

Increasing returns to scale

No matter what the price, the profit-maximizing firm always has incentive to increase output, demanding more and more labor, reaching a higher and higher isoprofit line.

This is an example of the nonconvexity problem we also identified in passing in the exchange economy. Under nonconvexity our local arguments (tangency) are rendered globally insufficient.

But what are ‘increasing returns to scale’? Is such a thing remotely plausible?

Generalizing the Robinson Crusoe model

- Just as we can generalize the exchange economy up from two consumers (with some mathematical effort), we can generalize the economy with production upward too.
- Consider an economy with two consumption goods, one firm and two consumers. As a preemptive caveat: we will stretch both the geometry of our illustrations and the modeling construct of the price system almost to breaking point to do so.
- To maintain geometric tractability we will assume that factors of production are fixed and costless - say there is a room full of constantly running machines, and we want to decide how machines to devote to good 1 and how many to good 2.
- Call the price of good 1 $p_1$ and the price of good 2 $p_2$ (again called by the auctioneer). We are interested in the existence of equilibrating prices.
The firm’s problem

- First we want to figure out the production possibility set.
- Say that some of the machines are better at producing good 1 and some are better at producing good 2. Then if we devote all machines to a single good we will have a smaller pile of goods than if we devoted some to each.
- The production possibility set will then be concave.
- This is not a huge departure from Robinson, where the goods were coconuts and leisure.
- The boundary again captures the marginal rate of transformation.

Joint production possibilities

Say there are two producers, A and B and two outputs, $y_1$ and $y_2$.

Producer A has an absolute advantage in the production of both outputs.
- To have absolute advantage means to be able to produce more of some output in the same time.
- Here A can produce 10 units of good $y_1$ in the same time as B can produce 4.
- And A can produce 5 units of good $y_2$ in the same time as B can produce 4.

But producer B has a comparative advantage in the production of good 2.
- To have comparative advantage means that to produce an extra unit of the good, the producer has to give up less production of the other good.
- Here A would have to forgo 2 units of $y_1$ to get an extra unit of $y_2$.
- But B would only have to forgo 1 unit of $y_1$ to get an extra unit of $y_2$.
- Conversely A has a comparative advantage in producing good 1.
Marginal rate of transformation

- Comparative advantage is therefore defined by the *marginal rates of transformation* for each producer.

\[
MRT = \frac{\Delta y_2}{\Delta y_1}
\]  
(3)

How much extra good 2 could I produce if I produced one fewer unit of good 1?
- In our example, \(MRT_A = -\frac{1}{2}\) and \(MRT_B = -1\).
- This is the ratio at which the producer can trade off production.

Joint production possibilities

The firm’s problem

- Given prices, the firm’s profit at some arbitrary production level \(y_1, y_2\) is

\[
\pi = p_1 y_1 + p_2 y_2.
\]  
(4)

Rearranging, we get isoprofit lines defined by

\[
y_2 = \frac{\pi}{p_2} - \frac{p_1}{p_2} y_1.
\]  
(5)

- The lines have a slope \(-\frac{p_1}{p_2}\).
- The solution to the firm’s profit maximization problem is, again, to pick the production plan that is profit-maximizing - on the highest isoprofit line.
Consuming the output

Now consider our two consumers. The total amount of goods available in the economy is the output: let’s imagine the Edgeworth box defined by the amounts $x_1^*$ and $x_2^*$ and some initial allocation this output.

Disequilibrium

- The price ratio for the consumers is of course the same as that faced by the firm.
- At that price ratio the consumers will have some demands. If the markets do not clear, we again have disequilibrium.
- That is: we cannot find a production plan $y$ and consumption plan $x$ such that are supported by this price system $p$ and endowment $\omega$ as a competitive equilibrium.
- Why? One way to view of disequilibrium is that one of the goods is being oversupplied: at the given price ratio, the good is “too expensive”, and the total amount that both consumers want of that good is less than the amount being produced.
Disequilibrium

If there’s a production plan \( y \), consumption plan \( x \) and prices \( p \) such that everyone’s at an optimal choice and markets clear, we again have a competitive equilibrium. And again marginal rates of substitution on the consumers’ side is equal to the marginal rate of transformation on the producer’s side. We are starting to push informally towards at a mechanism by which the price system creates feedback between the consumption and production side of the economy. This is in spite of the still-valid fact - discussed last week - that the idea of competitive equilibrium is definitional from a postulated price ratio rather than a description of a realistic mechanism by which prices are formed.

Equilibrium

We’ve looked at a graphical analysis of a simple one-input, one-output economy (albeit one in which the inverse of the input was itself a consumption good), and a two-output economy with an exogenous production technology. We can go on to imagine a more complicated economy with many output and inputs - which are not necessarily distinct categories - and in which consumers have an endowment and a share of firm profits. Graphical analysis is no longer available, but the spirit of these illustrative models survives.
Recalling the welfare theorems

FTWE
If
i. preferences are locally nonsatiated,
ii. a market exists for all commodities which enter into production and utility functions, and
iii. all markets are competitive with prices publicly known,
then every general equilibrium involves a Pareto efficient allocation.

STWE
If
i. all consumers have convex preferences, and
ii. all firms have convex production possibility sets,
then any Pareto efficient allocation can be achieved as the equilibrium of a complete set of competitive markets, given some suitable reallocation of the endowment.

Moving forward

- There are two big issues from the models of general equilibrium we have developed in the exchange economy and the economy with production. These are going to form the basis of the rest of the course.
  - First: where do prices come from?
    - Are they products of a process like the one we have skirted around in these models?
    - How can the characteristics of a good, or its consumers, or its producers, or its production technology, influence the price?
    - And in turn what does this do to the system of general equilibrium?
  - Second: are these market outcomes ‘good’?
    - How do we evaluate quality?
    - How can we collectively choose among different allocations if more than one is attainable?