1 Consumer choice

a) (10 points) \( MRS = \frac{MU_1}{MU_2} = \frac{1}{x_1} \). At tangency, \( MRS = \frac{p_1}{p_2} \), which implies \( 2x_1 = 4 \) or \( x_1^* = 2 \).

From the budget constraint (which holds with equality since the consumer likes both goods), \( p_1x_1^* + p_2x_2^* = 20 \), or \( 2*2 + 4*2 = 20 \), and so \( x_2^* = 4 \). Optimal choice is the bundle \((2,4)\).

b) (12 points) We know that optimal choice for a consumer with perfect complement preferences is the highest available point at which \( x_1^* = x_2^* \). Call this \( x^* \). So we can solve using the budget line: \( p_1x^* + p_2x^* = m \) implies \( x^* = \frac{m}{p_1 + p_2} \). The optimal choice of bundle is \((\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2})\).

Both goods are normal since as \( m \) increases, \( x_1^* \) and \( x_2^* \) increase. And both goods are ordinary, since as \( p_i \) increases, \( x_i^* \) decreases.

2 Optimal choice with a nonstandard budget

a) (10 points)
The consumer’s optimal choice $x^* = (2, 3)$. The consumer’s marginal rate of substitution is given by $MRS = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$. So at the corner point, the slope of the indifference curve is $-\frac{3}{2}$. This is steeper than the upper part of the budget set (where slope is $-1$) but less steep than the lower part (where slope is $-3$). So we know that the indifference curve through $(2, 3)$ looks as in the picture, and we can see that anything better than that is not in the budget set.

3 Good sandwich, bad sandwich

a) (10 points) Edgeworth box.

b) (5 points) Endowment.

c) (5 points) Indifference curves.
d) (6 points) Pareto improvements.

e) (6 points) Pareto efficient allocations (the whole left side axis of the Edgeworth box).

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\text{\textbf{Pareto improvements}}
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\text{\textbf{Pareto efficient allocations}}
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f) (8 points) The competitive equilibrium is at the top left corner. That is \( x_J = (0, 1) \) and \( x_A = (2, 0) \). The price ratio that supports this is \( \frac{p_1}{p_2} = 1 \). You can check that the bundles are utility maximizing for each consumer at these prices, and the market certainly clears since the allocated amounts add up to the endowment.

Intuitively, Jim will always sell all of his \( s \) and use the proceeds for \( i \); at the price ratio 1, he gets one \( i \) back. Avram is just happy to swap \( i \) for \( s \) at the price ratio 1.

4 Welfare theorems

The Second Theorem of Welfare Economics says that, under some conditions that guarantee the existence of competitive equilibria, every Pareto efficient allocation can be achieved as a competitive equilibrium, if we make a particular change to the starting conditions of the economy.

a) (5 points) We need to be able to move the endowment. The Second Theorem says you can find a price ratio to support any Pareto efficient point as a competitive equilibrium. That is, both consumers when faced with those prices will choose the bundles in that Pareto efficient allocation. But naturally we need those bundles to be on the budget line for each consumer, and so we need to be able to move the endowment.

This is hard in reality since endowments are what we ‘arrive’ to the economy with. Things like time, health, natural ability, and opportunity are all very hard to reallocate. Reallocation money is not necessarily the same thing, since money is typically the product of previous choices. The endowment really has to be something innate to the individual, but this makes it hard to imagine being able to reallocate it.

b) (10 points)
Here the allocations $x$ and $x'$ are both Pareto efficient (they’re on the contract curve; there is no area that both consumers prefer starting from either point). The Second Theorem gives us that (since preferences are convex) we can actually achieve either such point as a competitive equilibrium. We need price ratios such that the slope of the budget line at the $x$ or $x'$ matches the slopes of the (tangential) indifference curves.

We see that to move from one to the other requires moving the endowment to a point on the required budget line, something like $\omega$ or $\omega'$. Why? The endowment must always be on a consumer’s budget line, since they are always free to consume exactly what they were endowed with if they want.
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