1 Drawing indifference curves (20 points)

2 Marginal utility and marginal rate of substitution (20 points)

a) \( MU_1 = \frac{\delta u}{\delta x_1} = 3x_1^2x_2 \)

b) \( MU_1 = \frac{\delta u}{\delta x_2} = x_1^3 \)

c) \( MRS = \frac{MU_1}{MU_2} = \frac{3x_1^2x_2}{x_1^3} = \frac{3x_2}{x_1} \)

d) \( MRS = \frac{3}{4} \). At this point, Jim is willing to give up \( \frac{3}{4} \) of an hour per day of teaching microeconomics for an extra hour per day of watching baseball. Equivalently, Jim is willing to give up hours per day of teaching microeconomics to get hours per day of pinball at the rate \( \frac{3}{4} \) to 1.

e) \( MRS \) is exactly the same at all points for the old and new utility functions. We know this because the new utility function is an order-preserving transformation of the old one: take the natural log of the first (which preserves order) and then multiply by 2 (which preserves order). Therefore the two utility functions represent exactly the same preferences.

3 More MRS (20 points)

Find the marginal rate of substitution of good 2 for good 1 for each of the following utility functions:

a) \( MRS = \frac{MU_1}{MU_2} = \frac{4x_1}{x_2} = \frac{2x_2}{x_1} \)

b) \( MRS = \frac{MU_1}{MU_2} = \frac{\sqrt{x_1}}{2} = \frac{1}{\sqrt{x_1}} \)

4 Drawing budget sets (20 points)

a) \( 4x_1 + 2x_2 \leq 60 \)
b) \( x_1 + 5x_2 \leq 30 \)

5 The shape of preferences (20 points)

a) Let’s say the goods are “hours per day teaching micro” and “hours per day teaching math”. Jim likes to teach so his preferences are monotonic, but he prefers to teach one or the other each day so as to be more focused, rather than teaching both.

\[ \text{Good 2 (x_2)} \]
\[ \text{0} \]
\[ \text{Good 1 (x_1)} \]

\[ \text{Good 2 (x_2)} \]
\[ \text{0} \]
\[ \text{Good 1 (x_1)} \]

\[ \text{Good 2 (x_2)} \]
\[ \text{0} \]
\[ \text{Good 1 (x_1)} \]

\[ \text{Good 2 (x_2)} \]
\[ \text{0} \]
\[ \text{Good 1 (x_1)} \]

\[ \text{Good 2 (x_2)} \]
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\[ \text{Good 2 (x_2)} \]
\[ \text{0} \]
\[ \text{Good 1 (x_1)} \]

\[ \text{Good 2 (x_2)} \]
\[ \text{0} \]
\[ \text{Good 1 (x_1)} \]

c) A consumer with upward-sloping preferences in a two-good setting must derive positive marginal utility from one of the goods and negative marginal utility from the other. Informally: she must like one of the goods and dislike the other. Indifference curves will then slope up, because a little more of each good (in the right proportion) will leave the consumer on the same level as before, since the utility gain from the thing she likes is balanced by the utility loss from the thing she dislikes.