Due Wednesday, Feb. 17th in class or to the drop-off box in the basement of Robinson Hall.

1 Optimal choice by tangency (20 points)

In each of these cases in which the consumer has well-behaved preferences, find the consumer’s optimal choice. Show your intermediate steps.

a) \( u = \ln x_1 + 4 \ln x_2, \ p_1 = 3, \ p_2 = 2, \ m = 30 \)

b) \( u = x_1 + \sqrt{x_2}, \ p_1 = 8, \ p_2 = 2, \ m = 12 \)

2 Optimal choice by picture (20 points)

In each of these cases the consumer does not have well-behaved preferences. Find the consumer’s optimal choice, sketching a picture (with the budget set and a couple of indifference curves) to illustrate.

a) \( u = \min\{x_1, x_2\}, \ p_1 = 3, \ p_2 = 2, \ m = 20 \)

b) \( u = \max\{x_1, x_2\}, \ p_1 = 2, \ p_2 = 1, \ m = 10 \)

3 Explaining optimal choice (20 points)

a) Explain in a few sentences what the rational choice model in economics is, as if you were talking to a friend at Brown who had never taken an economics course.

b) Take the following example. There are two goods. A consumer chooses to spend all of his money on one of the goods. Suggest three different utility functions, each representing a different preference ordering, that would rationalize this choice behavior.

4 Comparative statics I (20 points)

A consumer has utility \( u = x_1 x_2^2 \), has income \( m \), and faces prices \( p_1 \) and \( p_2 \). Use the tangency method to find the consumer’s demand functions. For each of the two goods, is the good normal or inferior? Ordinary or Giffen? Are the two goods substitutes or complements?

5 Comparative statics II (20 points)

Consider a two good model with a consumer whose utility function is \( u = \ln x_1 + 2x_2 \). Prices are \( p_1 = p_2 = 1 \) and the consumer has income \( m \). Derive the income expansion path for this consumer and sketch a diagram to illustrate it. Describe her choice behavior in words, and suggest an example of two goods that might fit this type of preferences and choice behavior.

To think about

This is for you to think about only, not to write an answer. Try to understand why the tangency method works to find optimal choice when we have well-behaved preferences and a standard budget set, but does not necessarily work without both of these things. If you have time, play around with counterexamples to get a sense of what’s going on.