1 Explain the welfare theorems (20 points)

Using a couple of non-technical sentences for each, explain the main message of the first and second welfare theorems.

2 Working with the second welfare theorem (20 points)

Consider a two-consumer, two-good economy. The total endowment of is 4 units of each good. Consumers $A$ and $B$ have utility functions:

\[ U_A = \ln(x_A^1) + 3\ln(x_A^2), \]
\[ U_B = \ln(x_B^1) + \ln(x_B^2). \]

a) Find an equation for the contract curve in this economy.

b) Using your answer to a), and invoking the second welfare theorem, show that the allocation $x_A = (2, 3), x_B = (2, 1)$ can be supported as a competitive equilibrium. What must the auctioneer do to implement this allocation as a competitive equilibrium?

c) Similarly, show that $x_A = (1, 2), x_B = (3, 2)$ can be supported as a competitive equilibrium. What must the auctioneer do to implement this allocation as a competitive equilibrium?

3 Production (20 points)

A firm produces a single output $y$ using a single factor of production $x$, according to the production function $y = 4\sqrt{x}$. The price of $y$ is $p$ per unit and the price of $x$ is $w$ per unit.

a) Find expressions for the average product and the marginal product of $x$.

b) Set up and solve the firm’s profit maximization problem to find its optimal choice of input, $x^*$, as a function of the prices $p$ and $w$.

c) With your answer to b) and the production function, write the firm’s optimal choice of output, $y^*$, as a function of the prices $p$ and $w$.

d) Say that $w = 1$. Using your answer to c), sketch the firm’s supply curve.

4 Jim no function well coffee without (20 points)

Jim Corp. produces lecture slides $y$ using two inputs, coffee ($x_1$, measured in cups) and labor ($x_2$, measured in hours). Its production function is $y = 10\min\{\sqrt{x_1}, \sqrt{x_2}\}$. That is, it needs one cup of coffee and one hour of labor to achieve any output; coffee is useless without time to work, and time to work is useless without coffee. Jim Corp. sells lecture slides on the thriving black market for economics for a price of $12 per unit. Coffee costs $2 per cup and labor costs $10 per hour.

Does Jim Corp.’s production function display increasing, decreasing, or constant returns to scale? Assuming that Jim Corp. maximizes profits, what is its optimal choice of production plan?
5 Costs and perfect competition (20 points)

A firm in a perfectly competitive industry produces an output $y$ that sells for $p = 35$ per unit. Its cost function is $c(y) = 25 + 5y + y^2$.

a) What are the firm’s fixed costs, and what are the firm’s variable costs?

b) Find expressions for marginal cost and average cost.

c) Write the firm’s profit function. Find the firm’s optimal output $y^\ast$. What profit does the firm make?

d) Find the price in this industry in the long run.

e) Find the firm’s optimal output if the price is at the long run value you found in c). What profit does the firm make?