1 Cournot (20 points)

Two firms produce an identical product and simultaneously choose what quantity to produce, as in the Cournot model. The demand curve for the product is given by \( p = 10 - y \), where \( y = y_1 + y_2 \) is the sum of the outputs chosen by each firm. The cost to produce output is \( c(y_i) = 2y_i \) for each firm \( i = 1, 2 \).

a) Write the profit function for firm 1 and the profit function for firm 2.

b) Find reaction functions that define each firm’s optimal output for any output choice by the other firm.

c) Find the Nash equilibrium, an output for each firm that is a mutual best response.

d) If this market had been served by a single monopolistic producer, what would its output choice have been? How does the outcome in the Cournot equilibrium compare to the monopolistic outcome?

2 Hotelling (20 points)

Consider the basic version of the Hotelling spatial duopoly from the lecture notes. Explain the setup of the model in a few non-technical sentences using an illustrative example of your choice. Explain why the unique mutual best response is for both firms to locate at the midpoint.

3 Bertrand (20 points)

Jim runs a lemonade stand. There are two lemonade stands on his street. Jim (firm 1) and his competitor (firm 2) must simultaneously set the price for which they will sell lemonade this summer. The lemonade stand with the lower price will attract and serve all 200 customers; if the two stands have identical prices they will each serve half the market. The cost of producing lemonade is a constant 25 cents per cup.

a) Carefully characterize what Jim’s best response would be for each possible price that his competitor could set.

b) What pair of prices set by Jim and his competitor would be a mutual best response (that is, each has set the best possible price given the price set by the other)?

Now assume that each lemonade stand can serve a maximum of 100 customers. Further assume that each customer is willing to buy precisely one cup of lemonade so long as the price does not exceed $1.

c) Explain why the pair of prices in b) is no longer a mutual best response.

d) Explain why for each firm to set a price of $1 is a mutual best response in this modified game.
4 Sustaining collusion (20 points)

In each of period \( t = 0, \ldots, \infty \) each of two firms must choose whether to set a ‘high’ or ‘low’ price. The payoff to firm 1 in period \( t \), denoted \( \pi_{1,t} \), depends on the choice made by both firms as follows:

\[
\begin{array}{c|cc}
\text{Firm 1} & \text{Firm 2} \\
\hline
\text{High} & 2 & -2 \\
\text{Low} & X & 0 \\
\end{array}
\]

The total payoff to firm 1 is \( \Pi_1 = \sum_{t=0}^{\infty} \delta^t \pi_{1,t} \), where \( 0 < \delta < 1 \).

Say that firm 2 is following a strategy such that it will play ‘high’ in every period unless in some period firm 1 plays ‘low’, and then firm 2 will play ‘low’ in every subsequent period.

a) Write an expression for firm 1’s total payoff if it plays ‘high’ in every period.

b) Write an expression for firm 1’s total payoff if it plays ‘low’ in every period.

c) Find a condition on the parameters \( X \) and \( \delta \) such that firm 1 prefers to play ‘high’ rather than ‘low’? Interpret this result in words.

5 Winter is coming (20 points)

Jim has HBO and is very generous. He therefore allows his friends to come over to watch ‘Game of Thrones’, making his living room a common property resource for the group. There are no costs associated with people coming over. However, there is an externality: the more people are present, the harder it is to concentrate on the show, and so the benefit each person enjoys is decreasing in the number of people present. If \( n \) people show up, everyone present enjoys a benefit that is an equal share of a total value \( 10n - n^2 \). Assume that each person decides to show up if the benefit they get exceeds the cost they must pay.

a) How many people will show up if each decides privately?

b) Find the socially efficient number of people in the room by equating marginal social cost to marginal social benefit.

c) Jim is finding it hard to focus on the show, and is considering adding an entry fee: anyone who comes to watch has to pay a tribute of \( t \). What \( t \) should Jim set in order that the socially efficient number of people show up?