Due May 4th, by 5pm to the box in the basement of Robinson Hall.

1 A production externality (25 points)

There are two firms. Firm 1 produces plastics \((x)\). It can use a clean production process or one that produces noxious fumes \((f)\). Costs to produce some level of \(x\) are lower when the firm uses more the process that produces fumes: production costs for a given level of output \(x\) and byproduct \(f\) are given by \(c_1(x, f) = c(x) - f(1 - f)\).

Firm 2 produces computer software \((s)\). Its facilities are located next to firm 1’s plant. The more fumes firm 1 produces, the more firm 2 must pay its employees to compensate them for the unpleasantness. The cost to firm 2 is given by \(c_2(s, f) = c(s) + f^2\).

Let the price of plastics be \(p_x\) and the price of software be \(p_s\).

a) Write firm 1’s profit function. Find the level of \(f\) that firm 1 will choose.

b) Write firm 2’s profit function. What is the marginal effect of \(f\) on firm 2?

c) Combining your answers to a) and b), find the socially efficient level of \(f\).

d) Say the two firms were to merge. Write a profit function for the merged firm. Show that the profit-maximizing level of \(f\) for the merged firm is equal to the socially efficient level of \(f\).

e) Explain why the merged firm is more profitable than the two firms were separately.

2 A public good (25 points)

Jim and Martin are working on a project together. They each have an endowment of 20 hours that they can divide between a private consumption good ‘lounging around’ \((x)\) and a public good ‘working on project’ \((g)\): each hour they spend working on the project benefits both. Both Jim and Martin like lounging around but also want to do well on the project; their utility functions are as follows:

\[
U_J = 3x_J + g \quad (1) \\
U_M = x_Mg \quad (2)
\]

The marginal cost of 1 hour spent on the project is 1 hour of lounging.

a) Find each person’s marginal rate of substitution between hours spent lounging and hours spent on the project. Remember to make the public good ‘good 1’ in your calculations (see the lecture slides for details).

b) If Martin was working on the project alone, how many hours would he choose to spend on the project?

c) Show that if Jim sees Martin makes a privately optimal decision as in b), his best choice is not to contribute any more hours.

d) Write an equation that characterizes socially efficient allocations for the two-consumer case.
e) Characterize a socially efficient allocation in which Jim contributes no hours.

f) Explain why $g$ is higher in e) than in b).

3 Adverse selection and signaling (25 points)

Explain in a few non-technical sentences, ideally using an example of your choice:

a) What is an ‘adverse selection’? How can this issue lead to a downward spiral in prices in the presence of hidden type?

b) What makes something a credible signal of quality in a situation with hidden type?

4 An incentive contract (25 points)

Jim has had what might be a million dollar idea, but he is a busy guy and so has decided to hire an employee to undertake the project on his behalf.

If the project succeeds it will generate revenue of $1 million; if it fails it will generate no revenue. Whether the project succeeds depends on both luck and on how much effort the employee makes: the probability of the project succeeding is \( \frac{3}{4} \) if the employee exerts high effort, but only \( \frac{1}{4} \) if the employee exerts low effort.

Jim would like to maximize \( \pi = E(R) - E(w) \) (expected revenue minus expected wage). The employee has a utility function \( u = E(w) - c(e) \), where \( c(e) \) is the cost of effort and is equal to 50,000 for high effort and 0 for low effort. The employee can get \( u = \bar{u} = 0 \) by working elsewhere, and so will not accept any contract that yields less than this utility.

a) Write the employee’s participation constraint for high effort.

b) Write the employee’s incentive compatibility constraint for high effort.

c) Say that Jim can observe the employee’s effort and offers a contract that pays zero if the employee exerts low effort and $100,000 if the employee exerts high effort. Show that high effort satisfies the incentive compatibility and participation constraints.

d) Now assume that effort is unobservable. If Jim offers a contract that pays a flat wage of $100,000, show that high effort is not incentive compatible.

e) If effort is unobservable and Jim offers the contract in d), what is Jim’s expected profit?

Continue to assume that effort is unobservable to Jim. Say now that Jim offers a contract that pays the employee $200,000 if the project is a success and $0 if the project is a failure.

f) Is high effort incentive compatible? Why or why not?

g) Will the employee accept the contract? Why or why not?

h) What is Jim’s expected profit?