Production

- Back to basics: economics is all about resources, the things at our disposal that we hope to use to satisfy our wants and needs
- But we don’t have to eat resources directly: they can be changed, combined, and put to work to create new things
- This is production, a big part of the story of microeconomics
- Producers and consumers are often presented as two distinct groups in economic stories, but as we saw in the general equilibrium model, in some stories we can break down that distinction
- So production theory will also ultimately help us to put a bow on our general equilibrium model in two ways: we can let actors in the story turn stuff into other stuff, or own a part of something that does
- In your textbook (Serrano & Feldman) this is going to be a brief tour of chapters 8, 9 and 10
- Producer choice looks a bit messy when you first study it, but as you’ll see going forward it’s not that hard to apply

Production possibilities

A firm’s production function says: what is the most output $y$ the firm could get from a given combination of inputs $x$?

$$y = f(x_1, ..., x_n)$$ (1)

In our examples, we will typically work with cases with either one or two inputs to keep things simple.

The marginal product of an input is how much output increases when we increase the amount of an input:

$$MP_1(x_1, x_2) = \frac{\delta f}{\delta x_1}$$ (2)

$$MP_2(x_1, x_2) = \frac{\delta f}{\delta x_2}$$ (3)

This is the slope of the production function.

Firms

But what things will be produced?
- The construct we will adopt in production theory is the firm
  - A firm is an entity that exists to produce something, maybe many things
  - It uses inputs and returns outputs
  - Our assumption will be that a firm’s goal is to maximize profits
  - And it faces constraints: the firm can’t get unlimited output from a given set of inputs, and it has to pay for its input and find willing buyers for its output
  - This is just another model of objective and constraints, like consumer choice
- Thinking of production in this way can seem a bit odd
  - Not all production is done for profit; some producers might have different goals
  - And ‘firms’ can seem a bit value loaded in general
  - For our purposes we can think of it as an abstraction—we’re still building models after all
  - Another objective-constraint model than profit maximization might fit better your idea of how production takes place in a model you build
Production possibilities

We can also think about average product:

\[ AP_1(x_1, x_2) = \frac{y}{x_1} = \frac{f(x_1, x_2)}{x_1} \quad (4) \]

\[ AP_2(x_1, x_2) = \frac{y}{x_2} = \frac{f(x_1, x_2)}{x_2} \quad (5) \]

Sometimes we can think about a production possibility set \( y \leq f(x_1, x_2) \):

- This captures the idea that the firm could in theory produce less than the maximum for a given set of inputs
- Of course, this doesn’t seem like a great idea; why not go all the way to the boundary, the production function?
- But it completes an analogy: production possibilities for the firm are just like a consumer’s budget
  - The production function acts just like the consumer’s budget line: a boundary on what can be chosen

Diminishing marginal product

- In the previous picture, the slope of the production function gets smaller as we increase the input \( x \)
- This is a typical assumption called diminishing marginal product: each extra unit of input brings less and less output
- In the two input case, this means each extra unit of input brings less and less output, holding the amount of the other input fixed
- Of course it’s tougher to visualize the production function in the two input case
- One thing that can help is to draw isoquants (“same quantity”) which show different combinations of two inputs which all yield the same output quantity

Two inputs

These isoquants are convex and monotonic: more input gives more output, and balanced inputs work better than skewed
Substitution

In the two input case, an analog of the marginal rate of substitution from consumer theory is the firm’s technical rate of substitution (TRS). This measures the slope of isoquants:

\[ TRS_{x_1,x_2} = -\frac{\Delta x_2}{\Delta x_1} \]

The derivation proceeds the same way as for MRS:

\[ MP_1 \Delta x_1 + MP_2 \Delta x_2 = 0 \]

\[ \frac{\Delta x_2}{\Delta x_1} = \frac{MP_1}{MP_2} \]

\[ TRS = \frac{\delta f}{\delta x_1} \frac{\delta f}{\delta x_2} \]

Convexity of isoquants implies diminishing TRS.

The long run

The long run is defined as the time when all factors of production can be varied; the short run is when at least one factor of production must be used in a fixed amount.

- For example, one input could be ‘land’, and it might take longer for the firm to acquire more land than to put a new machine on its existing land.
- This gives us a link between the one and two input models—with a little creativity the one input model can be seen as the short run of a two input model.
- Notation for the case in which input 2 is fixed is

\[ y = f(x_1, \bar{x}_2), \]

where the bar means ‘constant’.

Returns to scale

Returns to scale is a (long run) concept to capture what happens when we scale up every single input by the same factor \( k \).

- **Decreasing returns to scale**
  - Output increases by a factor less than \( k \)
  - \( f(kx_1, kx_2) < kf(x_1, x_2) \)

- **Constant returns to scale**
  - Output increases by a factor of exactly \( k \)
  - \( f(kx_1, kx_2) = kf(x_1, x_2) \)

- **Increasing returns to scale**
  - Output increases by a factor more than \( k \)
  - \( f(kx_1, kx_2) > kf(x_1, x_2) \)

This is related to the spacing of isoquants.

If we take the concept quite literally, it can feel like constant returns is the only logical case...

Fortunately, a true ‘scaling up’ of every factor of production is a tough thought experiment.

Profit

The firm’s assumed objective, profit, is defined as total revenue minus total cost:

\[ \pi = \text{Revenue} - \text{Costs} \]

\[ = (\text{output price } \times \text{quantity sold}) - \text{cost to produce quantity sold} \]

For now in our course we are going to assume that the firm operates in competitive markets.

- This means that the firm takes prices of all inputs and outputs as given.
- This is also the sense in which the general equilibrium model we studied recently yielded a competitive equilibrium.
- Later in the course we’ll analyze cases in which markets aren’t competitive.
- Remember that competitive markets were a condition for the first theorem of welfare economics to hold...
Profit in the one input case

In the one input case:

\[ \pi = py - wx. \]  

(12)

Here \( p \) is the price of the output good \( y \) and \( w \) is the price of the input good \( x \).

- Why \( w \)? By analogy to labor: inputs are paid a ‘wage’ per unit
- The firm will try to maximize profit subject to its constraint—the production function
- This means finding \( x \) to maximize

\[ \pi(x) = pf(x) - wx \]  

(13)

- Notice that what we’ve done here is something akin to the ‘brute force’ method, since we substituted the constraint into the objective function

We can visualize the solution by first rearranging this profit function to give isoprofit lines:

\[ \pi = py - wx \]  

(14)

\[ y = \frac{\pi}{p} + \frac{w}{p} x \]  

(15)

This will give us a family of parallel lines with slope \( \frac{w}{p} \): one for each potential profit level

- This is precisely the same spirit as a consumer’s indifference curve
- And so we now have all the ingredients to view the firm’s problem geometrically

The firm’s objective, pictured

Isoprofit lines

The firm’s optimal choice, \((y^*, x^*)\), is the most profitable production plan that is technologically feasible
Comparative statics

If $w$ falls or $p$ rises, the slope of the isoprofit line gets flatter; concavity of the PPS implies the firm wants to produce more.

Optimal choice in the one input case

The math:

$$\max_x \pi(x) = pf(x) - wx \quad (16)$$

First-order condition:

$$\frac{d\pi}{dx} = p \frac{df(x)}{dx} - w = 0 \Rightarrow pMP(x) = w \quad (17)$$

Second-order condition for a maximum:

$$\frac{d^2\pi}{dx^2} = p \frac{d^2f(x)}{dx^2} \leq 0 \quad (18)$$

And the firm must make be profitable at this $x$:

$$\pi(x) = pf(x) - wx \geq 0 \Rightarrow pf(x) = pAP(x) \geq w \quad (19)$$

Profit, two inputs, short run

In the short run with more than one input, the story is very similar (albeit a bit harder to draw pictures):

$$\max_{x_1} \pi(x_1, \bar{x}_2) = pf(x_1, \bar{x}_2) - w_1x_1 - w_2\bar{x}_2 \quad (20)$$

The first order condition for this yields:

$$pMP_1(x_1^*, \bar{x}_2) = w_1 \quad (21)$$

$$\Rightarrow MP_1(x_1^*, \bar{x}_2) = \frac{w_1}{p} \quad (22)$$

Tangency between the production function and the slope of the isoprofit line.

Profit, two inputs, long run

In the long run with more than one input, the firm will have to choose the level of all inputs:

$$\max_{x_1, x_2} \pi(x_1, x_2) = pf(x_1, x_2) - w_1x_1 - w_2x_2 \quad (23)$$

The first order conditions are then:

$$pMP_1(x_1^*, x_2^*) = w_1 \quad (24)$$

$$pMP_2(x_1^*, x_2^*) = w_2 \quad (25)$$

These pick out inverse factor demand curves: the relationship between optimal input demand and input price.
Cost minimization

An equivalent way to model a firm’s decision is cost minimization:

\[
\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \text{ subject to } f(x_1, x_2) \geq \bar{y}. \tag{26}
\]

That is, what is the lowest cost way to produce a given level of output?

- If we are producing some output at a way that’s not cost minimizing, we could switch to a lower cost combination of inputs that yields the same output and do better
- Graphically, we can consider isocost lines that connect input combinations that cost the same amount
- The slope of these is the relative price of the inputs: \(-\frac{w_1}{w_2}\)
- This generates a picture that is even more similar to what we had in consumer theory

Isocost lines

At the cost minimizing method, the isocost line is tangent to the isoquant; if we’re not at a tangency point, the firm is not producing \(y\) in the cheapest way

Cost minimization

How do we feel about cost minimization?

- Here, it involves choosing input combinations—input prices are given by the market
- This gets a bit normatively thorny when we think about a particular input: labor
- Does the firm have an obligation to employ a certain number of humans?
- And later when we think about non-competitive markets in which the firm sets prices instead of taking them as given, this invites us to reflect on squeezing wages and squeezing supply prices
- Microeconomists have, however, also considered situations in which it might be profitable for a firm to pay a higher wage than the ‘going rate’
- Again, for now at the intermediate level we have to be patient with these very stark models

Cost minimization

We have tangency between isoquant and isocost lines at the cost minimizing choice:

\[
TRS = \frac{MP_1(x_1^*, x_2^*)}{MP_2(x_1^*, x_2^*)} = \frac{w_1}{w_2} \tag{27}
\]

This is the ratio of our two conditions from profit maximization

Our last stop on the producer choice model is cost functions, which explore more deeply where costs come from.

- The cost function \(c(y)\) is just precisely the solution to the problem we just saw: what is the lowest cash cost to produce output level \(y\)?
Types of cost

Costs come in different flavors:

- **Fixed costs**
  - Constant costs that must be paid by the firm regardless of output level

- **Variable costs**
  - Costs that depend on output level

Costs that are constant but only paid if the firm produces positive output are sometimes called *quasi-fixed costs*, really a special type of variable cost:

\[
c(y) = \begin{cases} 
1 + k + y^2 & \text{if } y > 0 \\
1 + y^2 & \text{if } y = 0
\end{cases}
\]  

(28)

In this example, fixed costs are 1, variable costs are \(y^2\), and quasi-fixed costs are \(k\).

Average cost

Let’s think of a general case,

\[
c(y) = c_V(y) + F,
\]  

(29)

where \(c_V(y)\) is variable costs and \(F\) is fixed costs.

- **Average cost** is the minimum cost per unit to produce \(y\):

\[
AC(y) = \frac{c(y)}{y} = \frac{c_V(y)}{y} + \frac{F}{y} = AVC(y) + AFC(y)
\]  

(30)

- Average fixed cost must slope downward, since fixed cost is being spread over more units.
- Typically we assume that average variable cost (eventually) slopes upward.
- In sum we often draw average cost with a U-shape.
- This is a bit odd since it seems to run counter to our diminishing returns idea...

Marginal cost

Marginal cost measures the change in cost given a small change in output:

\[
MC(y) = \frac{dc(y)}{dy}
\]  

(31)

- Fixed costs don’t change as output changes so don’t appear here.
- Marginal cost cuts average cost at its minimum point.
  - Why? Take an analogy with batting average. If you’re hitting .300 on the year and go 0-for-4 today, your marginal performance was below your average, so your average must fall. If you go 4-for-4 today, your average must rise.
- The area under a marginal cost curve up to some \(y\) sums up all marginal costs to yield total variable cost.

Cost curves

A ‘typical’ set of marginal and average cost curves.
Profit maximization again

Thinking about cost curves gives us a new way to think about profit maximization in terms of output rather than input:

\[
\max_y \pi(y) = py - c(y).
\]  \hspace{1cm} (32)

The first-order condition is:

\[
\frac{d\pi}{dx} = p - \frac{dc(y)}{dy} = 0 \Rightarrow p = MC(y).
\]  \hspace{1cm} (33)

Second-order condition for a maximum:

\[
\frac{d^2\pi}{dy^2} = \frac{p - MC(y)}{dy} \leq 0 \Rightarrow \frac{dMC(y)}{dy} \geq 0
\]  \hspace{1cm} (34)

And the firm must make be profitable at this \( y \):

\[
\pi = py - c(y) \geq 0 \Rightarrow \frac{py}{y} - \frac{c(y)}{y} \geq 0 \Rightarrow p \geq AC(y)
\]  \hspace{1cm} (35)

At \( p < \min AC \) the firm would lose money at any positive output, so supplies nothing.
This is going to be our workhorse producer choice model for the rest of the course. What’s on tap?

- We’ll dive deeper into the implications of competitive markets in three ways:
  - What do the dynamics of a competitive industry look like?
  - How does a competitive industry look when we combine the producer’s choice with the consumer’s demand for the industry’s good?
  - How does a competitive economy look when we combine all production and consumption decisions in general equilibrium?

- Then we’ll consider non-competitive industries, as the first part of our exploration of market failure, which is when the conditions of the first welfare theorem do not hold