Public goods

Last time we looked at goods with externality effects; today a special case of an externality called *public goods*.

- A public good is one which must be provided in the same amount to all consumers if it is provided at all.
- This means that the good is *nonexclusive*: even if I use it, it’s still there and usable by all other consumers.
- If I was to buy a public good, I therefore create a huge externality effect.
- We’ll figure out what the efficient amount of a public good is, and why the market might fail to deliver it.
- We can also cast back some of the lessons we learn here to the issue of negotiating over externalities.
- In your textbook this is Chapter 18.

Taxonomy

- Pure public goods are difficult to find since there is usually a degree of exclusivity to a good.
  - Street lights?
  - National defense?
- It’s fair to see all goods as lying somewhere along a spectrum from purely private to purely public.
- But we often see policymakers and societies try to make something public even if its inherent characteristics are not.
  - TV broadcast signals might be made freely available to all who have an antenna.
  - A road can be made public and open to anyone.
- ‘Public good’ doesn’t have to be the same as ‘provided by the government’.
  - Education and healthcare are goods that some societies try to make as nonexclusive as possible.
  - But the government of course also deals in private goods.
Whether to provide

First consider a *discrete* public good: it is either provided or not. When ‘should’ we provide this good?

- Say we have two consumers, $A$ and $B$. Call the benefit to consumer $i$ of having the public good $b_i$.
- If it costs $c$ to produce the public good, it will be socially beneficial to provide the good if $b_A + b_B > c$.
- If the sum of benefits exceeds the cost of provision, then by this criterion the good ‘should’ be provided.
- This is very different than from a private good. Since a private good can be consumed by only one individual, this criterion recommends provision if any *one* individual benefits more than the cost of provision.

How much to provide

What about a *continuous* public good, one that can come in different quantities?

- Say again there are two consumers, $A$ and $B$, and two goods, private consumption $x$ and a public good $G$.
- Assume that each consumer has some preferences represented by a utility function $U_i(x_i, G)$. Note that private consumption is specific to the consumer but $G$ is not.

Illustrating MC of provision

Illustrating MRS

Marginal cost of each unit of public good

Marginal rate of substitution for consumer A
Efficient provision for one consumer

- If we had just consumer A, it would be efficient to provide extra units of G up until the point where $MRS_A = MC$.
- Why? Below this point, the amount that A is willing to give up for extra G is more than the cost of extra G.

Socially efficient provision

- But we have more than one consumer.
- If an extra unit of G is provided, both consumers must consume that extra amount.
- The sum of the two consumers’ marginal rates of substitution at that point tells us how much ‘society’ was willing to give up to get the extra G.
Socially efficient provision

- The efficient level of provision of a public good satisfies:

\[ \sum_i MRS_i = MC \]  

This is the Samuelson condition.

- Public goods are a special case of an externality.

- To find the efficient level, we have to account for the effect of the public good on everyone, since everyone will be affected.

- So we have to add up the marginal benefits they receive (their \( MRS \); their willingness to pay) and balance them against the marginal cost of providing the next unit of the good.

Math

A quick note on the math of the Samuelson condition:

\[ \sum_i MRS_i = MC \]  

- When you apply the Samuelson condition, calculate \( MRS_i \) this way:

\[ \frac{MU(\text{public good})}{MU(\text{private good})} \]  

- This is because we need to find how many units of the private good the consumer is willing to give up for one more unit of the public good.

- Then we can add that up across consumers.
Another way to get the math of the Samuelson condition is to think only about quasilinear preferences:

$$u_i(G, y_i) = v_i(G) + y_i$$  \hspace{1cm} (4)

This says a consumer’s utility from the public good and ‘other consumption’ (a.k.a. cash) is a function of $G$ and linear in cash. $MRS$ is then just $v_i'(G)$, the consumer’s marginal utility from the public good.

Then we get the Samuelson condition like so:

$$\sum_i MB_i = 1$$  \hspace{1cm} (5)

The sum of the marginal benefits of the public good equals the marginal cost of the public good, which is $1$ since both goods are denominated in dollars.

**Explaining the Samuelson condition**

- Suppose we’re at a point that does not satisfy the Samuelson condition.
- Say $MC = 4$ so that it costs 4 to produce an extra unit of $G$.
- Say $MRS_A = 2$, $MRS_B = 3$. Consumer $A$ is willing to give up 2 units of private consumption for an extra unit of $G$; consumer $B$ is willing to give up 3 units of private consumption for an extra unit of $G$.
- If we take 2 from $A$ and 3 from $B$, we have more than enough to produce the extra unit of $G$. We can divide the remainder between $A$ and $B$, and both will be better off.
- It was therefore Pareto-improving to provide the extra unit of $G$.

**Implications**

- Whether it is socially efficient to provide the public good and how much it is socially efficient to provide don’t depend on how it is financed.
- But endowments matter: the value of $MRS$, our measure of a consumer’s willingness to pay, at some point of course depends in general on how much private consumption the consumer has available to give up at that point.
- For a private good, efficiency means $MRS = MC$ for each consumer: each person can consume a different amount of the good, but they must have the same valuation of the marginal good or else they could profitably trade.
- But for the public good, efficiency means $\sum_i |MRS_i| = MC$: each person must consume the same amount (by definition), but different consumers can value the marginal good differently.

**The price mechanism**

Can the price mechanism deliver efficiency?

- In our (exchange economy) general equilibrium model, the auctioneer calls prices, then the consumers report demands, then the auctioneer checks for market clearing.
- But when a good is public, everyone must consume the same amount.
- Purely independent decisions (as via the price mechanism) cannot directly capture this inescapable interdependence of the two consumers’ decisions.
## The free rider problem

A central problem with public good provision is the free rider problem. This means that a consumer would, all else equal, prefer that others purchase the public good, so that he can enjoy the benefits without having to buy himself.

- Consider an example with two consumers and a discrete public good.
- Index the consumers \( i = 1, 2 \) and call consumer \( i \)'s benefit from consuming the public good \( b_i \).
- Each consumer will independently decide whether to buy the public good or not.
- And say the auctioneer calls a 'cost' \( c \) for the good. We can think of this as capturing how much consumption of other things a consumer must give up if they choose to buy the public good.

### Table: Net Benefit to Each Consumer

<table>
<thead>
<tr>
<th>Consumer 1</th>
<th>Consumer 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>Don’t</td>
</tr>
<tr>
<td>( b_1 - \frac{c}{2} ), ( b_2 - \frac{c}{2} )</td>
<td>( b_1 - c ), ( b_2 )</td>
</tr>
<tr>
<td>( b_1 ), ( b_2 - c )</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

Say \( b_1 < c \) and \( b_1 > \frac{c}{2} \). Then it is socially beneficial for the good to be provided.

- But each consumer deciding independently would prefer not to buy.
- No matter what consumer 2 does, consumer 1 prefers not to buy.

What if \( b_1 > c \) so that each consumer has a private incentive to buy the public good?

- Each still would prefer the other to buy the good.
- There is still a free-rider problem.
The free rider problem

The same problem exists for continuous public goods.

- To see this informally, consider a situation in which consumer 2 has contributed some amount $g_2$ of the good.
- Assume for now that consumer 1 knows how much 2 has contributed.
- This makes consumer 1’s ‘endowment’ the amount of private consumption (say $\omega_1$) she can afford plus $g_2$, since she gets to consume $g_2$ of the public good for sure.
- But consumer 1 can’t decrease the amount of the public good, so her budget set will have a peculiar shape.
Failure of private provision

- In this example consumer 1 prefers to consume the ‘endowment’ than to buy any more of the public good.
- In this way the amount of a public good bought by others reduces incentives for others to buy.
- Note also that the price signal is no longer necessarily reflecting allocative scarcity.
- Private decisions thus do not necessarily lead to an efficient level of the public good.

Social choice

Can we do better?

- If we knew something about how each consumer valued a public good, we could use this as a proxy for their utility.
- Then we’d have a standard social choice problem in which we could weigh the various users’ interests against each other and decide what to do.
- But how can we know those valuations?
- We need a mechanism that either get the valuations or deliver the ‘right’ amount of the public good directly.

Voting, discrete public good

Consider a voting mechanism for a discrete public good.

- Let’s say a public good costs $120 to provide.
- There are three consumers, who each have some willingness to pay for the public good:

<table>
<thead>
<tr>
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<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
</tbody>
</table>

- Provision would be socially beneficial, but no-one has unilateral incentive to provide.

Majority voting, equal cost share

First say that we’ll say that if a majority vote for provision, the cost will be shared.

- The majority vote against: both 2 and 3 will have to pay $40 if the good is provided, more than they value the good.
- Majority voting is inherently ordinal, but whether provision is socially beneficial depends on willingness to pay.
Reporting WTP, equal cost share

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What if we ask each agent to report their willingness to pay, and if the sum of WTP exceeds $120, provide the good and share the cost?
- Consumer 1 has incentive to report an arbitrarily large WTP.
- Consumers 2 and 3 have incentive to report arbitrarily small WTP.

Reporting WTP, variable cost share

<table>
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What if we ask each agent to report their willingness to pay, and if the sum of WTP exceeds $120, provide the good and have each agent pay their stated WTP?
- The free rider problem returns.
- If some consumer believes that the others’ WTP will be enough to ensure provision, he has incentive to report zero WTP.

Aggregating preferences

First let’s assume we want to make a social choice that reflects somehow ‘collective’ preferences.
- Three people and three possible allocations, a, b and c. Say each agent’s preference ordering is like so:

<table>
<thead>
<tr>
<th>Agent 1</th>
<th>Agent 2</th>
<th>Agent 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>c</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>a</td>
<td>b</td>
</tr>
</tbody>
</table>

- If you are in charge, which allocation do you implement?

Aggregating preferences

Say we use simple majority voting over pairs of alternatives as an aggregation mechanism.
- a beats b, b beats c and c beats a: the social preferences is intransitive!
- This is Condorcet’s paradox.
Manipulation and revelation

In general respondents will have an incentive to lie when we ask them how much they want the good.

- **Majority voting** doesn’t account for intensity of preference so can be inefficient.
- **Asking for WTP** gives incentives to exaggerate unless we make people pay their reported value (and then we can’t account for people of high need and low means).
- **Paying what you report** brings the free rider problem back.

There are clever and complicated *revelation mechanisms* by which you can ask questions that induce truth-telling in situations like these.

- The central idea is trying to make people account for the effect of their reported desire for the good on other people.
- This is exactly in the spirit of externality correction: we want to try to have the chooser internalize the effect they impose on others.

Public goods

- Public goods must be provided in the same quantity to everyone.
- Private decisions can’t capture this interdependence: the price system does not deliver an efficient amount.
- The problem of how much to provide is therefore social.
- A social choice mechanism like voting is either inefficient or manipulable.
- It is possible to construct mechanisms that recover true preferences for the public good and so deliver efficient provision.
- These mechanisms force each agent to face a ‘price’ that reflects the effect of their decision on others.