Review: Trusses

- Each joint consists of a single pin to which the respective members are connected individually.
- No member extends beyond a joint.
- Loads are applied only at joints.
- Each Member is a 2-force member and carries only axial load.

Internal forces in 2-force members

The internal force in a straight 2-force member consists of a force acting normal to the surface of internal surfaces that are perpendicular to the member. Force can be positive or negative. Positive force is tension, negative force is compression.
Frames

- Structures consisting of structural members connected in arbitrary ways.
Calculating forces in structures made of 2-force members

Find the internal force in each member of the idealized bike frame. Assume all joints are hinges; all members have equal length.

Method of Joints Net force at each joint is zero.

8 Unknowns: Force in each member (5): $F_{AB}$, $F_{BC}$, $F_{AD}$, $F_{CD}$, $F_{BD}$ reaction forces (3) $A_x$, $A_y$, $C_y$

8 Equations: $\Sigma F_x = \Sigma F_y = 0$ (2 at each of the 4 joints)
Equations

All angles 60°

\[ \sum F_{A_x} = F_{A_x} + F_{A_y} \cos 60° + A_x = 0, \sum F_{A_y} = F_{A_y} \sin 60° + A_y = 0 \]
\[ \sum F_{B_x} = F_{B_x} + F_{B_y} \cos 60° - F_{A_x} \cos 60° = 0, \sum F_{B_y} = -W - F_{A_y} \sin 60° - F_{B_y} \sin 60° = 0 \]
\[ \sum F_{C_x} = -F_{B_x} + F_{C_x} \cos 60° = 0, \sum F_{C_y} = -F_{C_y} \cos 60° + C_y = 0 \]
\[ \sum F_{D_x} = -F_{A_x} - F_{D_x} \cos 60° + F_{D_y} \cos 60° = 0, \sum F_{D_y} = F_{B_x}, \sin 60° + F_{D_y} \sin 60° = 0 \]

\( \begin{align*}
    &ax = 0, cy = \frac{w}{3}, ay = \frac{2w}{3}, fd = \frac{2\sqrt{3}w}{9}, fab = \frac{2\sqrt{3}w}{9}, fad = -\frac{4\sqrt{3}w}{9}, fbc = -\frac{\sqrt{3}w}{9}, fbd = -\frac{2\sqrt{3}w}{9}.
\end{align*} \)
3D problem: \[ \sum F_x = \sum F_y = \sum F_z = 0 \]

At each joint

Equations: \[ \Sigma F_x = \Sigma F_y = \Sigma F_z = 0 \] at each joint (12)

Unknowns: Total of 12:
- Member forces \( F_{AC}, F_{AD}, F_{AB} \) (3)
- Reactions : \( B_x, B_y, B_z, C_x, C_y, C_z, D_x, D_y, D_z \) (9)

Forces act parallel to the members

\[ F_{AB} \mathbf{n}_{AB} + F_{AC} \mathbf{n}_{AC} + F_{AD} \mathbf{n}_{AD} + W \mathbf{i} = 0 \]

<table>
<thead>
<tr>
<th>Member</th>
<th>Unit Vector pointing away from Joint A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member AB</td>
<td>( \mathbf{n}_{AB} = (2a\mathbf{i} - 5a\mathbf{k}) / \sqrt{29a^2} = (2\mathbf{i} - 5\mathbf{k}) / \sqrt{29} )</td>
</tr>
<tr>
<td>Member AC</td>
<td>( \mathbf{n}_{AC} = (-3\mathbf{i} - \mathbf{j} - 5\mathbf{k})\sqrt{35} )</td>
</tr>
<tr>
<td>Member AD</td>
<td>( \mathbf{n}_{AD} = (-3\mathbf{i} + \mathbf{j} - 5\mathbf{k})\sqrt{35} )</td>
</tr>
</tbody>
</table>
Joint A

\[ F_{AB} n_{AB} + F_{AC} n_{AC} + F_{AD} n_{AD} + W\mathbf{i} = 0 \]
\[ F_{AB} (2\mathbf{i} - 5\mathbf{k}) / \sqrt{29} + F_{AC} (-3\mathbf{i} - \mathbf{j} - 5\mathbf{k}) / \sqrt{35} + F_{AD} (-3\mathbf{i} + \mathbf{j} - 5\mathbf{k}) / \sqrt{35} + W\mathbf{i} = 0 \]

\[ \Sigma F_{Ax} = 2F_{AB} / \sqrt{29} - 3F_{AC} / \sqrt{35} - 3F_{AD} / \sqrt{35} + W = 0 \]
\[ \Sigma F_{Ay} = -F_{AC} / \sqrt{35} + F_{AD} / \sqrt{35} = 0 \]
\[ \Sigma F_{Az} = -5F_{AB} / \sqrt{29} - 5F_{AC} / \sqrt{35} - 5F_{AD} / \sqrt{35} = 0 \]

\[ F_{AD} = F_{AC} = (\sqrt{35}/10)W \quad F_{AB} = -(\sqrt{29}/5)W \]

Joints B, C, D

\[ F_{AB} n_{BA} + B_1\mathbf{i} + B_1\mathbf{j} + B_1\mathbf{k} = 0 \]
\[ -F_{AB} n_{BA} + B_1\mathbf{i} + B_1\mathbf{j} + B_1\mathbf{k} = 0 \]
\[ \Sigma F_{Bx} = -F_{AB} 2/ \sqrt{29} + B_x = 0 \]
\[ \Sigma F_{By} = B_y = 0 \]
\[ \Sigma F_{Bz} = 5F_{AB} / \sqrt{29} + B_z = 0 \]

\[ F_{AC} n_{CA} + C_1\mathbf{i} + C_1\mathbf{j} + C_1\mathbf{k} = 0 \]
\[ -F_{AC} n_{CA} + C_1\mathbf{i} + C_1\mathbf{j} + C_1\mathbf{k} = 0 \]
\[ \Sigma F_{Cx} = -F_{AC} 3/ \sqrt{35} + C_x = 0 \]
\[ \Sigma F_{Cy} = F_{AC} / \sqrt{35} + C_y = 0 \]
\[ \Sigma F_{Cz} = 5F_{AC} / \sqrt{35} + C_z = 0 \]

\[ F_{AD} n_{DA} + D_1\mathbf{i} + D_1\mathbf{j} + D_1\mathbf{k} = 0 \]
\[ -F_{AD} n_{DA} + D_1\mathbf{i} + D_1\mathbf{j} + D_1\mathbf{k} = 0 \]
\[ \Sigma F_{Dx} = F_{AD} 3/ \sqrt{35} + D_x = 0 \]
\[ \Sigma F_{Dy} = F_{AD} / \sqrt{35} + D_y = 0 \]
\[ \Sigma F_{Dz} = 5F_{AD} / \sqrt{35} + D_z = 0 \]